O P I N I O N

by Prof. Nikolay Ivanov Yankov, DSc

of a thesis for acquiring the scientific degree of **,,Doctor of Sciences"**

author **Assoc. Prof. Danila Dmitrivich Cherkashin, PhD**,

thesis topic ,,Extremal problems in Euclidean combinatorial geometry"

Scientific area: 4. Natural Sciences, Mathematics and Informatics,

Professional sub-area: 4.5. Mathematics,

Institute of Mathematics and Informatics (IMI),

Bulgarian Academy of Sciences (BAS)

1. General assessment of the received materials in accordance to the Law

By order No 65/25.03.2024 from the IMI-BAS Director I'm appointed as a member of the scientific jury, and at its first meeting I was voted to write this opinion. I have received all required materials for the thesis defence on electronic medium.

In accordance to art. 35 of the Law for the development of the academic staff in the Republic of Bulgaria (LDASRB) and RALDASRB (Rules on the application of LDASRB) based on the documents submitted, it was established:

- reference from National centre for information and documentation (NACID) showing a certificate No 00102 issued 05.01.2023 for the educational and scientific doctoral degree ,,PhD" (satisfying art 12, para. 1 of LDASRB). From the reference it is evident that the PhD thesis was in the professional sub-area: 4.5. Mathematics (Discrete Mathematics). The PhD thesis title is ,,Extremal problems in hypergraphs coloring";

- a reference list for meeting the minimal national requirement in accordance to art. 2b, para. 2 and 3 of LDASRB (satisfying art. 12, para. 3 of LDASRB);

- I confirm that the requirement of art. 12, para. 5 of LDASRB is satisfied and the DSc thesis is applicant's independent work and does not repeat the theme and significant volume of the contents of the PhD thesis;

- a thesis is submitted (in accordance to art. 12, para. 2 of LDASRB) and also: a thesis abstract (11 pages); a ,,curriculum vitae" of the applicant both in Bulgarian and English; a list of applicant's scientific publications; a reference for the original scientific contributions of the thesis; and a list of citations for the publications included in the thesis;

- I think that the requirement of art. 12, para. 4 of LDASRB is satisfied: the thesis contains theoretical generalizations and solutions to major scientific and applied scientific problems, which correspond to contemporary achievements and represent a significant and original contribution to science.

2. Candidate's background

The dissertant has a PhD from the St. Petersburg Department of the V. A. Steklov Mathematical Institute (PDMI RAS) from 2018. During 2015-2023, he worked at St. Petersburg University. Currently, he is working at the IMI-BAS. In 2023, I was a member of the scientific jury, and I have written a review following the procedure for awarding Dr. Cherkashin the academic position of Associate Professor at IMI-BAS.

3. Meeting minimal national requirements

I accept the candidate's list of points for meeting the minimal national requirements:

- Groups A and B: Applicant has a PhD degree and has submitted a DSc thesis;

- Group G: 242 points from 7 scientific articles with impact factor and/or SJR rank and one paper indexed in Zentralblatt MATH (the required minimum is 100 pts.);

- Group D: a total of 24 citations in WOS and Scopus to the papers included in the thesis, totaling 144 pts. (the required minimum is 100 pts.);

4. Thesis contents

This thesis consists of an introduction and six chapters totaling 143 pages. Here is a summary of its contents:

- Introduction, which defines the goals and tasks of the thesis, as well as the relevance and importance of the scientific problems (pp. 4-6);

- Steiner trees (pp. 6-32);

- Gilbert-Steiner problem (pp. 34-40);

- Minimal distance minimizers (pp. 41-94);

- Johnson-type graphs (pp. 95-115);

- Chromatic numbers of 2-dimensional spheres (pp. 116-126);

- Chromatic numbers of 3-dimensional slices (pp. 127-139).

5. Aim and problems solved by the thesis

The main goals of this thesis are:

- Steiner tree problems;

- Finding the maximal independent set of a graph;

- Finding the chromatic number of a graph.

The main problems solved in this thesis are extremal problems in the intersection of Euclidean geometry and combinatorics:

- In Chapter 2, which is devoted to Euclidean Steiner tree problem, a new results strengthening a previous result of Ivanov and Tuzhilin is proved. That is when $n \geq 4, d \geq 2$, the set of planar ambiguous configurations in $\mathbb{P}_2$ is of Hausdorff dimension $2n - 1$. Another result of the author is proving the path-connectivity of the subset of $n-$point configurations in $\mathbb{P}_d$ for which there is a unique Steiner tree.

- The main results for the Gilbert-Steiner problem are Theorem 3.3.1 and 3.1.2, where assuming that the cost function C is admissible and the flow has a branching point of degree ≥ 4, it's proved that there exists a flow with smaller value of Gilbert functional. As a corollary this means that the planar Gilbert-Steiner problem has no branching point of degree at least 4.

- The advance on the topic of minimising the maximum distance to a given compact set among the sets of given length is presented in Chapter 4. The authors contribution is finding the distance minimizers for a closed planar curve, assuming that the curvature is small enough.

- The exact values of the independent sets for the Johnson-type graph $J_{\pm}(d, k, t)$ when $n > n_0(k, t)$: for odd negative t; and for $t = 0$; are found. Some new results when $k = 3$ and $t = -2$ are also included here.

- Lastly, the chromatic numbers of 3-dimensional slices are studied. In Theorem 7.1.2 it's shown that this chromatic number for $\mathbb{R}^3 \times [0, \epsilon]^6$ is at least 10.

6. Scientific and applied-science contributions of the thesis

In my opinion, the chosen methodology and research methods are in accordance with the stated purpose and tasks of the thesis. The author manages to solve many of the goals set in this work. The examples posed and analyzed in the thesis are contribution of the author himself and of other scholars working in this field.

When using other scientists's work Assoc. Prof. Cherkashin correctly and exactly refers to those works. I think that sufficient and responsible citations of work of other scientist working in this area are used.

7. Thesis abstract

The abstract has a total of 11 pages and fully complies with the requirements of the Law. I believe that the abstract correctly reflects the main goals and contributions of the thesis.

8. Scientific publications used in the thesis

Included in the thesis are 10 authors scientific articles related to the topic. Two of these works are preprints posted in arXiv. Out of the remaining 8 articles, 5 have impact factor (Web of Science), two have SJR and one is referenced in Zentralblatt MATH. The five works with impact factor have been published in journals in the following quartiles: in Q1 is one, in Q2 – two, and in Q3 also two. All of these papers are in journals listed in the field of Mathematics in WoS and Scopus.

9. Impact of the thesis and dissemination

A search in Scopus shows a total of 244 citations and candidate h-index of 7. The stats from WoS show the same h-index of 7 and 115 citations. These impressive results are

convincing and definite proof that the results of the thesis have become available to the broad scientific community and have made significant scientific contributions in the field.

The results included in this dissertation were presented in the 8 workshops: 2nd and 3rd Russian-Hungarian combinatorial workshops (Budapest, 2018 and Moskow, 2019), an Analysis seminar in Pisa (2023), a seminar on harmonics analysis in University Paris-Saclay, and some other venues organized in Bulgaria.

10. Assessment of candidate's personal contributions

Of the ten scientific papers that form the basis of this dissertation (including the two in arXiv): one is sole candidate's work, six have another co-author and three are with a total of four authors. The most collaborations are with Yana Teplitskaya, a total of four papers. I consider the contribution of Dr. Cherkashin in the co-authored articles as equal as the other authors.

11. Critical notes and thesis questions

I have no questions on the thesis content and no critical remarks.

12. Conclusion

I believe that Assoc. Prof. Dr. Danila Cherkashin is a respected world-level specialist in the field of mathematics and, in particular, in Euclidean combinatorial geometry as evidenced by his scientific publications, as well as citations by respected scientists. All mentioned above, and the fact that candidate satisfies all requirements of LDASRB, RALDASRB and IMI-BAS's rules, forms the basis for me to propose to the scientific jury to vote positive and Assoc. Prof Dr. Danila Cherkashin, to be awarded the scientific degree of **,,Doctor of sciences"** in scientific area: 4. Natural Sciences, Mathematics and Informatics, Professional sub-area: 4.5. Mathematics.

Shumen, 22.04.2024   **Scientific jury member**:

/prof. Nikolay Yankov, DSc/