OPINION

by Assoc. Prof. Dr. Angel Marchev, University of National and World Economy and Faculty of Economics of Sofia University "St. Kliment Ohridski",

regarding: the dissertation work of Tsvetelin S. Zaevski on the topic: "AN APPROACH FOR PRICING AMERICAN-STYLE DERIVATIVES" (Approach for valuing American derivatives) for the acquisition of the scientific degree "Doctor of Science" in the field of higher education 4. Natural Sciences, Mathematics and Informatics, professional field 4.5. Mathematics

1. Data about the procedure and the candidate

This opinion is prepared in accordance with order 457/03.12.2024 of the Director of the Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences. The author is affiliated with the Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences, where he has also defended his doctoral degree. The procedure has been announced in accordance with the requirements of the Act on the Development of Academic Staff in the Republic of Bulgaria and the relevant regulations of the Bulgarian Academy of Sciences. The text is accompanied by an abstract, a list of publications and citations, as well as the necessary documents certifying the fulfillment of the minimum scientometric indicators for acquiring the scientific degree.

2. Brief description of the dissertation work

The dissertation is dedicated to the valuation of American derivatives, in particular – options, as well as more complex financial instruments (such as cancellable or convertible options, strangle strategies, etc.), which give the right to early exercise of the transaction. The author proposes a general methodology that combines mathematical modeling, probabilistic approaches (Laplace transforms, first arrival of Brownian motion) and numerical algorithms for fast and relatively accurate valuation of such derivatives.

Although the thesis is developed in the context of the classical Black–Scholes model (log -normal dynamics of the underlying asset), it provides ideas on how the presented methodology could be extended to more general stochastic processes. The work also includes a series of practical implementations of the

methods in the MATLAB environment, which confirm the applicability and comparative advantages of the described approach.

In terms of structure, after the introduction and a review of the current scientific literature, the author presents theoretical results related to the Laplace transforms of the first arrival times of Brownian motion (for linear and partially linear barriers). In the following chapters, the optimal limits for early exercise are analyzed in detail, as well as approaches to obtaining closed formulas for perpetual options. The study also covers newly introduced classes of derivatives that combine features of several standard products. In conclusion, there is a summary of the scientific contributions and exemplary numerical results illustrating the effectiveness of the method.

3. Financial and applied significance of the topic

From a financial point of view, American options are among the most popular derivatives, as they allow early exercise when certain levels of the underlying asset are reached. This brings a higher degree of flexibility to market participants but also makes the task of valuation (pricing) more complex. In practice, algorithms are sought that determine the optimal exercise limit and, accordingly, the correct option price, in “almost real time”. It is here that the author’s innovative contribution is particularly visible: a fast method for approximating the optimal limit is proposed, which is then embedded in numerical approaches (Monte Carlo or finite differences) for a more precise calculation.

In conditions of market volatility, such decisions become critically important. A similar logic applies to other types of derivatives – for example, capped options , game options or “ strangle ” combinations, where both a call and a put option are bought to hedge market fluctuations . All of these instruments occupy a growing share of global markets, and the methods of the classic Black– Scholes model are often modified to cover leap (Levy) or fractal (fractional Brownian) characteristics. The dissertation has a high value, as it clearly shows how, starting from the log-normal case, we can build on to more general applications.

4. Scientific contributions and their financial interpretation

The dissertation formulates several main scientific contributions. The contributions are original and authorial.

Contribution 1: Development of a general methodology for estimating American-style derivatives based on Laplace transforms of Brownian motion arrival times .

The author shows an approach for calculating the Laplace transform of the time instant of “first touch” to a linear or partially linear barrier and how this helps to find distributions of the optimal exercise moment. These results are particularly important for financial derivatives, where the profit or loss depends on whether and when the underlying asset has passed a certain critical level. Borderline cases such as perpetuity options and options with finite maturity are considered in detail, where the flexibility of the approach is clearly demonstrated. The great value here is that through Laplace transforms, complex double numerical iterations are often avoided, and a faster algorithm is achieved. The method is universal enough to be applied to different variable interest rates or even in the presence of dividends (with the appropriate change of parameters).

Contribution 2: Faster schemes for numerically determining the optimal limit for early exercise of American options, as well as the corresponding price, have been derived.

The author introduces a sparse time network on which the early exercise boundary function is approximated and shows that in real conditions this gives sufficient accuracy for practice. The significance of such optimization is in real time – an investor or market maker must immediately know whether it is more profitable to exercise the option or to hold it. In order to avoid excessively heavy computational procedures, the dissertation proposes a numerical approach that achieves accuracy to the fourth decimal place. In the context of high-frequency trading Such an efficient algorithm is particularly useful because the market reaction time is within milliseconds. This group of contributions demonstrates an important connection between the theory of optimal stopping time and rapid realization in a market context.

Contribution 3: Classification and analysis of new or less frequently studied derivatives: strangle, cancellable, game and other hybrid instruments.

The dissertation proposes a clear model for considering different types of payoffs, divided into call and put type or two-sided (reaching an upper and lower barrier). This allows for the simultaneous pricing of strategies that combine two or

more options (e.g. strangle). This allows for the creation of flexible portfolios – particularly relevant when an investor expects large price movements but is uncertain about the direction. Power options allow for specific protection in “out of the money” positions, while cancellable options also allow the option writer to terminate the transaction against a previously agreed compensation. The consideration and evaluation of these instruments theoretically opens the way to new financial products that more accurately match individual risk profiles and expected returns.

Contribution 4: Application of the methodology to approximation and analysis of multiple situations with different parameters, including finite and infinite maturity structure, illustrated with real numerical examples in MATLAB.

The author demonstrates that the proposed analytical models and algorithms do not remain only at theoretical level but can be directly implemented and used in a software environment. The dissertation provides detailed codes and test examples that reproduce the results presented in the relevant chapters. This is a significant contribution, because validation through simulations reveals the stability and speed of the method and also indicates where possible numerical difficulties arise. The combination of analytical apparatus and practical experiments impresses with its completeness. I recommend publishing the program codes in a public repository, so that anyone who wishes to verify, upgrade or modify the approach has a good basis for experiments and for possible extension to alternative processes outside the Black– Scholes model.

Contribution 5: Additional theoretical contribution – extension of the main results on the Laplace transform to two-sided reaching problems and some approaches for exotic options (e.g. convertible lookback instruments).

In addition to the detailed analysis of one-sided reaching, cases are also considered in which the path of the Brownian motion must remain in a given area or exit it, which is typical for barrier and lookback options. Such consideration allows one to determine probabilities and moment functions under non-standard conditions - for example, where exactly the underlying asset is located if it has not passed a conditional barrier. The author thoroughly examines the mathematical details of these calculations, using variational-style generator operators, which allows for the general formulation of new exotic derivatives. The important thing for

finance is that such two-sided reaching problems arise in many American-style combinations, where both parties (buyer and seller) have exercise rights. Thus, in purely practical terms, financial institutions could more accurately price complex products and offer personalized strategies.

5. Implementation of requirements and recommendations

The submitted publications show that the candidate has published in referred and indexed journals with an impact factor and impact rank, citations significantly exceed the minimum requirements for this degree. The dissertation is an independent, original scientific work that is clearly the result of in-depth work and brings new contributions.

Recommendations:

- It is possible that in the future the author will extend his applications to other models – for example, Levi processes or stochastic volatility, to show the universality of the approach;
- Comparisons of the proposed fast method for finding an optimal boundary for early exercise against already well-known methods would be interesting;
- A more detailed economic interpretation of the simulation examples could be introduced in the texts to highlight where specifically the new method excels in a real market situation , which contributions are undoubtedly available.

6. Conclusion

In conclusion, the dissertation work of Assoc. Prof. Dr. Tsvetelin Zaevski " An Approach for Pricing American- Style Derivatives ” is an in-depth study with a clear theoretical and applied focus in the field of financial markets and mathematical modeling. The scientific contributions presented are undeniable, and the resulting method has significant added value for the rapid and precise valuation of American derivatives and their generalizations.

In view of the above, I categorically maintain the opinion that the dissertation meets the criteria of the ZRASRB and the relevant regulations for awarding the scientific degree "Doctor of Sciences". I strongly recommend that the esteemed scientific jury award Assoc. Prof. Dr. Tsvetelin

Zaevski the scientific degree "Doctor of Sciences" in professional field 4.5. "Mathematics".

February 20, 2025, Sofia

(Assoc. Prof. Dr. Angel Marchev)