REER REVIEW

for the competition for the academic position "Associate Professor"
for the needs of the Institute of Mathematics and Informatics
at the Bulgarian Academy of Sciences
announced in Newspaper of State, No. 65 of 02.08.2024
Area of Higher Education: **4. Natural Sciences, Mathematics and Informatics**
Professional Field: **4.5. Mathematics**
Scientific Speciality: **Geometry and Topology**
(**Homogeneous Spaces and Geometric Invariant Theory**)

The peer review is written by **Prof. D.Sci. Vesselin Drensky, Full Member of the BAS**, retired professor and associate member at IMI – BAS, member of the Scientific Jury for the competition by Order No. 347/01.10.2024 of the Director of IMI – BAS.
The only applicant who has applied for the position is **Dr. Valdemar Vasilev Tsanov,** researcher at the International Center for Mathematical Sciences of the Institute of Mathematics and Informatics – BAS.

1. Biographical data. Dr. Valdemar Tsanov was born in Sofia. He completed the bachelor's program in mathematics at FMI – SU, while in the meantime he stayed at Université de Nantes under the Erasmus program and defended his bachelor's thesis "Knot groups" under the supervision of Sylvain Gervais. After that, he graduated as a master in the program "Mathematics and Mathematical Physics" at FMI – SU with a master thesis "Geometry of toric knots" under the supervision of Petko Nikolov from the Faculty of Physics of SU. The applicant defended his doctoral thesis "Embeddings of flag manifolds and cohomological components of modules" at Queen's University at Kingston, Canada under the supervision of Ivan Dimitrov. The diploma was legalized by the BAS. After that, he defended his habilitation thesis (Habilitationsschrift) "Homogeneous projective varieties and invariant theory" at the Ruhr-Universität Bochum, Germany. The habilitation thesis is a mandatory requirement for habilitation in Germany. (It is a popular opinion that such a work is close to a thesis for a Doctor of Sciences in Bulgaria.) I am convinced that the excellent education that Valdemar Tsanov received, the stay in established mathematical centers and the opportunities for contacts with leading specialists in several fields of mathematics and physics are a good prerequisite for continuing his successful scientific activity.
Dr. Tsanov began his scientific and pedagogical career already during the master's program with problem solving classes for mathematical students. He was appointed as a mathematician (= instructor) at IMI – BAS immediately after graduating as a master. During his doctoral studies in Kingston, he led classes for undergraduate mathematicians and engineers. Subsequently, he was a research assistant at the Ruhr-Universität Bochum, Universität Göttingen and Jacobs University Bremen, again combining his scientific activity with teaching mathematicians and engineers. After returning to Bulgaria, he became a researcher at the International Center for Mathematical Sciences at IMI – BAS.
Together with Peter Heinzner, they supervised the PhD student Johanna Mareen Neuhaus, who successfully defended her thesis "The saturation problem for SL_2-subgroups of classical groups" at Ruhr-Universität Bochum in 2021. (Valdemar Tsanov was the "Thesis advisor" and Peter Heinzner was the "Degree supervisor".) In addition, Tomasz Maciążek, at that time a

doctoral student at the Center for Theoretical Physics of the Polish Academy of Sciences, specialized under the supervision of Tsanov.

Dr. Tsanov is the author of 9 papers published since 2013 and one accepted for publication (8 papers in journals with impact factor or impact rank and 2 papers in scientific conference proceedings), 2 preprints in arXiv and the habilitation thesis published by Lap-Lambert Academic Publishing. (One of the most important activities of this publishing house is the publication of dissertation works.) Of the papers and preprints, 4 are without co-authors, 6 are with one co-author, and one preprint is with two co-authors. Co-authors are Sawicki and Maciążek from Poland, Petukhov from Russia, Ivan Penkov and Seppänen from Germany (2 joint papers). Co-authors of the preprints are Maciążek and Elitza Hristova of one of the preprints and Yana Staneva of the other. The collaboration with Bulgarian mathematicians (two of whom work in Germany – Ivan Penkov and Yana Staneva and with Elitza Hristova from IMI – BAS) makes a pleasant impression. The presence of a large percentage of joint papers speaks for the possibility to work successfully in team. This is a quality that I personally value very much. In addition, Valdemar Tsanov participated in the preparation of lecture notes (alone or with other colleagues) of two courses of lectures given by other lecturers at conferences in Germany. (His credits as an author of the notes are stated in the published conference proceedings.) Dr. Tsanov has also presented a list of 49 scientific talks or series of lectures at seminars and conferences in Bulgaria, Germany, Canada, USA, France, Poland and Georgia, as well as a list of conferences and seminars of which he is the organizer or co-organizer.

2. Description of the presented documentation. The documentation presented by the applicant Dr. Valdemar Tsanov is in accordance with the requirements of the law and the accompanying rules of the Institute of Mathematics and Informatics. It contains: the statement that he wants to apply for the position, CV, copies of the diplomas for Bachelor, M.Sci., Ph.D. and habilitation, lists of all publications and the publications presented for the competition, a document confirming the scientific positions he had at the Universities of Bochum, Bremen and Göttingen and at IMI – BAS, the announcement for the competition in Newspaper of State, proofs that he covers the minimal scientific requirements of the law for the position, description of the scientific and pedagogical activity of the applicant. The scientific activity will be discussed below. The applicant has presented for participation in the competition 9 scientific papers in foreign journals and proceedings of conferences published by foreign publishing houses, as well as his habilitation thesis. The total volume of the 9 papers is 286 pages. These are all the publications of the applicant with the exception of the two preprints and one of the published papers, which is without co-author and is cited in the Ph.D. thesis. The publications are in the period 2013 – 2024, are in the field of the competition and I accept them for reviewing. The papers are published as follows: 2 in Journal of Physics A: Mathematical and Theoretical and one in each journal L'Enseignement Mathématique. 2e Série, Manuscripta Mathematica, Journal of Geometry and Symmetry in Physics, IMRN. International Mathematics Research Notices, Abhandlungen aus dem Mathematischen Seminar der Universität Hamburg. The proceedings of the two conferences are published by the European Mathematical Society and Springer. Three of the presented papers were used in the habilitation thesis, but this does not contradict the restrictions imposed on the publications for participation in the competition.

3. General characteristic of the scientific work and achievements of the applicant. According to the summary of the scientific contributions, the scientific interests of Valdemar

Tsanov are in three areas:

I. Geometric theory of invariants and flag varieties. Most of the research of the applicant is in this area. The founder of the geometric invariant theory is David Mumford, who pioneered it in 1965 using ideas of the publication of Hilbert from 1893 on the classical invariant theory. Groups acting on algebraic varieties or schemes are considered. Very soon after its creation, the geometric invariant theory found meeting points with symplectic geometry and topology, with applications in differential geometry and theoretical and mathematical physics. Flag varieties are complex projective varieties that are homogeneous under the action of a complex linear algebraic group. In his research, the applicant is interested in classical and actively studied problems at the meeting point of algebraic geometry, algebra, analysis, and theoretical and mathematical physics. In Bulgaria, there are mathematicians and theoretical physicists who work in some of the areas under consideration, and I expect that Valdemar Tsanov will find a way to colaborate with them.

II. Universal tensor categories and categories of tensor modules of infinite-dimensional Mackey Lie algebras. Category theory is a theory of abstract mathematical structures and the relationships between them. That is why it is often called "Abstract Nonsense". But it allows, for example, to derive general results that are valid not only for one class of objects (for example, for groups or rings), but are valid for all algebraic structures. Mackey Lie algebras are a new object in the theory of Lie algebras. They were introduced in 2014 by Penkov and Serganova. These are Lie algebras of linear transformations of a linear space that are related to a non-degenerate bilinear form of the space and another linear space. The topic is new for Valdemar Tsanov. (The paper with Ivan Penkov was submitted on 15.11.2023 and published online on 14.9.2024.)

III. Three-dimensional geometry and knot theory. Knot theory is a branch of topology that originates from the study of knots that appear in everyday life such as shoelaces or ropes. But mathematical knots are distinguished by the fact that their two ends are tied. In mathematical language, knots are embeddings of a circle in the three-dimensional Euclidean space. Valdemar Tsanov's interest in the theory of knots began as early as his student years, and his master thesis was devoted to questions from this theory.

4. Main scientific and scientific-applied contributions. I shall briefly discuss the main results contained in the submitted publications of the applicant, as well as my evaluation of them. I shall follow the order in which Valdemar Tsanov has divided his publications into groups in the summary of his scientific contributions.

I. Geometric theory of invariants and flag varieties (publications Nos 2 – 9 in the list in the summary of the scientific contributions). In this group of papers correspondences between homogeneous projective manifolds and the structure of their symmetry groups are studied from the point of view of the theory of invariants associated with subgroups of the groups of symmetries. The investigations include classic and continuing to be topical issues and objects. Many of the problems in invariant theory are due to the non-constructivity of Hilbert's approach. (The "king of invariant theory" Paul Gordon famously characterized Hilbert's results as "This is not mathematics, this is theology". ("Das ist nicht Mathematik, das ist Theologie.")) The cycle of papers explores the connections between the structure theory of reductive groups and the variation of some parameters of the generating sets of the invariants under some natural restrictions on the variety on which the group acts. The leading idea is that the properties of the algebra of invariants are related to the properties of natural geometric objects. Paper No. 2 (joint with Sawicki) studies the relations between three topics studied in representation theory,

algebraic geometry, and quantum information theory: spherical actions of reductive groups on projective varieties, secant varieties of homogeneous projective varieties, and quantum entanglement. Special attention is paid to the impact of the considered properties on the behavior of bosons and fermions.

Paper No. 4 (joint with Maciążek) uses methods from symplectic geometry and representation theory of compact Lie groups. Irreducible tensor representations with group action are classified, which appear in a number of important cases, for example in spherical actions.

The non-constructive theorem of Hilbert that the algebra of invariants of a connected semisimple complex algebraic group is finitely generated does not give an upper bound on the degrees of the minimal system of homogeneous generators. When such an upper bound is known, finding the generators can be solved in a finite number of steps. The exact upper bound for the degree of the generators of the invariants of a given group is called the Noether number. Paper No. 6 and the habilitation thesis No. 9 investigate invariants of low degree when the group acts on an irreducible module. Some divisors of the degrees of the generators are also found. It turns out that in some cases the divisibility constraints give exact values. The obtained results allow to find a lower bound on the Noether number.

The results on the restrictions on the degrees of the generators of the algebra of invariants are particularly useful in the cases of semi-continuity of the so-called rank function. The problem of classification of semicontinuous rank-function varieties was posed in 2007. In paper No. 3 of 2015 (joint with Petukhov, submitted in 2014) a list of all irreducible representations of reductive groups with semicontinuous rank function.

Papers No. 5 and No. 7 (joint with Seppänen) are devoted to a geometric invariant theory of an action of a connected reductive subgroup on flag varieties of a semisimple connected complex linear algebraic group. The standard research methods of the considered problems are based on cohomological methods and are associated with a number of technical difficulties. The two papers propose solutions that replace cohomological methods with combinatorial ones, leading to substantial advances in research. Furthermore, both papers open up possibilities for further research.

Paper No. 8 is devoted to the properties of convex hulls of (co)adjoint orbits of compact connected groups. The research is oriented towards applications in invariant theory and decompositions of tensor products of irreducible representations. The resulting numerical invariants give lower bounds on the degrees of the invariant polynomials, in the spirit of article No. 6.

II. Universal tensor categories and categories of tensor modules of infinite-dimensional Mackey Lie algebras (publication No. 10 in the list in the summary of the scientific contributions). The paper continues the research of the co-author Ivan Penkov of the applicant and his co-authors. Two categories depending on an integer parameter are studied: the tensor category generated by two dual spaces and the universal tensor category generated by two objects with finite filtrations. Objects whose description was not known even for a zero value of the parameter were found in their explicit form. It has turned out that the obtained results have direct relations with the combinatorics developed by Littlewood and Richardson.

III. Three-dimensional geometry and knot theory (publication No. 1 in the list in the summary of the scientific contributions). This is the first paper of Valdemar Tsanov. The results in it originate from his master thesis. A geometric structure is constructed in the complement of a toric knot in the three-dimensional sphere. Connections between classical objects are explored: triangular Fuchsian groups, $\mathbb{C}^{\times}$-equivariant singularities of plane curves, and

complements of toric knots in the 3-sphere. The proofs use a variety of techniques from different areas of mathematics. As a consequence, explicit constructions of objects are given, the existence of which follows from results of other authors. The presence of at least 14 citations and the large volume of the article – 41 pages in a journal of high scientific level is an additional proof of the quality of the obtained results.

In conclusion of my comments on the scientific contributions of the applicant, I want to note that he knows very well the main problems in the field and the literature on the objects under consideration and uses a rich arsenal of methods. I am convinced in the correctness of the arguments in the proofs. I have not noticed any significant inaccuracies.

The summary of the results correctly presents the main contributions in the publications submitted for participation in the competition.

The applicant has presented a list of 30 citations of 7 of his papers published in the period 2013 – 2019, with 29 of the citations being of 6 papers submitted for participation in the competition, and 16 of these citations being included to meet the minimum requirements. This list can be expanded. For example, searching https://scholar.google.com for paper No. 1 in the list of publications of the applicant gives 6 new citations of the paper or of the arXiv preprint: 2 citations in papers, 1 in a preprint in arXiv, and 3 in theses defended in France, Turkey and the USA:

1. Charles F. Doran, Terry Gannon, Hossein Movasati, Khosro Monsef Shokri, Automorphic forms for triangle groups, Communications in Number Theory and Physics 7(2013), No. 4, DOI: 10.4310/CNTP.2013.v7.n4.a4.
2. Tali Pinsky, Templates for geodesic flows, Ergodic Theory Dyn. Syst. 34 (2014), No. 1, 211-235.
3. Murray Elder, Arkadius Kalka, Logspace computations for Garside groups of spindle type, arXiv:1310.0933 [math.GR].
4. Pierre Dehornoy. Invariants topologiques des orbites périodiques d'un champ de vecteurs. Ecole Normale Supérieure de Lyon, Ph.D. Thesis 2011.
5. Bilal Demir, Genişletilmiş genel Hecke grupları, Balıkesir Üniversitesi, Ph.D. Thesis, 2015.
6. K. Boyle, On Symmetries of Knots and Their Surgeries, University of Oregon, Ph.D. Thesis, 2019.

From my observations of the procedures for academic positions at IMI, I believe that the number of citations does not differ significantly from that of most of the procedures.

It can be concluded from the applied documentation that:
a) The scientific publications satisfy the minimal requirements of the law and the accompanying rules of IMI – BAS for the academic position "Associate Professor" in the scientific field of the competition. With minimal requirements for the groups of indicators C, D, E and F respectively of 100, 220, 70 and 20 points, the applicant has submitted evidence of 100, 248, 96 and 70 points;
b) The scientific publications submitted for the completion have not been used in previous applications;
c) No plagiarism has been established in the presented for the competition works.

5. Significance of contributions to science and practice. The results obtained in the research papers of the applicant are interesting and meaningful. They contain new facts about objects

which appear naturally in a number of areas of mathematics and theoretical and mathematical physics and many of which have been previously studied by other authors. The results and the methods for obtaining them have been used and can be successfully used in other studies of this kind in the future.

6. Critical remarks and recommendations. I do not have essential critical remarks to the publications of the applicant. I want especially to mention that the documentation for the competition is prepared extremely accurately.

7. Personal impressions for the applicant. My personal impressions of Dr. Tsanov as a colleague and a scientist are very good and are based primarily on his talks at the seminars I have attended and the positive feedback I have heard about him from my colleagues.

CONCLUSION

In the presented scientific publications Dr. Valdemar Tsanov has obtained interesting results in current areas of mathematics and theoretical and mathemaical physics. Most of the results have already been used or can be used in a similar kind of research by other authors. A substantial part of the results have been published in respectable journals and reported at high level scientific forums. I have every reason to confidently propose Dr. Valdemar Vasilev Tsanov for the academic position of "Assiciate Professor" in the Area of Higher Education: 4. Natural Sciences, Mathematics and Informatics, Professional Field: 4.5. Mathematics, Scientific Speciality: Geometry and Topology (Homogeneous Spaces and Geometric Invariant Theory)

Sofia, November 17, 2024   Referee:

                                            (Prof. D.Sci. Vesselin Drensky, Full member of the BAS)