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SEEING THE INNOVATIONS AS AN OPPORTUNITY, NOT A THREAT: LESSONS FROM THE *InnoMathEd* EUROPEAN PROJECT

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The main outputs of the *InnoMathEd* European project on innovations in mathematics education are presented. Some dynamic virtual environments and their applications are described together with a number of didactical resources and methods for their implementation in mathematics classes. The emphasis is on possible strategies for organizing the inquiry-based learning in mathematics. Specific examples appropriate for the junior-high and secondary school are considered. The main dissemination activities and initiatives in the frames of the project together with the evaluation of experts are presented.

1. Introduction. The *innovation* is sometimes defined as *the ability to see the change as an opportunity, not a threat; to consider the complexity as a chance, not an obstacle.* It is that type of change that is envisaged in the frames of the *InnoMathEd* European project [1, 2]. A change not necessarily in the curriculum, the time schedule, and the standards but rather in the attitude to the learning process – from one of acquiring knowledge transmitted by the teacher to one of building up knowledge based on explorations and inquiries. Such an attitude is usually associated with studying natural science but *InnoMathEd* promotes *substantial innovation*s also in studying mathematics. Such innovations do not happen automatically once we supply the schools with "good software" or "goo learning environments"– it is necessary to keep the key role of teachers in mind [3]. And this role should be determined by the awareness that *the art of teaching is the art of (assisting) discovery.*

Thus the main goals of the project embraced the development and practical evaluation of innovative learning environments and didactic concepts together with a specific methodology for in-service teacher education for demonstrating the inquiry-based learning at a meta-level. Now, at the end of this two-year project we would like to share our experience in Bulgarian setting together with the lessons learned.

2. Dynamic mathematics – the focus of *InnoMathEd*. *Dynamic mathematics* is a complex notion often referring to software for dynamic geometry but recently also to cross-platforms integrating geometry, algebra and calculus. It could embrace even the concept of simple *dynamic systems* where a fixed rule describes the time dependence of a point in a geometrical space. Below we describe in brief the specifics of the dynamic mathematic software concentrating on platforms used by teachers in Bulgarian schools (viz. *GeoGebra*, GEONExT, *The Geometer's Sketchpad, Comenius Logo* and

Elica-DALEST applications: Origami Nets, Cubix Editor, Math Wheel, Bottle Design, Potter Wheel). Software for dynamic mathematics allows the user to observe continuous changes of a mathematical construction or surface through re-orientating its perspective. The users could explore what this surface look like when they move the object or what properties of the specific construction stay invariant under transformations in which all user-defined mathematical relationships are preserved. Some examples are seen in Fig. 1.

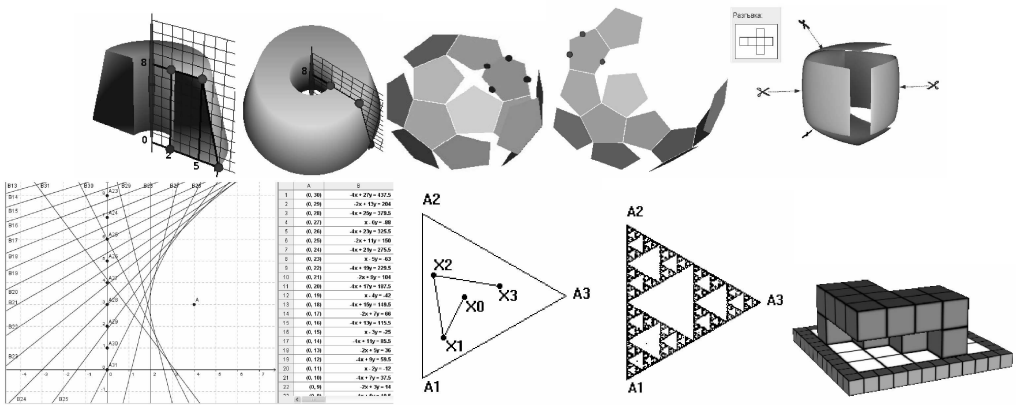

Fig. 1. Various computer environments for dynamic explorations

In addition to constructing, the dragging of well-constructed objects offers another dynamic perspective on geometric diagrams and is referred to as a "drag test" [4]. For instance, if A, B, C are constructed as points on a circle and C is dragged along it, then the software environment updates the configuration and shows that the respective inscribed angles ACB remain unchanged (Fig. 2).

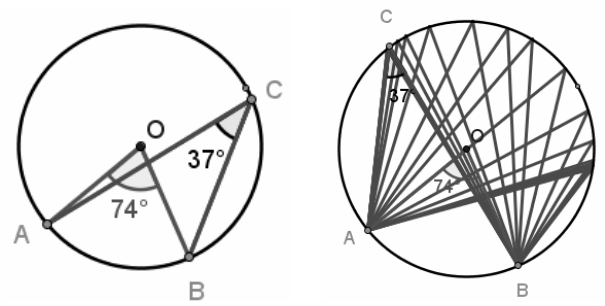

Fig. 2. The *drag test* for the angle ACB

The dynamic mathematics environments have been chosen as a platform for the inquiry-based style of learning since when working with them the students can make conjectures, verify them, generalize and study special cases based on various iterations of the mathematical construction (dynamically re-drawn on the screen with updated information). Because of their dynamic nature such environments have been found to be

valuable for studying mathematics since they force students to reason at higher levels [5]. The *learning environments* as defined in *InnoMathEd* include also didactic scenarios promoting the inquiry-based approach.

3. Didactic scenarios promoting the inquiry-based approach. The materials prepared for the *InnoMathEd* workshops are published on the site of the Mathematics and Informatics Education Department of the Institute of Mathematics and Informatics, Bulgarian Academy of Sciences [6].

3.1. Enhancing the *good old* classics. An *InnoMathEd* inspired series of scenarios on the geometric transformations *Reflection in line*, *Dilation* and *Inversion* was developed based on the methodology presented in Bulgarian mathematics textbooks for the secondary school dating from 20 years ago [7]. The aim has been to present the material in a "classical" way, but using the potential of the new information technologies in terms of visualization, interactivity and dynamic explorations (Fig. 3):

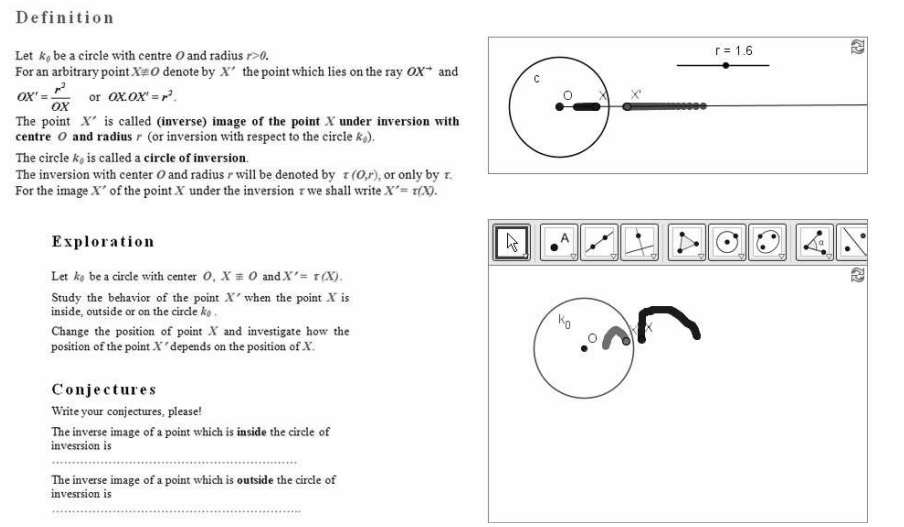

Fig. 3. A fragment from the *Inversion* scenario

The learning session consists of five basic phases:

- **Exploration** – the students make experiments moving objects in a dynamic construction. They answer questions and make hypothesis about the properties of the objects.
- **Sketch (Dynamic construction)** – the students make new dynamic constructions.
- **Investigation**– the students investigate the construction they have made.
- **Conjectures** – the students write down all the conjectures they have reached.
- **Theory** – the teacher gives in a classical way the definitions, the theorems, the corollaries, and facilitates the proofs which now have been motivated by the students' conjectures.

The scenarios end with problems, dynamic constructions, investigation or proofs meant for all students, as well as some more difficult problems formulated as challenges.

3.2. When working with younger students. The exploratory style should be promoted as early as possible. With this idea in mind we have developed new scenarios for working in 3D with the Elica-DALEST applications dealing with *rotational solids, nets of Archimedean and Platonic solids, layers in configurations of colored unit cubes* [9]. Other scenarios meant for the junior-high school make use of *GeoGebra – Cartesian coordinate system, Problems and games with matches*, *Elementary and basic geometric constructions* [7]. A typical example of the inquiry-based style in the context of rectangles is to find the one with the maximal area among those with fixed perimeter (Fig. 4).

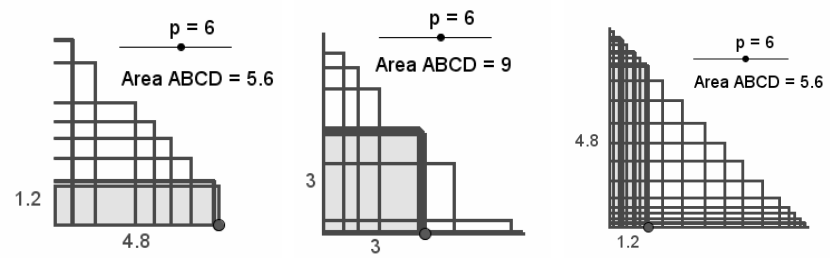

Fig. 4. Explorations with rectangles of fixed perimeter

3.3. Extra-curricular scenarios. An old problem by the Bulgarian mathematician Salabashev found by a teacher in the context of his 100^{th} commemoration gave rise to interesting investigations becoming a basis of an extra-curricular scenario. The problem reads: *Find the locus of the center of a circle tangent to two given circles.* Although the formulation is laconic, this problem provides very rich platform for explorations (Fig. 5).

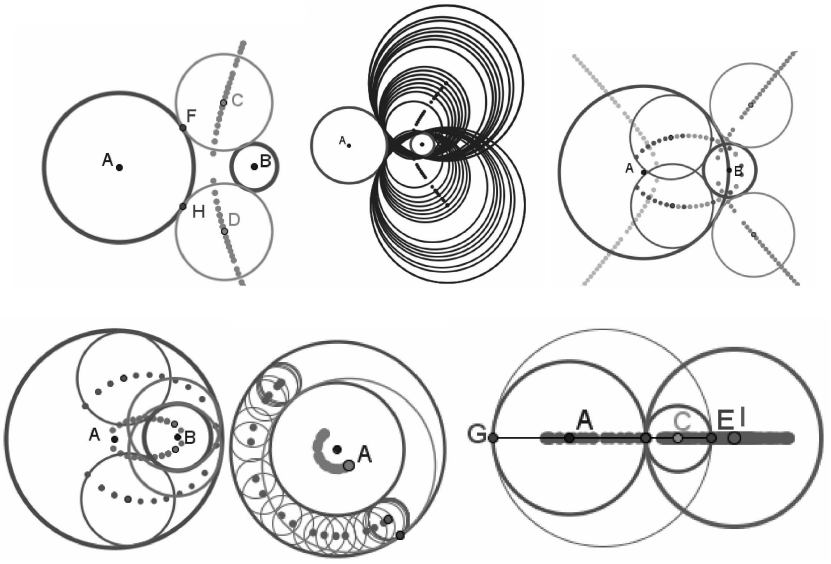

Fig. 5. Studying the locus sought in Salabashev's problem for various initial configurations of circles

4. *InnoMathEd* Workshops for in-service teachers. After intensive correspondence with dozens of potential participants we organized series of workshops in Sofia and in a number of towns in the country. The help of experts in mathematics and local enthusiasts in mathematics education was valuable. The participants in the first workshop (held at the Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences) were teachers from Montana, Vidin, Shumen, Veliko Turnovo, Varna, Plovdiv, Troyan, Rousse, Lovech, Vratza, Gorna Oryahovitza and Sofia. The materials for the workshop included a brochure of six exemplary didactic scenarios associated with *GeoGebra, Elica Applications* and *Elica Geomland*. Three of the scenarios were in Bulgarian, the rest – in English. The recommendations of the participants for having more resources in Bulgarian were taken into considerations and we created a Bulgarian site for the project where pilot versions of the learning environments were published in Bulgarian. After testing them with teachers, we translated them in English reflecting their suggestions.

The *InnoMathEd* ideas and the resources created in its frames were further disseminated during workshops in Blagoevgrad, Dimitrovgrad, Montana, Plovdiv, Pravets, Ruse, Shumen, Sofia, Stara Zagora, Uzana, Vratsa.

The workshops embraced more than 250 participants (in-service teachers in mathematics, informatics and IT from the junior-high and the secondary school). These forms turned out to be very effective for sharing experience with dynamic mathematics, ideas for inquiry-based scenarios and planning further joint activities within the project. In harmony with the *InnoMathEd* teacher training methodology, the lecturers did their best to *stimulate acts* (Fig. 6).

Fig. 6. Stimulated for acts

Special seminars were organized in Ohrid in the frames of the MASSEE congress, 16–20 September 2009, and in Albena, in the frames of the 39th spring conference of the Union of the Bulgarian Mathematicians, 6–10 April 2010. Professors, teachers, students – all had the chance of actively participating in these workshops by solving specific problems. The goal was to involve the audience independent on their preliminary knowledge on the dynamic software used. The projects developed by the high-school students showed that the young researchers do not need additional motivations for using dynamic

software – they are convinced in its applicability for solving complex problems and generating new hypotheses.

4.1. Impressions and recommendations of the teachers involved in the project.

At the end of each workshop the teachers were asked to answer a short inquiry:

- *What was the most useful thing for you?*
- *What were the main problems and difficulties you experienced?*
- *What do you suggest for a better dissemination of the project materials?*
- *What options do you see for applying in your work some of the learning environments considered at the workshop?*
- *Do you have any ideas for a scenario of your own or a modification of the scenarios we considered at the workshop?*
- *Any suggestions for further collaboration?*

It was with a great satisfaction that we would read the feedback of the teachers – not only in the inquiry forms, but also – in e-mails sent immediately after the workshops:

- *We are very grateful to the InnoMathEd project and its team for loading us with energy and enthusiasm at the beginning of the school year and for offering us new challenges for creative work.*
- *We greet the idea for breaking the classical frames of the lesson and involving the students in active learning which would make them feel like explorers and discoverers and will change crucially their attitude to the math classes. Our main concern is the limited number of classes according to the syllabus of the professional schools.*
- *Find attached my first attempt to show mathematics in action. I made it by the trial and error method and I liked it! It is not directly related to the traditional curriculum but I had fun!*
- *The two days of the workshop were full of novelties for me and this was the best part. Everything was at an accessible level and I was happy to create something on my own (however immodest this may sound) and this was something I wanted to have a long time ago!*

The most frequent recommendations dealt with:

- publishing all the materials also in Bulgarian;
- the necessity of additional stimuli for the innovative teachers;
- periodical meetings for disseminating the good practices;
- organizing small working groups which would develop specific topics;
- organizing a competition among teachers/students for developing didactic scenarios on a specific topic;
- formulating open problems to be investigated by student teams ;
- teacher training on a large-scale in various towns with various teachers;
- maintaining an active network among the teachers and the project team.

4.2. Some lessons we learned. All those recommendations were taken into account – the journal *Mathematics and Informatics* has published resources developed by the Project team and by teachers involved in *InnoMathEd* in six consecutive issues [10]. A competition for the best scenario developed by a teacher was organized. The follow up of the workshops are active virtual communications and face-to-face presentations of teachers' projects for didactic scenarios.

In a *nut shell*, the impressions of the Bulgarian members of the Project team are that the prevailing number of teachers getting familiar with the *InnoMathEd* are motivated to try out the ideas presented at the workshops and to be involved in implementing, evaluating and possibly modifying the proposed scenarios, as well as in developing some new ones.

The discussions (in formal and informal setting) showed that the question: *What software should be used in the math classes and to what extent?* doesn't have a unique answer. It depends on multiple factors including the level of the students and their mathematics and informatics culture. In any case it is the nature of the mathematical problems that matters, and the environment has just an auxiliary role.

The specific options of the computer environment should be used only after the students have acquired the standard methods for geometric constructions. Thus the learners would realize that the basic geometric constructions or graphs are not a matter of simply pressing a button or selecting a menu option. And even when they use a button for a standard construction it wouldn't be a *black box* for them...

As far as the technical skills are concerned, the teachers acquire them relatively fast. What is still a challenge for us as promoters of the inquiry-based learning is to encourage them to apply the full potential of the dynamic mathematics software in support of that style in their class setting. The prevailing part of the teachers are still seeing this software as a means for visualization of mathematical facts (envisaged within the curriculum) rather than for organizing experiments and explorations, for discovering patterns, for making conjectures. Even though being themselves engaged in exploratory activities during the workshops, some teachers were skeptical about applying such style of learning in class due to the limited time envisaged for covering a significant number of mathematical topics.

The presentation of teachers' projects for dynamic mathematics scenarios demonstrated that teachers still see it as a great challenge to develop original resources (since it is *difficult and time consuming*). They expressed great interest in having access to ready-made dynamic constructions, or even to complete lessons (in Bulgarian) that could be used directly or after small modifications. At the same time, the participants were fascinated from the scenarios relating mathematics in a natural way to other fields – art, dance, nature, architecture, etc. They shared with us their wish to make the students familiar with aspects of mathematics which show that *it is much more than computing, that it is part of our culture* [11].

This directed our efforts in two main streams:

- embedding lessons from classical math textbooks in HTML structures;
- enriching the existing curriculum with scenarios for extracurricular activities.

In addition, we encouraged the teachers to use the homework for organizing specific stages of the inquiry-based process. To work actively and on their own is crucial for the learners to achieve the goals behind the inquiry based learning strategies. The mere demonstrations would never accomplish these goals. The time dedicated to exploring, observing, conjecturing, generalizing and studying special cases does not only lead to a better understanding of the subject matter, but enhances the thinking and the imagination of the learners, the acquirement of research skills and contribute to a relevant attitude to science in general.

Examples of what we considered to be *successful dynamic approaches* were sent to an

external evaluator (Kenneth Ruthven) together with the respective learning environments and recommendations how to use them in practice. It is with pride and satisfaction that we share the feedback from him and from our Project partners:

- Kenneth Ruthven (University of Cambridge): *Thank you for sending such rich range of examples of successful dynamic mathematical approaches.*
- Tamara Bianco (University of Augsburg): *Thanks a lot (this goes to the whole team of Bulgaria) for the work you have put into this.*

5. Conclusions. In conclusion, the development of resources making use of dynamic constructions is just an element of the dynamic mathematics education. The discoveries, the representations and the implementation of mathematical objects and ideas could be related to the enhancement of the creative potential of learners by providing appropriate conditions and our on-going efforts are in this direction.

A positive impact in this direction is expected from the current implementation of the *Fibonacci* European project [12] whose main focus is on disseminating the inquiry based education. Even though it is hard for a single organization or a research team to do very much to affect the overall system directly, addressing the problems of the current educational system with InnoMathEd-like strategies will hopefully influence the global situation. The innovations in mathematics educations in harmony with the knowledge/creativity-based society are extremely important since the very mathematics is seen *as the language for innovation* [13].

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ИНОВАЦИИТЕ КАТО ВЪЗМОЖНОСТ, А НЕ КАТО ЗАПЛАХА: УРОЦИ ОТ ЕВРОПЕЙСКИЯ ПРОЕКТ *InnoMathEd*

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Представени са основни реализации на целите на европейския проект *InnoMathEd*. Описани са някои динамични виртуални среди, техни приложения, дидактически материали и методика на използването им в обучението по математика. Поставен е акцент върху възможностите за организиране на математически изследвания. Представени са примери за среден и горен курс, за задължително и избираемо обучение. Описани са проведени конкретни инициативи за разпространение на резултатите на проекта и получените в резултат експертни оценки.