STUDYING FINE-ART COMPOSITIONS BY MEANS OF DYNAMIC GEOMETRY CONSTRUCTIONS

Evgenia Sendova, Toni Chehlarova

Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Sofia 1113, Acad. G. Bonchev Str., Block 8, Bulgaria jenny@math.bas.bg toni.chehlarova@gmail.com

Abstract

The paper deals with integrating the study of art and mathematics by exploring the balance and the logical emphasis of paintings by means of dynamic geometry constructions developed in mathematics classes. A dynamic scenario developed and experimented in the context of the DynaMAT Comenius project is discussed as an illustration of how students could be encouraged to apply their mathematical knowledge for gaining a deeper insight in art compositions. Several methods suggested by art specialists are considered (e.g. rabatment, the rule of thirds, the golden section) together with appropriate dynamic geometry implementations. Ideas for further activities with students formulated in terms of long-term projects are offered. Although the considered scenario is still in its early phase of experimentation (mainly at teacher training courses) the first impressions are shared as being promising – the teachers become aware that they could attract more students to mathematics when showing its application in various contexts. The experience gained by the authors on a broader scale ─ in the context of visual modeling, is reported to have contributed to building new strategies in teacher education, which could prepare teachers for their changing role of partners in a creative process.

Keywords

dynamic geometry software, art, rabatment

INTRODUCTION

Seeing is not as simple as it looks
Ad Reinhardt

Many artists claim that they could explain nothing about their works, that their paintings came upon them by inspiration. The founder of the abstract art however, expresses in his book (Kandinsky, 2011) his theory of painting and sums up ideas that influenced his contemporaries. Kandinsky makes the brave prediction that *we are fast approaching the time of reasoned and conscious composition, when the painter will be proud to declare his work constructive*.

To motivate better the study of geometry for students with interests in art we could reveal for them the strong relation between the esthetics of artistic compositions and some geometric principles. When reading the works of art critics we come across notions such as *harmony, style, rhythm, balance* (not necessarily the better defined *rules, symmetry, geometry*). Perhaps they think that if revealed the rules behind a balance composition would trivialize the art. To us, revealing certain patterns and rules would in contrast raise the level of appreciation of an observer. The modern fine art tries to speak about things which *will be seen*, that is why its language is not understandable for many. But this language could be better learned if we try to study it together with the language of geometry.

VISUAL MODELING

Exploring the properties of geometric shapes in a computer environment has proven to be more exciting for students of different age if made part of a visual modeling of some works of art (Sendova and Girkovska, 2005, Nikolova et al, 2011). By building computer models of a given painting the students can gain deeper insight in its structure and motivation to elaborate their knowledge in mathematics and informatics.

When analyzing an abstract painting from mathematical point of view it is interesting to discuss its basic elements and to classify them. From an artistic point of view, though, the problem is not only to understand the elements of a composition, but also to understand its balance. In pre-service and in-service teacher training courses on using language-based computer environments for education we used a specially designed Logo microworld (Sendova, 2001) in which it was easy to experiment with figures of various sizes, colors and degrees of complexity, i.e. to verify different definitions of *balance*. In addition, the participants in the courses could play with Kandinsky's ideas concerning the relation between geometric shape and color and study the effect of both components in various combinations.

We could qualify the following factors as the most relevant ones in the study of an abstract painting:

- The character of the objects and their composition in terms of clustering, overlapping, isolation, balance, relationship between size, shape and color
- Main categories of the objects
- Establishing hierarchy related to the distance of the center, the size, the color, etc.
- Functional associations (which objects occur in combination in the work of a given author).

The visual modeling could be used not only to study a specific painting, or a specific artist, or more general – the style of a certain artistic movement, but also to bring possibly new creative ideas. Thus, products of the visual modeling should be judged with respect not only to the closeness between the original and the generated works but also to their potential to generate works bringing the spirit of the original together with new, unexpected ideas – a potential that depends on the user, of course. After leaving the frames of the strict imitation some of the future teachers were inspired by new combinations of forms and colors and got new insight, which in turn led to new formalization.

These visual modeling activities were carried out by means of programming which might create certain psychological problems among the typical mathematics teachers. Still combing art with geometry seems a very natural way of motivating the students to enhance their understanding in both fields. Some inspirational sources for integrating mathematics and art include (Ghyka, 1946, Livio, 2002, Hemneway, 2005, Olsedn, 2006, Skinner, 2009). More recent developments offer various dynamic geometry constructions as tools for analyzing works of art and appreciating the esthetics of well known paintings (Sánchez, 2013). The ideas presented below are based on a dynamic scenario (Sendova and Chehlarova, 2011) developed and experimented in the context of the *DynaMAT* Comenius project (http://www.dynamatadmin.oriw.eu) with the intention to encourage students in applying their mathematical knowledge for gaining a deeper insight in art compositions.

CREATING DYNAMIC CONSTRUCTIONS OF COMPOSITION TOOLS

The dynamic scenario deals with several relatively simple geometric constructions which have proved useful in creating and studying the balance of the fine-art compositions. After describing them we present their implementation in a dynamic software environment (*GeoGebra* in our case) so as to illustrate how they could be applied to exploring various paintings (classical and more modern alike). A further step offered to the students is to apply their newly gained art-evaluation competencies in the context of taking and editing photographs.

RABATMENT

The first (relatively less known) compositional method we introduce is *rabatment* which has been broadly used in the 19th century. This method is applicable to paintings in rectangular shape. It consists of taking the shorter side of a rectangle and placing it against the longer side (rotating the shorter side along the corner), creating points along the edge that can be connected directly across the canvas as well as a diagonal from these points to the corners. In a rectangle whose longer side is horizontal, there is one implied square for the left side and one for the right; for a rectangle with a vertical longer side, there are upper and lower squares. In traditions in which people read left to right, the attention is mainly focused inside the left-hand rabatment, or on the line it forms at the right-hand side of the image (Fig. 1).

Fig. 1 The left rabatment and its appearance in the Monet's painting *Red Poppy-field*

To achieve a more powerful composition one could add the diagonals of the rectangle and the two squares. Here is how the rabatment applied to the painting *A Sunday Reading in a Village School* of Bogdanov-Belsky looks like (Fig. 2).

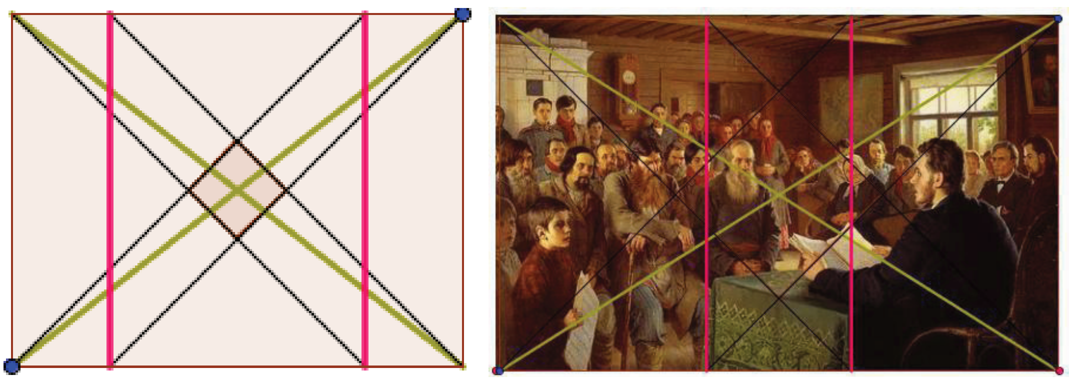

Fig. 2 The rabatment as applied to a Bogdanov-Belsky painting

More information about the rabatment method can be found at http://emptyeasel.com/2009/01/27/how-to-use-rabatment-in-your-compositions/ (Mize, 2013).

To enable the analysis of paintings of any rectangular shape it is convenient to have a universal rabatment tool. It could be created as a *dynamic geometrical construction* which means a construction which saves its main properties under movement of some of its objects defined as independent. The rabatment construction is introduced in our dynamic scenario by means of *GeoGebra* as follows.

CREATING A DYNAMIC RABATMENT TOOL

We start with constructing a dynamic rectangle. If the students are novices to using the software they should be encouraged to suggest and try out various ways of constructing a rectangle and discuss which of their constructions are in fact dynamic ones. After the discussion we could consider the following construction as appropriate for our purpose.

We construct one of the corners of the rectangle as an independent object – point *A*, and two variables (sliders) for its base *a* and height *b* specifying the range of their values. Next we construct point *B* - the opposite corner of the diagonal through *A* as a point whose coordinates depend on the coordinates of *A*, *a* and *b*). Then we construct lines through *A* and *B* parallel to the coordinate axes. The remaining two vertices of the rectangle could be obtained as intersection points of these lines. Then we connect the four points to get a rectangle (which is dynamic with respect to the size of its sides but always with a horizontal base, the normal position of a painting's frame).

Let $b < a$. Now we construct circles with centers the four vertices of the rectangle and a radius b – the length of the shorter side of the rectangle. We find the intersection points of the four circles with a side of the rectangle and construct two of the rabatment segments. Then we complete the construction with the diagonals.

What is left is to construct the square in the center which appears when $b < a < 2b$. We hide the auxiliary objects (the lines and the circles) and we explore the construction for various values of *a* and *b*.

Similarly, we make a workable construction for a rectangle with a shorter base.

It is worth mentioning here that a good educational quality of *GeoGebra* is the opportunity for the users to enrich the toolkit with their own tools. This facilitates the implementation of our rabatment tool, viz. we show to the students how to make the rabatment construction a part of the toolbar. For the purpose a suitable name, an icon and the inputs of the construction (a point and two numbers in our case) should be specified.

In our scenario we have considered in fact two constructions appropriate for a rabatment tool, the second one having as inputs the ends of one of the rectangles diagonals. Thus the students could create and use two rabatment buttons in the same *GeoGebra* file (named RabatmanPNN and RabatmanPP after the necessary inputs for the respective construction). As a further step in our scenario we show how images could be studied by means of the composition tools being created, i.e. how to display and how to resize (if necessary) the inserted image by preserving its proportions. Now the ground for explorations is set – the students could use the rabatment button, place the rabatment construction on the image and look for interesting properties of the composition of a specific painting (Fig.3).

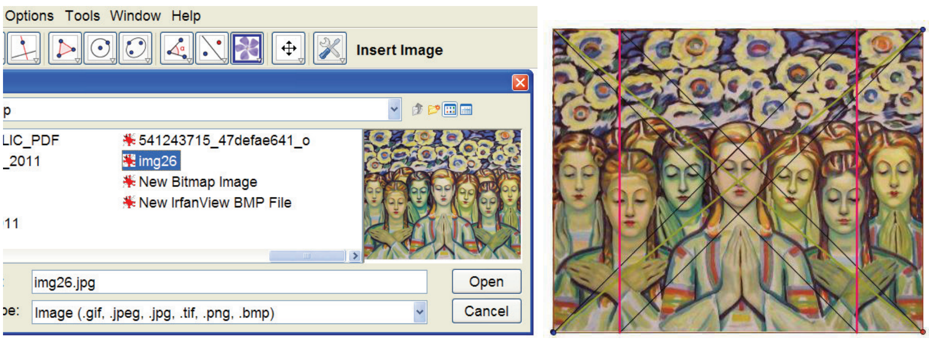

Fig. 3 Inserting the painting *Prayer* by Maystora and applying the Rabatment button to it

Depending on the emphasis the teacher would like to make, s/he could encourage the students to continue with exploring the created dynamic tool with other paintings and to formulate their findings. Alternatively s/he could enhance their mathematics skills of implementing other composition tools. Here is what we have suggested further in our scenario.

THE RULE OF THIRDS

The *rule of thirds* is a simple method that can be used not only as a tool for exploring the paintings of famous artists but also to enhance and improve our own compositions (when we draw or take pictures). In the diagram below, a rectangle has been divided horizontally and vertically by four lines. The rule of thirds states that the points of interest for any rectangle are determined by those lines. The intersections of the lines are considered by some specialists (Maze, 2013) to be *power points* (the black dots in Fig. 4).

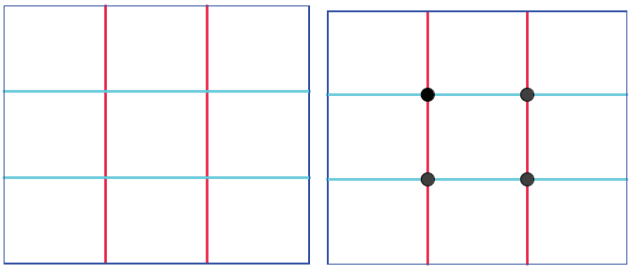

Fig. 4 The rule of thirds and the *power points*

Here is the rule of thirds in action (in horisontal version and in vertical one):

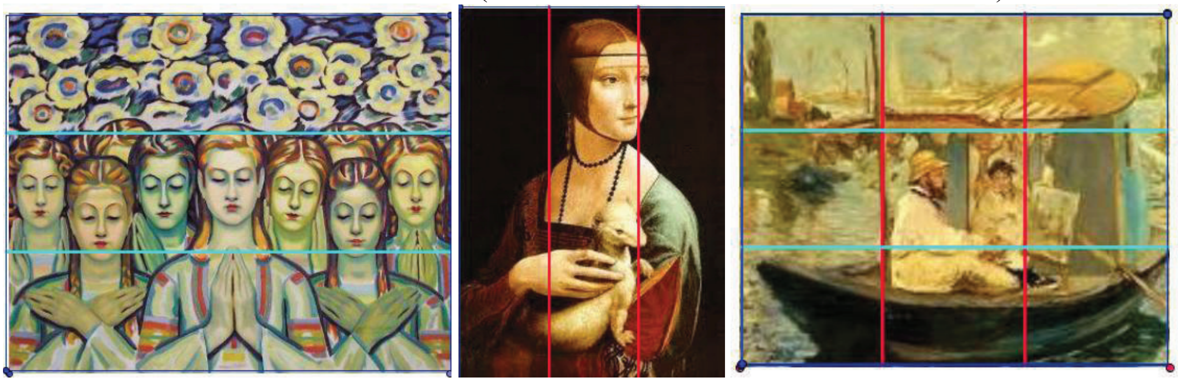

Fig. 5 The *rule of thirds* applied to Maystora's *Prayer,* to Leonardo's *Lady with an Ermine,* and to Manet's *Monet painting in his floating studio*

For the students it is of essential importance to use the rule of thirds not only when studying famous paintings but also when taking (or editing) photographs of a scenery (Fig. 6).

Fig. 6 The *rule of thirds* in photography

The teachers could create a list of tasks for the students taking into account their interests and mathematics background. Some examples of tasks we have offered in our dynamic scenario deal with

- Making several digital pictures of a scenery by applying the rule of thirds in just one of them and explain which version seems to be the most balanced one
- Creating *Thirds* buttons (a vertical and a horizontal versions)
- Exploring some classical and some modern paintings by various composition buttons.

In addition to using some classical composition tools the mathematics teachers could suggest geometric constructions of their own, possibly jointly with the art teachers. Here is an example from our scenario.

THE CENTRAL RHOMBUS

The logical emphasis of a painting is often located in a rhombus with vertices the midpoints of the sides of the rectangle:

Fig. 7 The central rhombus applied to Maystora's *The Girl with the Dahlias*, and to Mrkvička's *Ruchenitsa*.

To make a dynamic construction and turn it into a rhombus button in *GeoGebra* for exploring images is another activity offered to the students.

Of course, the most popular notion combining art and mathematics is the *golden section*.

A DYNAMIC GOLDEN SECTION CONSTURCTION

The most famous mathematical composition tool, though, is the *Golden Section* (also known as the *Golden Mean* or the *Golden Ratio*) defined as the point at which a segment can be divided in two parts *a* and *b*, so that $a/a+b = b/a$. We introduce the notion of a *golden rectangle* as a rectangle whose side lengths are in the golden ratio. The golden ratio is often depicted as a single large rectangle formed by a square and another rectangle. What is unique about this is that we can repeat the sequence infinitely within each section. If in addition we draw an arc of 90º in the consecutive squares we get the so called *golden spiral* (Fig. 8).

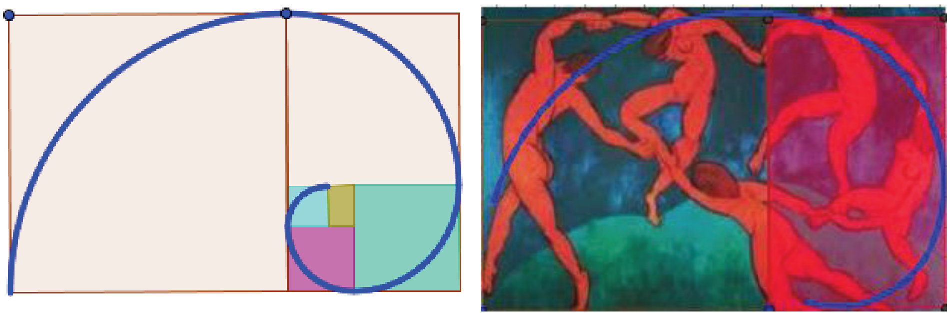

Fig. 8 A sequence of golden rectangles and the golden spiral applied to *The Dance* by Matisse

We give in our scenario the following algorithm for constructing a dynamic golden spiral

- *Construct a unit square (blue).*
- *Draw a segment from the midpoint of one side to an opposite corner.*
- *Use that segment as the radius of an arc that defines the longer dimension of the rectangle (Fig. 9).*
- *Construct an arc of 90° in each square so as to get a golden spiral:*

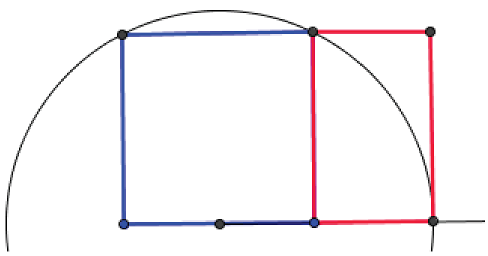

Fig. 9 Constructing a dynamic golden rectangle

Then we suggest to students to construct *GeoGebra* buttons based on the golden ratio and to explore various paintings with all the composition tools they have created.

It is very important to extend these activities by assigning long-term projects to the students. Here are some examples of *dynamic mini-projects* included in the scenario:

- *Take a picture of a scenery in two ways so that they reflect specific goals. Explore the result by means of dynamic constructions and edit the pictures correspondingly by cutting out.*
- *Arrange for a picture in two ways (according to two composition methods): 6 persons at a birthday party sitting around a round table; a class of 24 pupils and their teacher; flowers and fruits; perfumes and an advertisement. Explore the result with dynamic constructions and make corrections if necessary.*
- *Make an advertisement in two ways of: your school; your hobby; natural juices; an old town. Explore the result with dynamic constructions and make corrections if necessary.*
- *Make in two ways a design of an invitation cart for: a fest of mathematics (physics, music, the flowers, athletics); a ball with masques; a birthday party. Explore the result with dynamic constructions and make corrections if necessary.*
- *Explore the rotational dynamic constructions by means of the sliders so as to create models similar to the pictures of rotational objects (Fig. 10).*

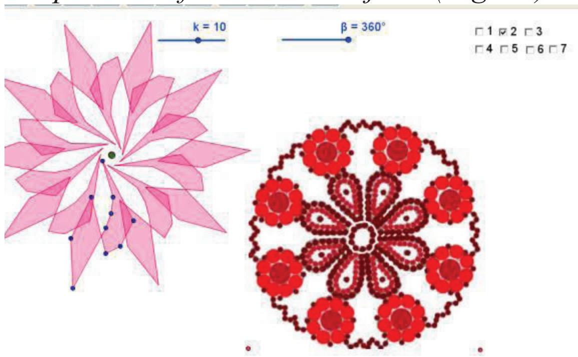

Fig. 10 Rotational dynamic construction

- *Create models of objects around you based on rotational symmetry (wood carved ceilings, embroidered table clothes, etc.)*

CONCLUSIONS

Our overall experience in educating students and teachers alike shows that the integration of the learning and creative processes by means of visual modeling could contribute to a new learning style in mathematics education. Such type of activities sensitizes students to looking at not only the art but also at the world around them in a more meaningful way.

Although the considered scenario is still in its early phase of experimentation (mainly at teacher training courses) the first impressions are promising – the teachers become aware that they could attract more students to mathematics when showing its application in various contexts (often unexpected for them as art is).

A famous quote by the american poet Robert Frost reads: *Writing free verse is like playing tennis with the net down.* We could extend this quote to art in general. But an important point we make to the teachers is that every rule can and should be broken for artistic effect, from time to time. This should be done however not because we don't know the rules but rather when we are looking for new ideas. This is for example how some stunning photographs are made.

The experience gained leads us naturally to building new strategies in teacher education, which could prepare teachers for their changing role of partners in a creative process.

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