THE HISTORICAL AND LOGICAL METHODS IN MATHEMATICS EDUCATION

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ABSTRACT

In the following the nature of the historical and logical methods is examined. Its role as the means and ends in mathematics education is pointed out.

"There is need of a method for investigating the truth about things.'
René Descartes
Rule IV of "Rules for the Direction of the Mind."

The use of gnosiology as a model is an effective approach in examining the process of mathematics education. "Cognition in the process of mathematics education also has the three basic congnitive categories: the cognitive subject (the subject that can know), the cognitive object, and the cognitive (gnosiological) image. A cognitive process takes place in education as a result of the object-subject relationship whereby a cognitive image is formed in the mind of the cognitive subject". ([2], p. 153.)

The cognitive target (Π) (the part of the object to which the attention of the subject is directed) establishes the connection between the **object (O)** and **the subject (S)**. Having reached the conclusion in [2] that the cognitive subject **S(P;U),** object **O(Z;U)** and target **Π**($Z_1;U_1$) have a binary structures, we introduce the following model of the process of mathematics education (Fig.1).

In this model the two-way arrow between the subject and the object (S↔O) denotes their interaction that creates the cognitive process. This interaction is further specified by the direct and indirect interaction (showed by the relevant arrows) among the remaining elements of the model: **P** – educator (professor, teacher); **U**- learner (student, pupil); **Z** – knowledge, skills, methods, beliefs and norms of conduct that the learners need to acquire in order to be able to develop their personal qualities.

It becomes clear from the model in Fig.1 that one of the possible way to improve the process of mathematics education is related to the components of the

cognitive target **Π** $(Z_1;U_1)$. That is to say, it is related to that part of **Z**, denoted with Z_1**,** which will form the relevant qualities U_1 of the learners **U.**

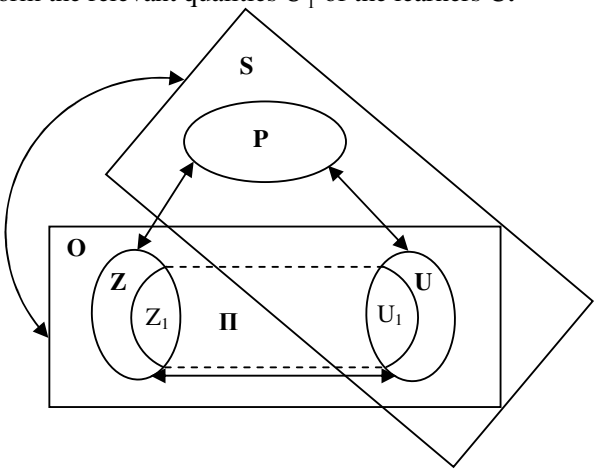

Fig.1 Object-subject model of the process of mathematics education

The following examination deals with the students studying to become teachers and the formation of their professional qualities U_1 by knowledge Z_1 of the university subject of Methodology of Mathematics Education (MME). In order to improve the quality of education in MME, it is necessary to increase the "spectrum" of gnosiological methods taught in Z_1**.** This means that the education of MME needs to be transformed from being predominantly fact-based to being more methodological. 'Methodological' here is understood in both of its aspects – as a theory of the methods and as an aggregate of the methods of investigation and education in a given science.

In this connection, our **purpose** is to **popularise** the nature of the historical and logical methods (HL methods) and to identify their **place** and **role** in the system of methods treated in MME. We will naturally begin with a brief **history** of their conception and development.

In the general philosophy, the classic philosophers have developed the HL –methods. The latest philosophical literature and the didactical literature treating the methodological aspects of education have enriched the methods with new contents. Although the issue of the contents of the HL methods issue is rooted in Aristotle's philosophy (384-322 BC), Hegel is the first philosopher who introduces this issue. Marx and Engels subsequently apply the HL methods as a unity specifically in political economy.

Until recently the so-called **traditional** understanding of the nature of the HL methods has been prevalent in the philosophical and pedagogical literature.

Gnosiology can reflect the objective history of things and phenomena in two different ways. It can reflect the transformation as it is with all its specific

details, meandering and deviations and thus turn it into a subjective copy of the objective process. This very same objective historical process can also be reflected in such a way, that the most noteworthy and regular aspects are emphasised, and all insignificant details and deviations characterising the specific manifestations of the process are left out.

In the first case, the picture of the real and specific history of the process is fathomed. This characteristic is primarily for the historical approach in scientific research. In the second case, the aim is to reflect the general, essential, significant, and regular aspects in the same history of the process. This is typical for the logical approach to scientific research. Thus, these characteristics represent the traditional understanding of the HL methods.

The HL methods are applied not only to scientific research but also to the presentation and acquisition of scientific knowledge, research results as well as to the development of the theory of knowledge. Of interest here is the application of the HL methods in mathematics education when specific scientific knowledge is presented and gained. The connection between the contemporary understanding of the HL methods and the presentation and acquisition of scientific knowledge in mathematics will be explained.

Applying the **historical** method does not simply serve the purpose of legitimising the object's history, its conception, existence, development and possible demise. Knowing the history of mathematics and the reforms in mathematics education [1] is a necessity. The method is characterised by **analysing** the concrete history of the object and its evolution. This analysis is the history of human cognition of this object. For example, when dealing with the topic of "Function" during a lesson in Particular Methodology, the notion of function is analysed from two perspectives. Firstly, from the perspective of its historical evolution in scientific cognition. Secondly, from the perspective of the history of mathematics education in relation to studying the notion. In these analyses different forms of the utilised approaches are applied.

When the analysis takes the form of **dismembering a whole into its components,** these components are compared and contrasted with the purpose of finding the analogy and disparity between them. One determines which of the components are 'primary' and which are 'secondary', i.e. the condition for the existence of the other one. Finally, the initial component among the 'primary', and thus the undeveloped beginning of the developed history of the object, is identified. This type of analysis is a process of 'descending' from the concrete to the abstract. In this way the historical method is realised through many other methods of scientific knowledge.

The specific history of the object can also be analysed from a 'Descartes' perspective seeking the reasons for the conception of the 'created history'. That is to say, the purpose is to find the necessary or sufficient conditions for the conception, existence and development of the object.

Finally, the historical method of research can be realised through an analysis based on the **quantitative** learning of the qualities of the object by utilising the notions of 'number' and 'measure'.

The addition to the current understanding of the historical method presented above, paves the way also for a more contemporary role of the method in mathematics education.

From a didactic point of view, the role the historical method has in mathematics education is no longer simply to incite the interest and motivation of the students or to nurture virtues by following examples from stories about the lives and works of famous mathematicians. Instead, the historical method influences the formation of students' appreciation of the role of mathematics in the development of other branches of science. Students learn that mathematics is a powerful tool for exploring methodological issues of mathematics itself such as the issues of the origin of notions and the influence of practice on the development of mathematics.

The historical method not only aides learning about the discoveries but also helps **analyse the method utilised by the discoverer**, which brought about the formulation and proof of the conclusions. **Exploring the methods** of remarkable examples in science, **examining the process of their discovery** as well as applying these methods leads to gaining knowledge of the discoveries.

Describing the logical method as a shortened repetition of historically conceived knowledge is enhanced by the understanding that the logic has undergone **a qualitative transformation**, that it is the **'rectified'** historical from the perspective of the **most superior stage** of development of knowledge. More specifically this means that the old notions are changed or complemented with new content, that new notions are created, as well as that the chronological order, in which the notion apparatus and theory are explored, is reconsidered and 'revised'. A case in point is the **historical and logical** scheme for expanding the notion of 'number' examined in MME.

As mentioned above, an important aspect of the current understanding of the nature of the historical method is the process of 'descending' from the concrete to the abstract, which takes place when examining historical knowledge. This is the most important process since it establishes the preliminary abstraction, which is the source of extraction, development and reproduction of new concrete knowledge. This new concrete knowledge is different from the initial concrete in the first 'descending' process, where it is the beginning of the cognitive process hidden in a unity of varieties.

The logical method begins where the establishment of the preliminary abstraction of the historical method ends. Characteristic for the logical method is that process of 'ascending' from the abstract to the concrete begins by means of induction, synthesis and deduction. This new concrete is the end of the cognitive process and not the beginning. It is a result of the mental process and a unity of all aspects and features of the examined object. It is a variety in the unity, a compilation of multiple definitions. It is clear that the 'ascent' characterising the

logical method is impossible without the 'descent' characterising the historical method. In this sense, when discussing the unity of the two methods we believe that one of the forms of this unity is the unity between 'descent' and 'ascent'. Another important feature of the unity between the two methods is that in the objective process of gnosiology development the historical becomes logical, and on the other hand, the logical becomes in due course historical in order to be further the cognitive development later on in time.

In addition, another significant characteristic in the nature of the logical method is the fulfilment of the requirements for logical strictness when gaining scientific knowledge introduced by Aristotle. These requirements are as follows:

1. All notions, which are not primary and are introduced in the theory, must be defined only by previously defined notions and if necessary by preliminary notions.
2. All statements formed, which are not preliminary (axioms), must be proved by previous statements and if necessary by the axioms.

It is not a question of personal preference which of the two HL methods will be applied. As pointed out in [3, p. 180], it depends on: "1) the level of development of cognition; 2) the maturity stage of the examined object; and 3) the character of the historical era."

In mathematics education the two methods are not applied in pure form, isolated from one another, but in such a unity that the logical method has a leading role.

The HL methods have to be a aim of the education of future teachers of mathematics and consequently a part of the contents of university subject of MME. They need to be taught and studied together with the other gnosiological methods, which are a tool for the realisation of the HL methods.

The place of the HL methods in mathematics education for pupils and university students studying the subject of MME is related to the problems of MME as a university subject. These problems are numerous but can be summarised into the three following groups:

I group – problems related to the choice of subject purpose and contents;

II group – problems connected to the structure of the subject contents (the system of the construction and presentation of the presentation);

Group I and II are the two fundamental problems of MME.

III group – problems related to the principles, methods, and tools of education, the so-called problems of the process.

In sum, presented above are the nature, role and place of the HL methods in mathematics education and in MME. This is the beginning of a number of investigations specifying the ideas presented above.

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