CUBE CONSTRUCTIONS

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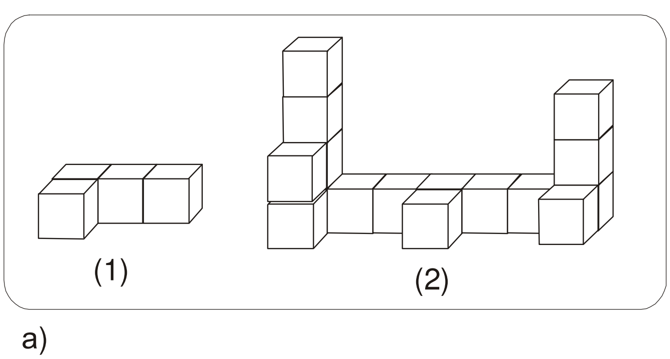
The development of great power spatial imagination remains still insufficiently solved problem in Bulgarian Mathematics Education. According to actual school programs some knowledge of prism, pyramid, cone, cylinder, ball and sphere should be formed during the compulsory education in 6th grade. Superficial knowledge of Solid Geometry is learned in the frames of the compulsory education by the choice of Mathematics as a profiling subject in 11th and 12th grades. The new school program provides for cube and rectangular parallelepiped teaching in 5th grade. But even sporadic problems connected with spatial imagination development are missing in the Bulgarian textbooks for the other grades.

A possible way to overcome the situation is to include cube constructions as entertainment elements of Mathematics lessons. Cube constructions are used in the most famous psychological tests to study spatial ideas. Examples are connected with Shepard' s tests (see Djaldetti, 1999). Games with colored cubes are created for the purpose too (see Nikitin, 1989). Problems with cube constructions appear also in the International Mathematics Competition "European Kangaroo" (see Grozdev, 2002).

The present paper considers some cube constructions for which the corresponding solving process will help students to develop great power spatial imagination. The elements of the constructions are called to be "unit cubes" and the assumption is that all unit cubes are equal with respect to dimensions and weight.

Problem 1. *Weighing the unit cubes from construction (1), Neda has established that the total mass is equal to 26 g. What is the total mass of the unit cubes from construction (2)?*



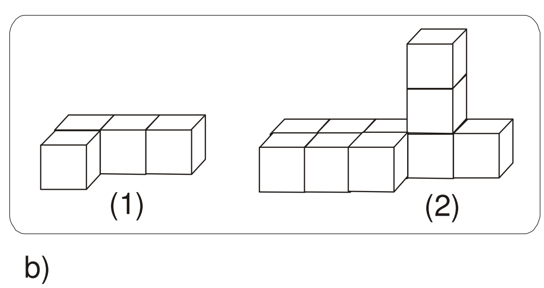


Fig. 1

Solution:

a) Construction (2) could be considered as consisted of 4 constructions (1) (Fig. 2). Thus, it is enough to find the product 26.4, which gives 104 g.

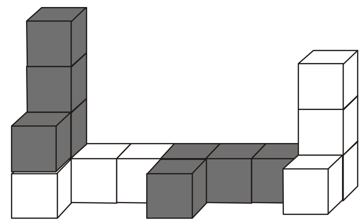


Fig. 2

In case that the possibility to divide construction (2) into 4 constructions (1) has not been noticed, another way of reasoning is the following: construction (1) consists of 4 unit cubes, while construction (2) consists of 16 unit cubes and consequently the mass of the last is 4 times greater

b) It is possible to propose this problem to students after the 5^{th} grade when they are already acquainted with fractions. Such students will find the problem not difficult. But still, if knowledge of fractions is not assumed, then the following solution could be proposed:

Since 4 unit cubes weigh 26 g, then 2 unit cubes will weigh 13 g. Construction (2) contains 10 unit cubes and therefore its mass is 5.13 = 65 g.

Note that the second solution of a) and the given one of b) uses the number of the unit cubes in each of the constructions while the first solution of a) applies the form of the constructions only.

<u>Problem 2.</u> *How many of the unit cubes of construction (2) are invisible if the masses of construction (1) and construction (2) are 30 g and 70 g, respectively?*

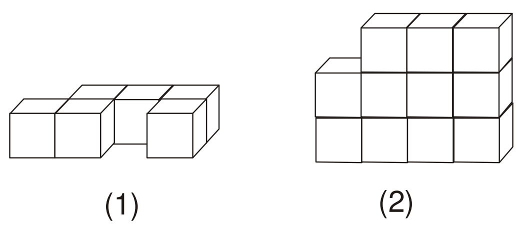


Fig. 3

Answer: 3

Problems similar to the next one are suitable to give meaning of visibility and invisibility.

<u>Problem 3.</u> *How many are the invisible unit cubes if the construction is a cube:*
a) 2 x 2 x 2; b) 3 x 3 x 3; c) 4 x 4 x 4?

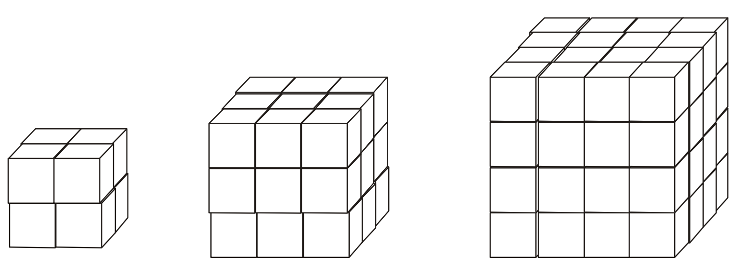


Fig. 4.

Answer: a) 0; b) 8; c) 27

For a correct visualization concerning the first example of problem 3 it is useful to enumerate the visible unit cubes as shown in Fig. 5.

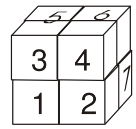


Fig. 5

One of the main problems in cube constructions is connected with the visibility of some of their unit cubes. Different kinds of invisible unit cubes may occur. In the above examples there are such which are situated on the supporting plane directly. Others are situated on unit cubes which are already situated on the supporting plane. Here the feeling of symmetry plays an important role, i.e. symmetry is the leading argument. But it may happen, that one or several unit cubes are stuck to one of the invisible sides of the construction. In this case different situations are possible and a clear formulation of the problem is needed. When necessary we will indicate the exact disposition of the invisible unit cubes.

<u>Problem 4.</u> *A cube 3 x 3 x 3 can be constructed by exactly 4 of the constructions from Fig. 6, consisted of 11, 4, 3, 5 and 7 unit cubes, respectively. Which of the constructions should be eliminated?*

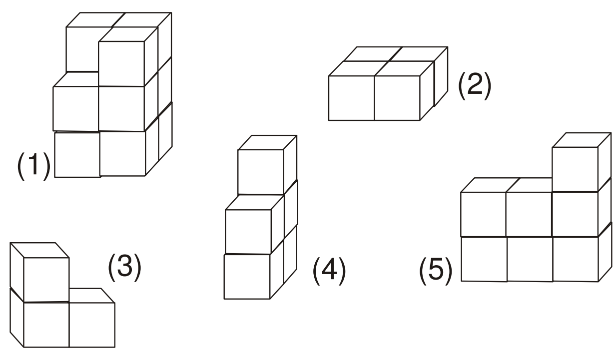


Fig. 6

Two ways of reasoning are outlined clearly.

<u>Solution 1</u>: The cube 3 x 3 x 3 contains 27 unit cubes while the sum of the unit cubes from the given constructions is 11 + 4 + 3 + 5 + 7 = 30. It follows that a construction with 3 unit cubes should not be used. The only such is construction (3).

<u>Solution 2</u>: Use the corresponding assembling of the given constructions (Fig. 7).

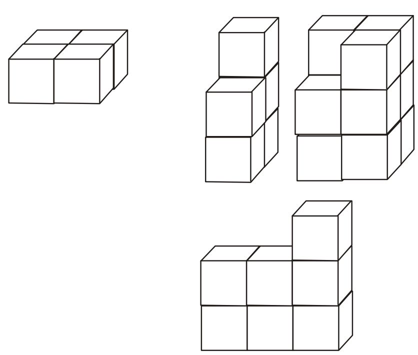


Fig. 7

If the problem is formulated in the following way: *Using some of the shown constructions, construct a cube 3 x 3 x 3*, then the solution consists of both the given solution in the same order.

<u>Problem 5</u>. *A cube 3 x 3 x 3 can be constructed by exactly 4 of the constructions from Fig. 8, consisted of 5, 9, 5, 8 and 5 unit cubes, respectively. Which of the constructions should be eliminated?*

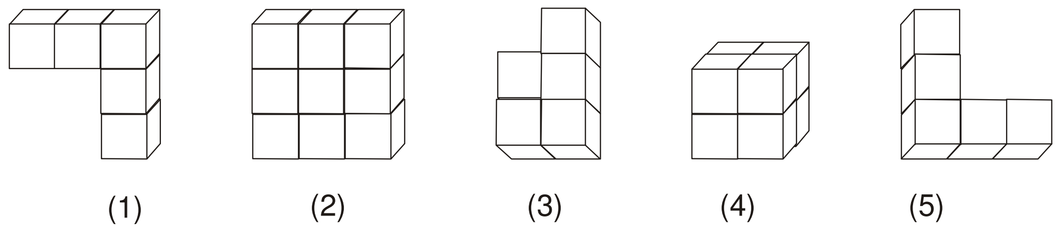


Fig. 8

<u>Solution:</u> Doing similar calculations as in the previous problem, we conclude that a construction with 5 unit cubes should be eliminated. We have 3 possibilities but the checking could be reduced to 2 only since two of the constructions with 5 unit cubes are identical.

Answer: (3)

<u>Problem 6.</u> *One of the unit cubes of a cube 2 x 2 x 2 is eliminated (Fig. 9). What is the area of the surface of the new construction with respect to the area of the surface of the initial one:*

a) greater; b) smaller; c) equal?

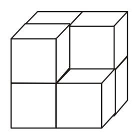


Fig. 9

Answer: c)

The last problem is analogous to plane geometry problems for comparing lengths. An example is shown in Fig. 10 where the length of the black line and the length of the grey one should be compared. What is important here is to give meaning of the general idea when solving such

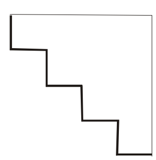


Fig. 10

problems. Naturally, this idea goes to the "parallel projection" method.

<u>Problem 7.</u> *Two unit cubes of a cube 3 x 3 x 3 are eliminated (Fig. 11). What is the area of the surface of the new construction with respect to the area of the surface of the initial one:*

a) greater; b) smaller; c) equal?

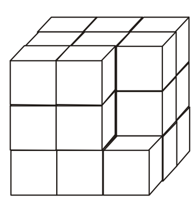


Fig. 11

Answer: c)

Problem 8. *Several unit cubes of a cube 4 x 4 x 4 are eliminated as shown in Fig. 12. Find the area of the surface of the new construction if the edge of a unit cube is 1 cm.*

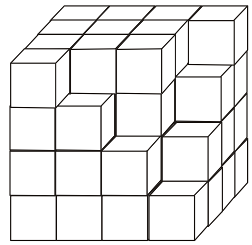


Fig. 12

Answer: 96 cm^2

Problem 9. *Eliminate 3 unit cubes from a cube 3 x 3 x 3 in such a way that the area of the surface of the new construction is:*

a) greater; b) smaller; c) equal

with respect to the area of the surface of the initial one.

Problem 10. *Draw the front elevations of the three constructions, the right side ones and the upper ones if the number of their unit cubes is 10, 14 and 6, respectively.*

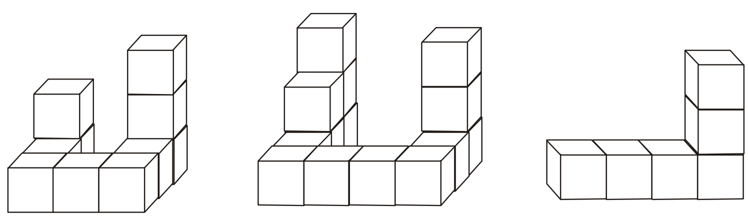


Fig.13

Problem 11. *Three plane elevations of a cube construction are shown in Fig. 14: the front one, the right side one and the upper one, respectively. Find the number of the unit cubes of the construction.*

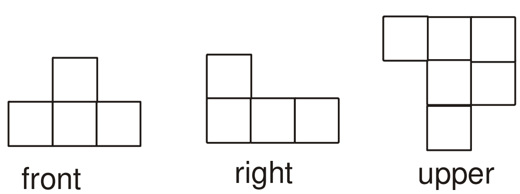


Fig. 14

Solution: One could fix the counting results by writing down the defined number of the unit cubes from the first elevation on the third one and then writing down the defined number of the unit cubes from the second elevation on the third one again. Thus, the front elevation shows that each of the left and the right columns of the construction contains no more than one unit cube. The information is fixed as shown in Fig. 15 a). The same elevation shows also that each of the middle columns of the construction contains 2 unit cubes at most but at least one of these columns contains exactly 2 unit cubes. The right side elevation shows that such a middle column is exactly one. Also, this column is situated on the last row at the back. Thus, the position of the digit 2 is defined in a unique way and the information is fixed as shown in Fig. 15 b). Consequently, the answer is 7.

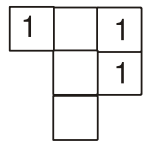


Fig. 15 a)

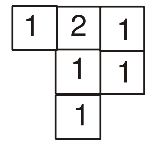


Fig. 15 b)

Problem 12. *What is the mass of a unit cube if the balance is in equilibrium and the shot weighs 100 g? Consider only the case when the shown constructions are cubes.*

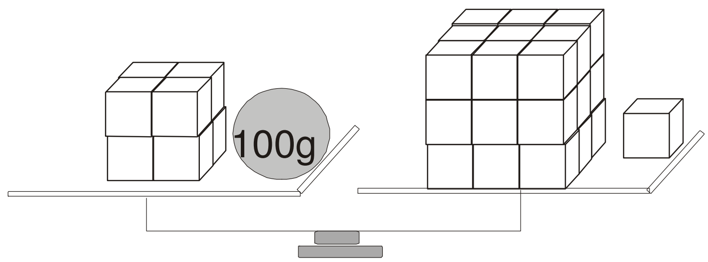


Fig. 16

Answer: 5 g

Problem 13. *All unit cubes of a cube 3 x 3 x 3 are used for the construction of a rectangular parallelepiped. The length of any edge of the parallelepiped could not be equal to:*

a) 3 units; b) 6 units; c) 9 units.

Answer: b)

Problem 14. *It is not possible to construct a cube using all unit cubes of several constructions containing 4 unit cubes each (Fig. 17) if the edge of a unit cube is 1 cm and the edge of the cube under construction is:*

a) 2cm; b) 4 cm; c) 6 cm; d) 8 cm; e) 9 cm.

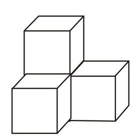


Fig. 17

Answer: e)

Problem 15. *Several unit cubes are arranged in a row and are packed by a ribbon (Fig. 18). If the knot is not considered, then the length of the ribbon is always divisible by:*

a) 3; b) 4; c) 5; d) 6

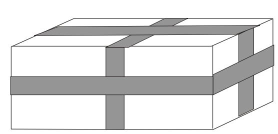


Fig. 18

Answer: b)

The problems under discussion in the present paper are directed to the development of spatial imagination of students. The authors believe that their solutions use the possibilities of both brain hemispheres despite the general opinion in the psychological literature (Granovska, Munzerth) according to which school education assumes the using of the left hemisphere mainly. Different aspects in this direction will be discussed in another paper of ours. The most important is that the entertainment character of cube constructions rises student interest and increases the efficiency of mathematical education.

R E F E R E N C E S

1. Granovska, R. Elements of the Practical Psychology. Science and Art, Sofia, 1989. (in Bulgarian)
2. Grozdev, S., A. Vassileva, N. Sabeva. European Kangaroo. Union of Bulgarian Mathematicians, Sofia, 2000. (in Bulgarian)
3. Djaldetti, A., V. Vassilev, R. Stamatov. Psychology. Education, Sofia, 1999. (in Bulgarian)
4. Munzerth, A. Test your intelligence. Actbull, 1996. (in Bulgarian)
5. Nikitin, B. Levels of creativity or games for development. Education, Moscow, 1989. (in Russian)