DOMINOES AND FRACTIONAL NUMBERS

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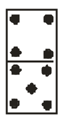
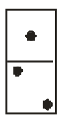
Abstract: *According to the Bulgarian Curriculum in Mathematics, fractions are studied in the 5th grade. Arithmetic operations with them are not attractive for pupils and lead to fatigue and repulsiveness. The paper proposes amusing elements to be involved in the process of teaching and learning. It turns out that the use of dominoes is suitable for the purpose. A didactical system of mathematical problems is developed applying dominoes as models of fractional numbers.*

To learn fractional numbers and arithmetic operations with them is an essential part of the aims in the Curriculum of Mathematics for the 5th grade of the Bulgarian school. On the other hand, a long-lasting manipulation with fractional numbers is an obligatory activity in the process of forming of knowledge and skills but still leads to fatigue and to a decreased level of attention stability. This hinders fruitful learning itself. Not occasionally, in various textbooks in Psychology the activity under consideration is assumed as a suitable example of monotony, which hides a danger to cause ennui and discourage for those, who execute the activity. It seems to be possible to use dominoes as models of some fractions aiming at the elimination or at least the reduce of the negative effects of monotony and the resulting fatigue. The very fact, that the topic concerns a play, creates arguments for liberation from the eventually accumulated tension. It is well known that dominoes are suitable for demonstration of various mathematical relations, for the solutions of combinatorial and geometrical problems but in the present note they are applied to learn the topic "Fractional Numbers" namely. The main idea is that dominoes could be used in presenting common fractions and decimals. This could be done in the following way:

Common fractions

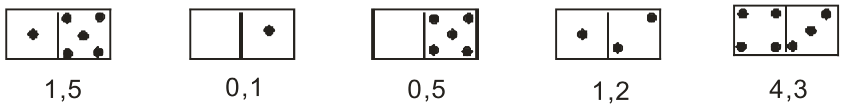
The separating line plays the role of a vinculum.

 $\frac{4}{5}$

 $\frac{1}{2}$

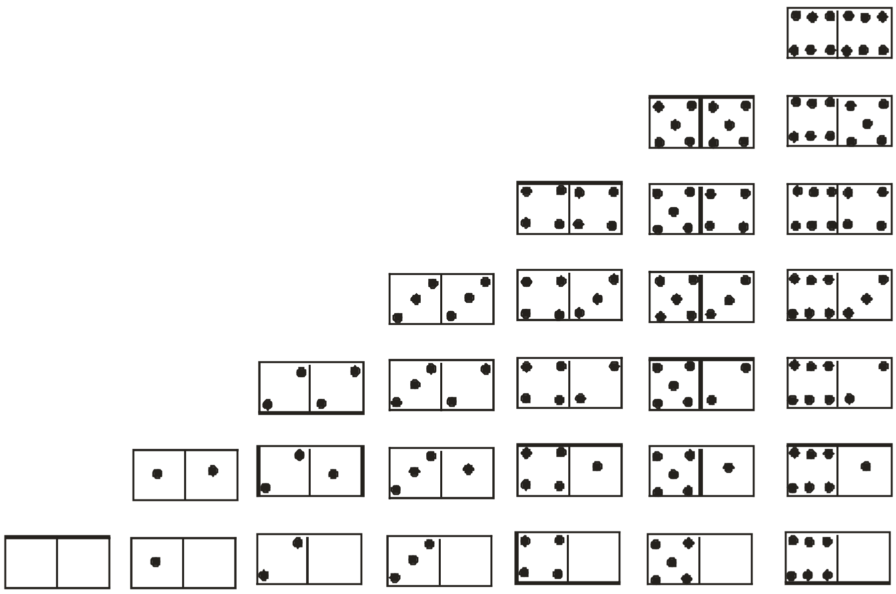
Decimals

The separating line plays the role of a decimal comma.



The initial acquaintance with the suggested instruments includes an establishment of the number and the type of the dominoes from a full collection. This is a completely combinatorial problem, which could be solved by an exhaustion of variants respecting pupils' age. It is enough to start with the number 0 for example and to establish that this number could be combined with 0, 1, 2, 3, 4, 5 and 6 obtaining 7 dominoes in total. The number 1 comes next and it could be combined with 1, 2, 3, 4, 5 and 6 (6 dominoes in total). The turn is of the number 2, by means of which 5 new dominoes are obtained (with 2, 3, 4, 5 and 6), then the number 3 comes obtaining 4 new dominoes (with 3, 4, 5 and 6), by 4 – three dominoes (with 4, 5 and 6), by 5 – two dominoes (with 5 and 6) and finally by 6 – one domino (with 6). Thus, the total number of all dominoes in a full collection is equal to 7 + 6 + 5 + 4 + 3 + 2 + 1 = 28.

All the dominoes in a collection

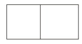


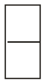
Some other combinatorial problems are proposed in the sequel (see problems 1, 2 and 3). Additional problems are possible too including ones which are connected with the rules of the play. We do not consider them. The basic aim of the paper is to present a didactical system of problems, which is directed to the improvement of the skills in manipulations with fractional numbers.

Problem 1. Are there dominoes by which it is not possible to obtain:

a) a common fraction; b) a decimal?

Solution: a) Taking in mind that a common fraction is well defined when its denominator is different from zero, we conclude that if a domino does not contain a zero then it represents two different common fractions. (Realizing that dominoes could be turned “up-down” is essential for the further problems too.) The domino with two zeroes is the only one by which no common fraction could be formed. Equally valuable common fractions could be obtained by all the others with a zero. In this case the corresponding value is zero.

b) Decimals could be formed by each domino. This time the domino  is not excluded and the decimal 0,0 corresponds to it (its value is equal to zero).

Answer: a) the domino  is the only one by which no common fraction could be formed;

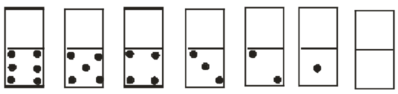
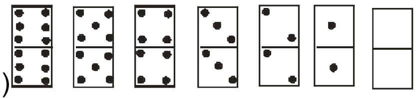
b) decimals could be formed by each domino.

Remark. The solution of the last problem is connected with the necessity of becoming conscious about the definitions of a common fraction and a decimal including the fact that the denominator of a common fraction is a number, which is different from zero.

Problem 2. By which dominoes is it not possible to obtain:

a) an improper common fraction; b) a proper common fraction?

Solution: The dominoes from a collection can be divided into three groups. They are in the first group those dominoes (in total 15), by which two different common fractions can be formed. For example, the domino with five and four points on it represents the common fractions $\frac{4}{5}$ and $\frac{5}{4}$. What is possible to do by each domino from the second group is to represent a unique common fraction. Such a domino contains only one zero and the number of the dominoes of this type is equal to 6. Also, they are in the same group the dominoes with one and the same number of points on their both sides. The number of such dominoes is equal to 6 too. It remains only one domino (third group) with no points on its both sides. As mentioned in the previous problem, this domino is special with respect to the formation of common fractions. Further, it is enough to remind the definitions of a proper common fraction and of an improper one: a common fraction is called to be proper if its nominator is less than its denominator; if the nominator is greater or equal to the denominator then the corresponding common fraction is called to be improper.

Answer: a) ; b).

Problem 3. Find the number of all different common fractions which could be obtained by a full collection of dominoes. How many of them are proper and now many improper?

Answer: 24 (the digits 0, 1, 2, 3, 4, 5 and 6; the common fractions $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{2}{3}, \frac{2}{5}, \frac{3}{2}, \frac{3}{4}, \frac{3}{5}, \frac{4}{3}, \frac{4}{5}, \frac{5}{2}, \frac{5}{3}, \frac{5}{4}, \frac{5}{6}$ and $\frac{6}{5}$).

Problem 4. Find:

a) the greatest common fraction, which corresponds to a domino;

b) the smallest common fraction, which corresponds to a domino;

c) the greatest decimal, which corresponds to a domino;

d) the smallest decimal, which corresponds to a domino.

Answer: a) $\frac{6}{1}$; b) $\frac{0}{1};\ldots;\frac{0}{6}$; c) 6,6; d) 0,0.

Problem 5. How many are the dominoes by which one could represent:

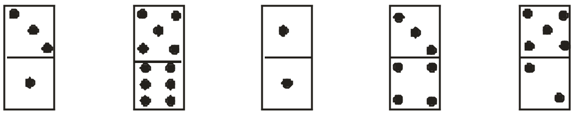
a) a common fraction, equal to $\frac{1}{2}$; b) a common fraction, equal to $\frac{1}{3}$;

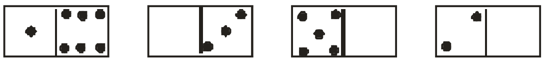
c) a common fraction, equal to $\frac{2}{3}$; d) a decimal, equal to 1,2;

e) a fractional number, equal to $\frac{1}{2}$; f) the digit 0?

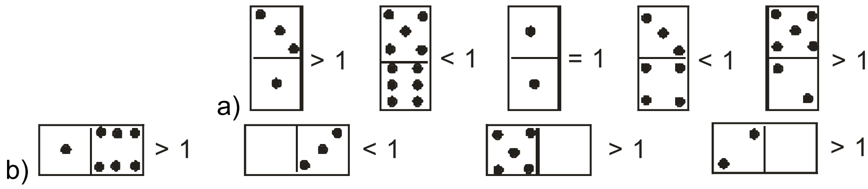
Answer: a) three: $\frac{1}{2}=\frac{2}{4}=\frac{3}{6}$; b) two; c) two; d) one; e) four (the representation of the decimal 0,5 should be added to the representation from point a)); f) 7 (six common fractions and one decimal).

Problem 6. Compare the number 1 with all fractional numbers, which correspond to the dominoes below:

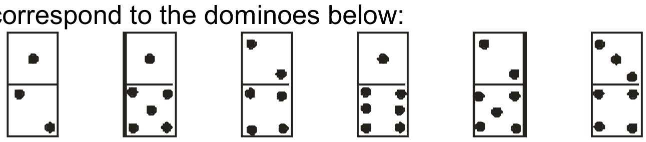
a) 

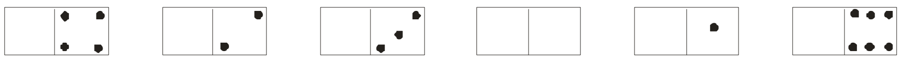
b) 

Answer:



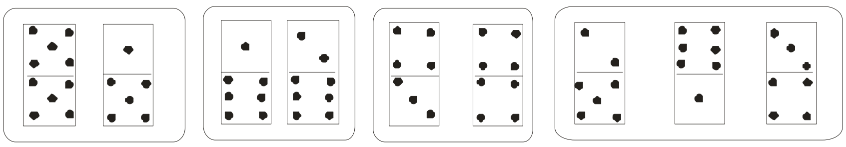
Problem 7. Find the difference between the number 1 and all fractional numbers, which correspond to the dominoes below:

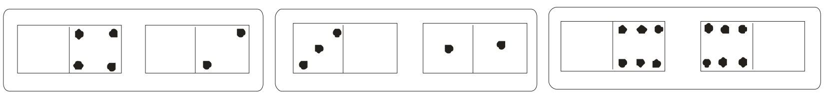
a) 

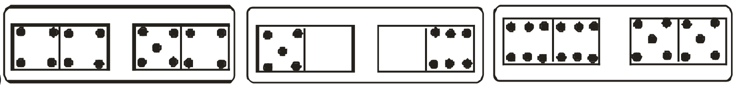
b) 

Answer: a) $\frac{1}{2};\frac{4}{5};\frac{2}{4};\frac{5}{6};\frac{3}{5};\frac{1}{4}$; b) 0,6; 0,8; 0,7; 1; 0,9; 0,4.

Problem 8. Find the sum of the fractional numbers, which correspond to the dominoes below:

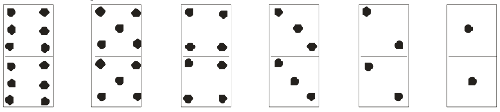
a) 

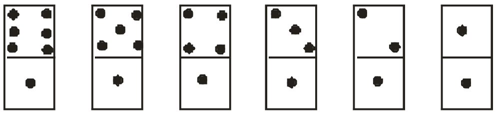
b) 

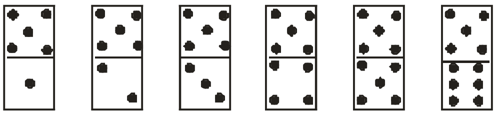
c) 

Answer: a) $\frac{6}{5};\frac{1}{2};2\frac{1}{3};7\frac{3}{20}$; b) 0,6; 4,1; 6,6; c) 9,8; 5,6; 12,1.

Problem 9. The sum of the fractional numbers, which correspond to the given dominoes is equal to:

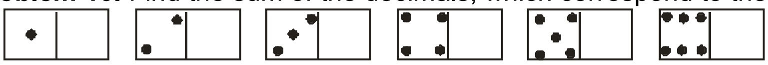
a) 

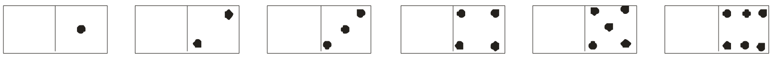
b) 

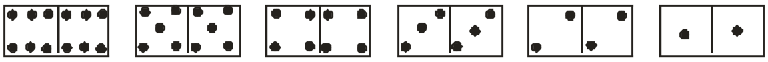
c) 

Answer: a) 6; b) 21; c) $12\frac{1}{4}$.

Problem 10. Find the sum of the decimals, which correspond to the dominoes:

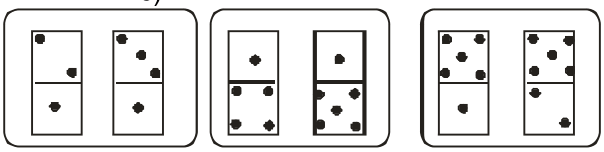
a) 

b) 

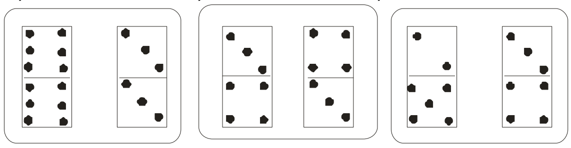
c) 

Answer: a) 21; b) 2,1; c) 23,1.

Problem 11. Compare the common fractions, which correspond to the dominoes: a) b) c)

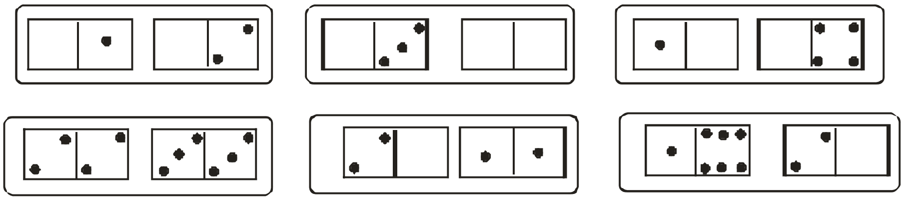


d) e) f)



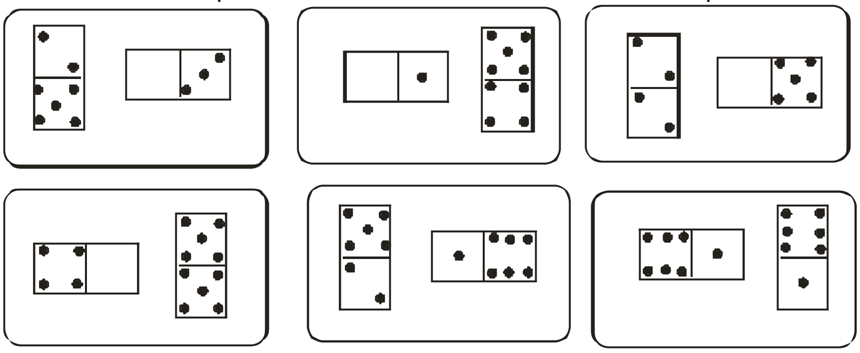
Answer: a) 2 < 3; b) $\frac{1}{4} > \frac{1}{5}$; c) $5 > \frac{5}{2}$; d) 1=1; e) $\frac{3}{4} < \frac{4}{3}$; f) $\frac{2}{5} < \frac{3}{4}$.

Problem 12. Compare the common fractions, which correspond to the dominoes.



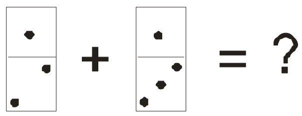
Answer: 0,1 < 0,2; 0,3 > 0; 1 > 0,4; 2,2 < 3,3; 2 > 1,1; 1,6 < 2.

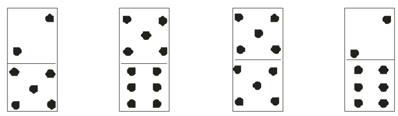
Problem 13. Compare the fractional numbers, which correspond to the dominoes.



Answer: $\frac{2}{5} > 0,3$; $0,1 < \frac{5}{4}$; 1 > 0,5; 4 > 1; $\frac{5}{2} > 1,6$; 6,1 > 6.

Problem 14. By which of the listed dominoes should be replaced the question mark in order to obtain an exact equation?

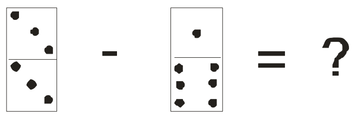


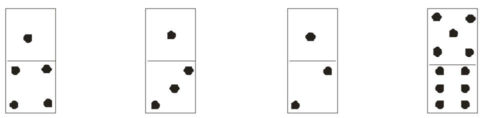


A) B) C) D)

Answer: B).

Problem 15. By which of the listed dominoes should be replaced the question mark in order to obtain an exact equation?

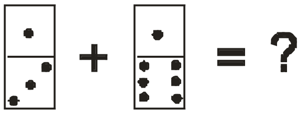




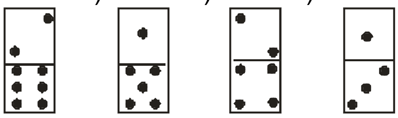
A) B) C) D)

Answer: D).

Problem 16. By which of the listed dominoes should be replaced the question mark in order to obtain an exact equation?



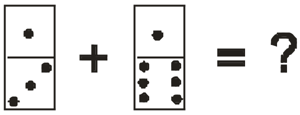
A) B) C) D)



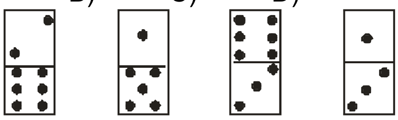
Solution: Add the two common fractions with different denominators $\left(\frac{1}{3}+\frac{1}{6}\right)$ and get $\frac{3}{6}$. Represent the obtained fraction using an irreducible fraction $\left(\frac{3}{6}=\frac{1}{2}\right)$. Then, compare $\frac{1}{2}$ with the common fractions, which correspond to the given dominoes. The conclusion is that $\frac{1}{2}=\frac{2}{4}$.

Answer: C).

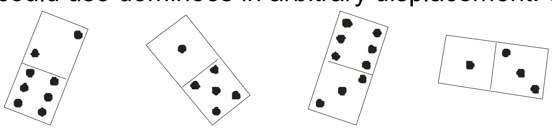
Problem 17. By which of the listed dominoes should be replaced the question mark in order to obtain an exact equation?



A) B) C) D)



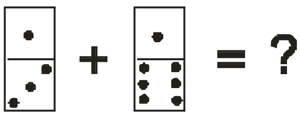
Solution: It is essential in the present problem to discover a domino by which $\frac{1}{2}$ could be represented. Such is the domino in C), but it should be turned up-down. (As an intermediate stage one could use dominoes in arbitrary displacement. See the example.):



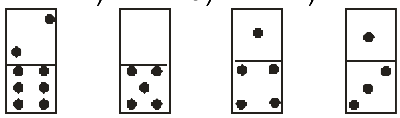
A) B) C) D)

Answer: C)

Problem 18. By which of the listed dominoes should be replaced the question mark in order to obtain an exact equation?

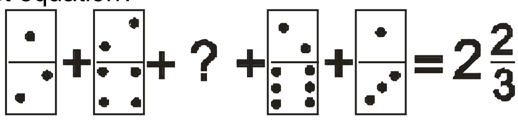


A) B) C) D)

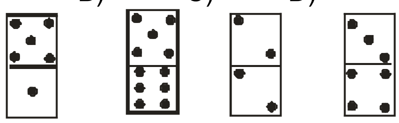


Answer: B) (this domino is used as a model of the decimal 0,5).

Problem 19. By which of the listed dominoes should be replaced the question mark in order to obtain an exact equation?



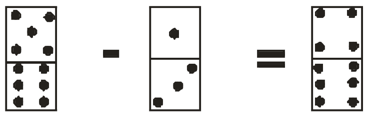
A) B) C) D)



Solution: It is convenient here to require an oral solution. In such a way pupils are directed to look for rational ways of getting the correct answer. In the concrete case one should realize that $\frac{1}{2}+\frac{2}{4}=1$ and $\frac{2}{6}+\frac{1}{3}=\frac{2}{3}$. We get $1\frac{2}{3}$ at the left hand side of the equality and $2\frac{2}{3}$ at the right hand one. Consequently, the question mark should be replaced by a fractional number, which is equal to 1.

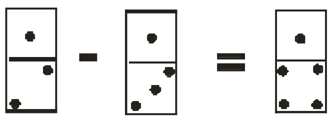
Answer: C).

Problem 20. Replace one of the dominoes by another one in order to obtain an exact equation.



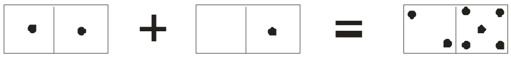
Answer: one of the possibilities is to replace $\frac{5}{6}$ by $\frac{1}{1}$, $\frac{2}{2}$, ... , $\frac{6}{6}$ or by 1,0; a second possibility is to replace $\frac{1}{3}$ by $\frac{1}{6}$; a third possibility is to replace $\frac{4}{6}$ by $\frac{3}{6}$, $\frac{1}{2}$, $\frac{2}{4}$ or by 0,5.

Problem 21. Replace one of the dominoes by another one in order to obtain an exact equation.



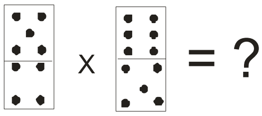
Answer: $\frac{1}{4}$ is replaced by $\frac{1}{6}$ or $\frac{1}{3}$ is replaced by $\frac{1}{4}$.

Problem 22. Replace one of the dominoes by another one in order to obtain an exact equation.

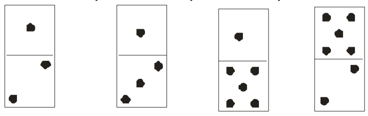


Answer: 2,5 is replaced by 1,2 or by $\frac{6}{5}$; also, it possible to replace 0,1 by 1,4 or 1,1 by 2,4.

Problem 23. By which of the given dominoes should be replaced the question mark in order to obtain an exact equation?



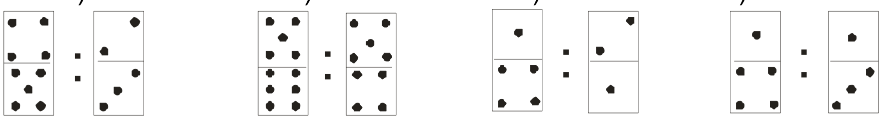
A) B) C) D)



Answer: C) (this domino is used as a model of the decimal 1,5).

Problem 24. Which of the quotients could not be represented by a domino?

A) B) C) D)



Answer: C).

The list of the proposed problems could be extended. The aim of the present paper is to turn the attention to the basic ideas, which are connected with teaching and learning of the topic “Fractional Numbers”. As mentioned at the beginning, dominoes could be used in the solutions of other mathematical problems. Some of their various applications will be considered in another paper.

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