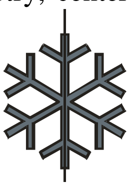
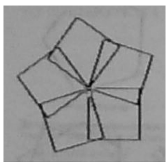
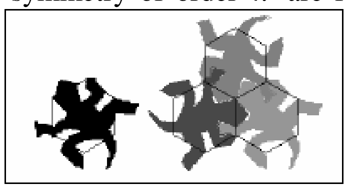
# FORMATION OF AN IDEA OF A SYMMETRIC SOLID

**Sava Grozdev, Toni Chehlarova**

**Abstract**

A model is proposed in the paper to form an idea of a symmetric solid. Cube constructions and rotation solids are used for the purpose.

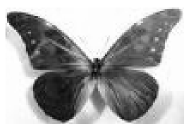
In School Mathematics Curriculum congruences of figures are studied including ax and central symmetries. Using various approaches in some textbooks and handbooks [3], [4], [5] notions of plane figures with ax of symmetry, center of symmetry or rotational symmetry of order $k$ are formed

too.

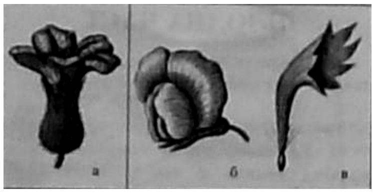
In teaching plane figures with ax of symmetry more frequent examples in the textbooks look like the following:





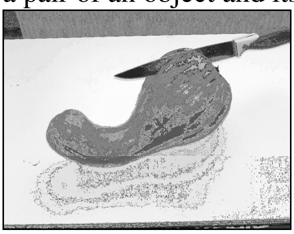
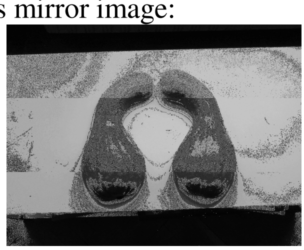

In fact, all these are plane images of solids. Many objects in the surrounding world have plane of symmetry. To illustrate some of them it is suitable to use 2D images but there are cases in which 2D images do not give sufficient information. Sophisticated spatial imagination is necessary in conscious understanding of next objects that are used in Chemistry, Biology (DNA molecules), Arts Education and others:


In the Bulgarian textbook in Biology for $6^{th}$ grade (Kozhuharov, S. and al., Biology for $6^{th}$ grade, Sofia, Syela, 2003) the figure below illustrates the following text: “According to the symmetry (the one to one position of the color parts) the colors are regular or irregular. The regular ones are symmetric radially while the irregular are two-side symmetric or non-symmetric.

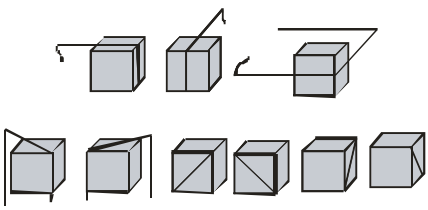


Spatial congruences of solids are not included in School Mathematics Curriculum. For this reason the authors of the present note have proposed a model in [2] for the formation of an idea of 3D congruence which is convenient for out-of class Mathematics education. In the sequel a model for the formation of an idea of symmetric solids is proposed. Cube constructions, and rotation solids are used for the purpose, as well as other 3D models, which are based on software programming from http://www.elica.net.

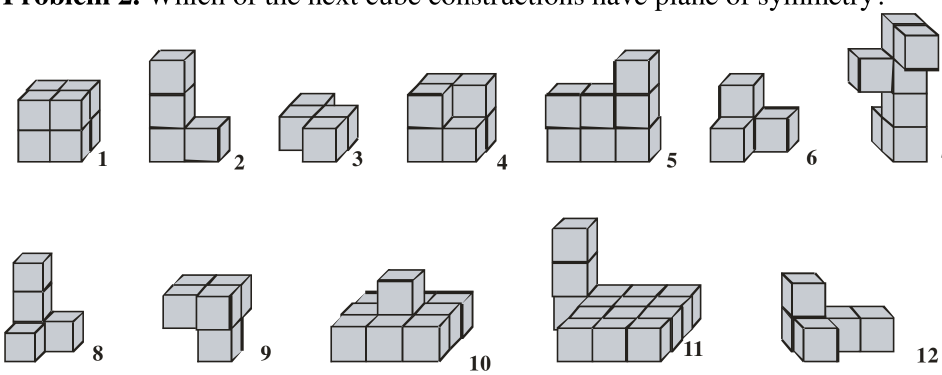
Cutting a suitable object is a possible instrument to illustrate a solid with a plane of symmetry. After the cutting two parts are obtained which are considered as a pair of an object and its mirror image:

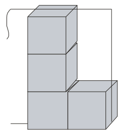

**Problem 1.** Find the number of the planes of symmetry of the cube.
*Answer:* 9.



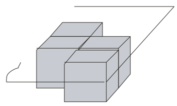
**Problem 2.** Which of the next cube constructions have plane of symmetry?



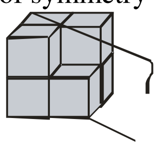
*Solution:* Construction 1 has nine planes of symmetry (the figure is a cube). Note that one of the dimensions of construction 2 is equal to 1. In similar cases when at least one of the dimensions of a cube construction is equal to 1, then always a plane of symmetry exists which divides the construction into two symmetric parts with dimension 0,5 each.



The situation with constructions 3 and 5 is analogous.

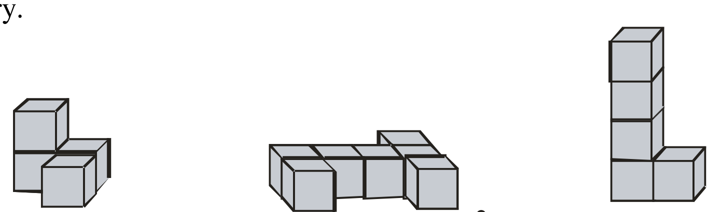


The next picture shows the plane of symmetry of construction 4:



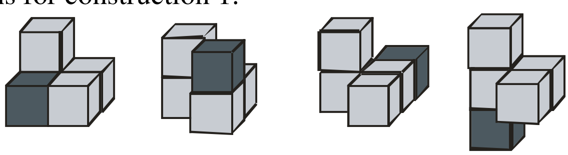
The planes of symmetry of constructions 6, 8, 9, 10 and 12 look similarly. Note that 12 and 8 can take one and the same position in the space while construction 10 has four planes of symmetry. Constructions 7 and 11 have no planes of symmetry.

**Problem 3.** Add one unit cube to obtain a construction with a plane of symmetry.

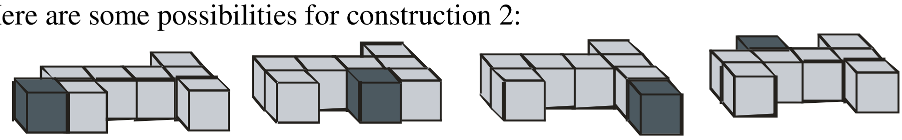


1 2 3

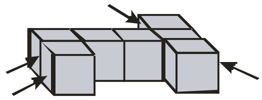
*Solution:* The added unit cubes are colored in black in the figures below. There are four solutions for construction 1:



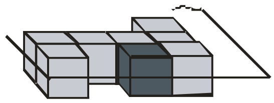
Here are some possibilities for construction 2:



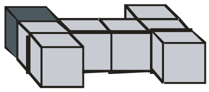
In fact, the new unit cube could stick on 13 sides in a way that one of the dimensions of the obtained construction remains equal to 1.



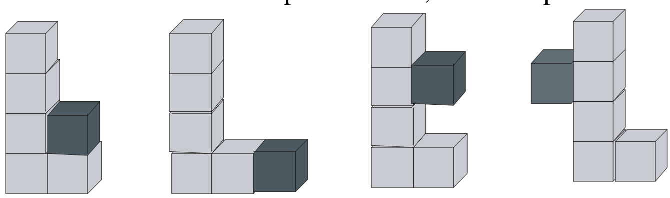
In this case one of the dimensions of the construction is equal to 1 and a plane of symmetry exists.



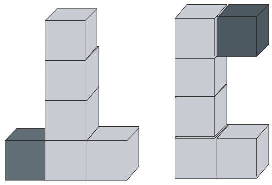
There is a case when the obtained construction has three planes of symmetry:



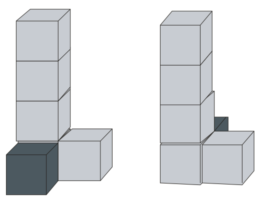
For construction 3 there are several possibilities, for example:



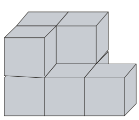
In some cases the planes of symmetry are two:



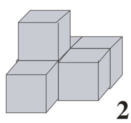
There are two possibilities when the obtained construction has no dimension equal to 1:



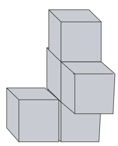
**Problem 4.** Eliminate one unit cube to obtain a construction with a plane of symmetry.



**1**

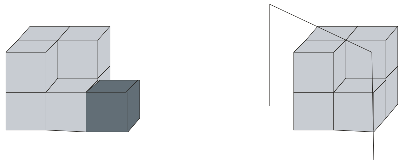


**2**

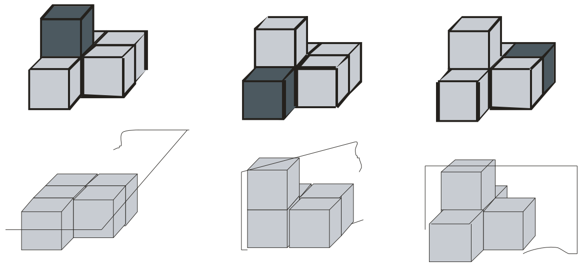


**3**

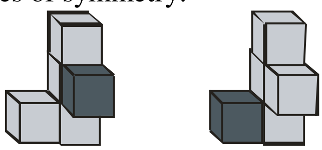
*Answer:* The unit cube which should be eliminated is in black in the figure below. There is one possibility for construction 1.



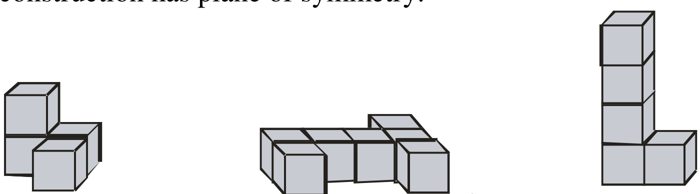
The possibilities for construction 2 are three.



For construction 3 there are two possibilities and each of the obtained constructions has two planes of symmetry.

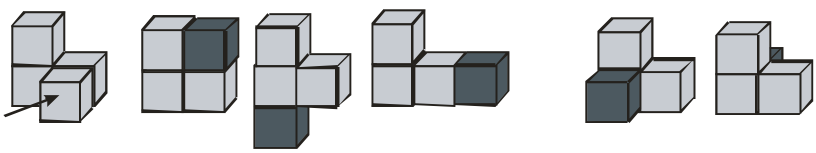


**Problem 5.** Change the position of one of the unit cubes in a way that the obtained construction has plane of symmetry.



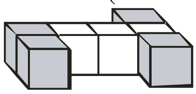
1 2 3

*Solution:* For the first figure the marked unit cube could be positioned to 9 places. There are several planes of symmetry in some of the cases. Two of the cases are when the obtained constructions have no dimension equal to 1.

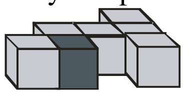


An important remark is that according to the definition of a cube construction which is used in the present note each unit cube has at least one common side with another unit cube in the construction (an exception is the unit cube when regarded as an individual construction). That is why for each of two unit cubes there is only one possibility for displacement and the obtained constructions have no plane of symmetry.

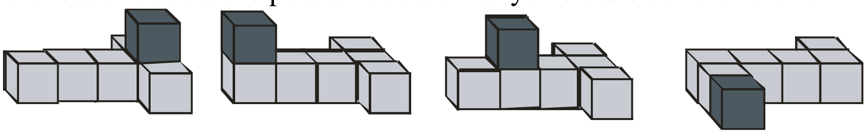
Three of the unit cubes in construction 2 (in white) could not be displaced:



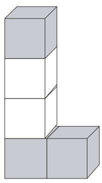
For one of the unit cubes there is only one possibility for displacement:



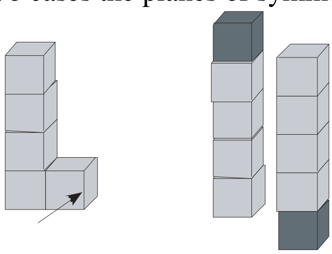
For the rest unit cubes the possibilities are many. Here are some of them:



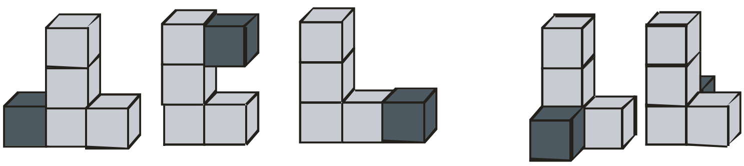
In construction 3 two of the unit cubes could not be displaced:



If the noted unit cube sticks to any other side then a construction is obtained with a plane of symmetry. In two cases the planes of symmetry are 5.

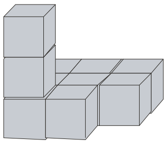


Also, for the most upper unit cube the possibilities are many. What are shown are those with more than one plane of symmetry and those without dimension equal to 1.



For the most right unit cube in construction 3 there is only one possibility for displacement.

**Problem 6.** Find the minimal number of unit cubes which should be eliminated in a way that the obtained construction has plane of symmetry.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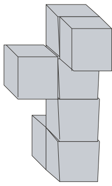
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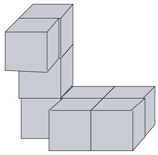
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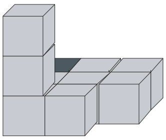
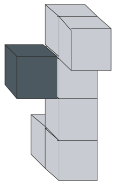
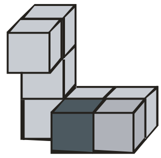
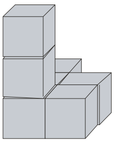
*Answer:* One unit cube for all the three constructions:







**Problem 7.** Find the minimal number of unit cubes which should be added in a way that the obtained construction has plane of symmetry.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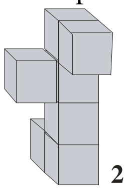
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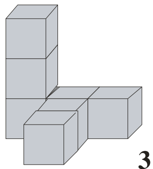
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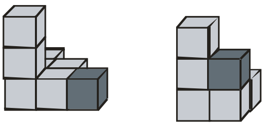
*Answer*: For constructions 1 and 2 – one unit cube, for construction 3 – two unit cubes: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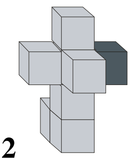
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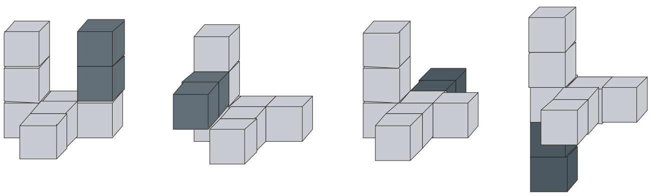
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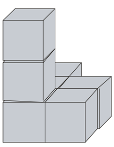
**Problem 8.** Find the minimal number of unit cubes which should be displaced in a way that the obtained construction has plane of symmetry.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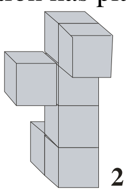
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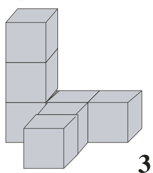
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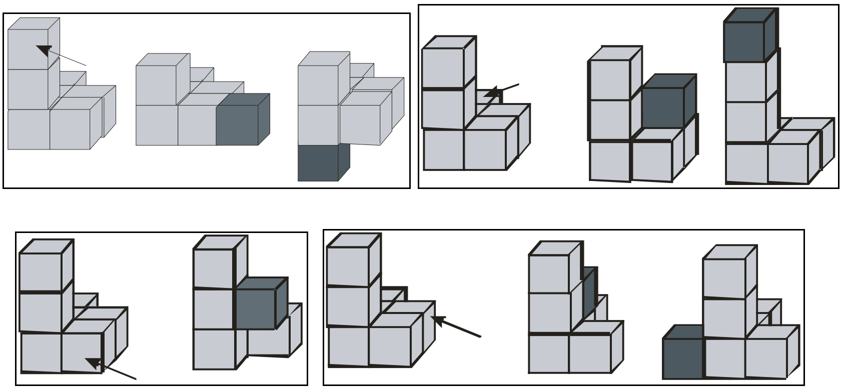
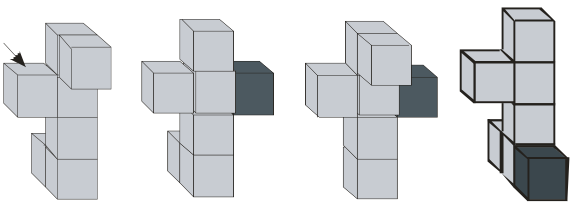
*Answer*: One unit cube for the three constructions. A grouping is proposed accounting for the displaced unit cube:



1

If the noted unit cube is eliminated from construction 2, the obtained construction has a dimension equal to 1. Thus, after displacing this unit cube in a way that the obtained construction has a dimension equal to 1, then a plane of symmetry is created. There are 12 possibilities for this. Some additional solutions are connected with the displacement of the noted unit cube and other unit cubes:



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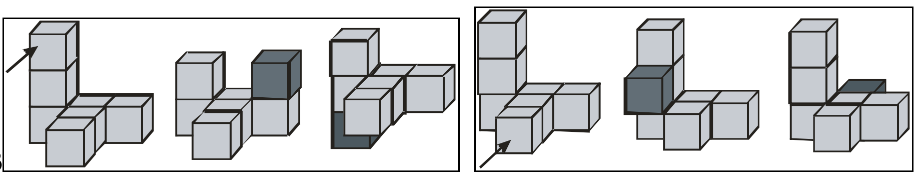


3

**Problem 9.** How many planes of symmetry has a regular $n-$:

a) prism if $n=3$; $n=4$; $n=5$; $n=6$; $n=8$;

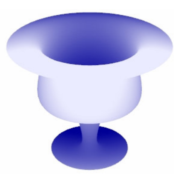
б) pyramid if $n=3$; $n=4$; $n=5$; $n=6$; $n=8$?

**Problem 10.** How many planes of symmetry has:

a) the right circular cylinder; b) the right circular cone;

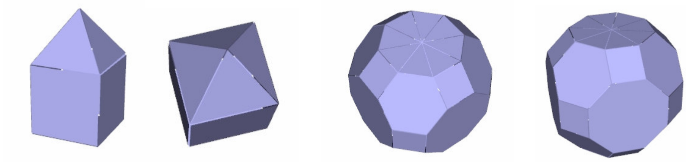
c) the right circular truncated cone; d) the ball;

e) the rotational solid of a figure?

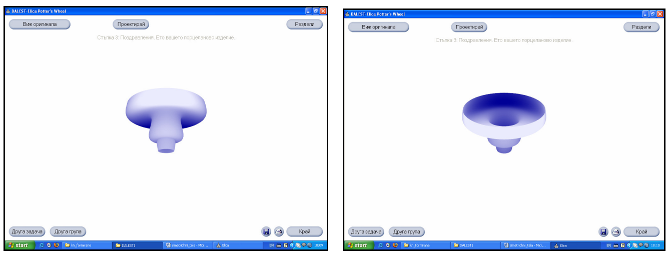


**Problem 11.** How many planes of symmetry has:

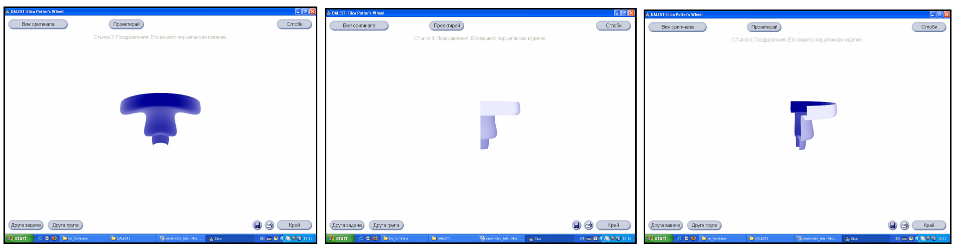
a) the octahedron; b) the dodecahedron; c) the „house” (two views); d) the solid.



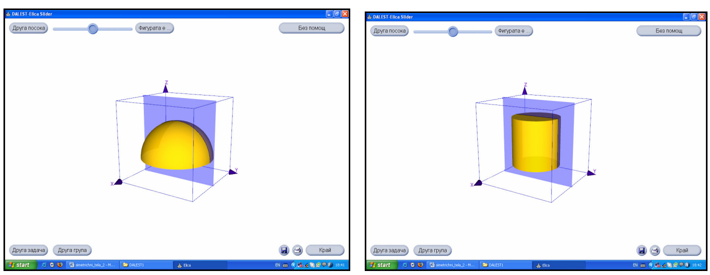
Two situations of each of the last two solids are shown. If necessary the software ***Elica Dalest Potters Wheel*** could be used. The main aim of using this software is to create rotational solids and to observe cross-sections, which makes suitable to carry out demonstrations. Here are two views of a rotational solid:

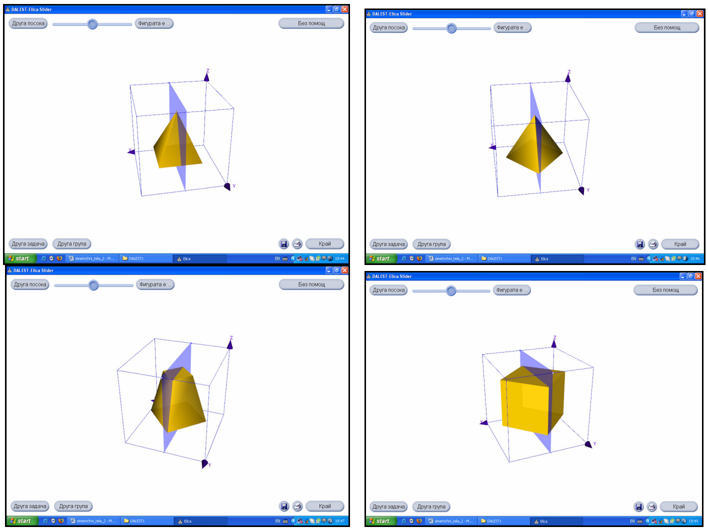


By the push-button *disjoint* a cross-section is presented, which could be rotated and observed from different directions.

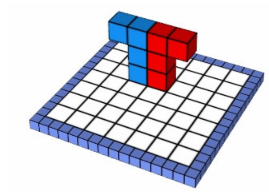


A specific application of the software ***Elica Dalest Slider*** to illustrations of some figures and their planes of symmetry is presented too.

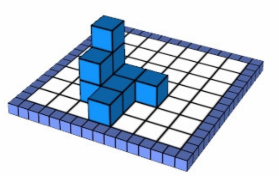




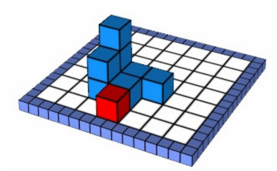
For formulation or problem solving like the cases of the problems from 2 to 8 it is suitable to use the software ***Elica Dalest Cubix editor***. Sometimes the plane of symmetry does not affect any unit cube in the construction. To illustrate the solution in such cases different coloring could be used concerning the two parts into which the construction is divided.



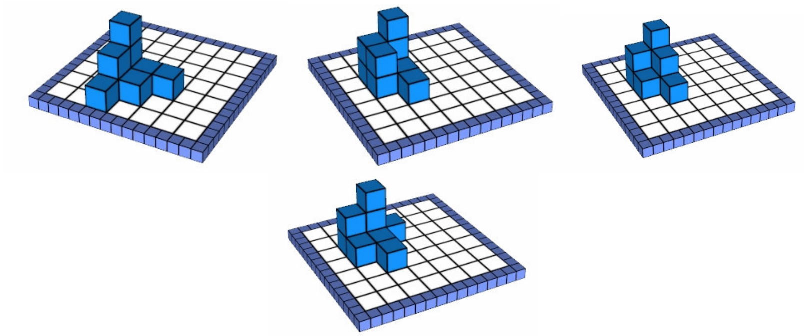
The next problem and its solution is presented by ***Elica Dalest Cubix editor***. One of unit cubes should be displaced in such a way that the obtained construction has plane of symmetry.



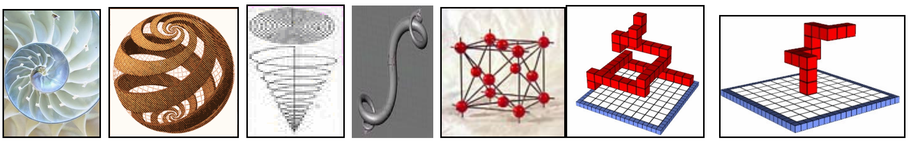
The unit cube which is to be moved is colored differently. In such a way the remaining part of the construction is distinguished and in fact the problem is reduced to the adding of a unit cube to the intermediate construction which is obtained after the elimination of the red unit cube.



The possible solutions are:



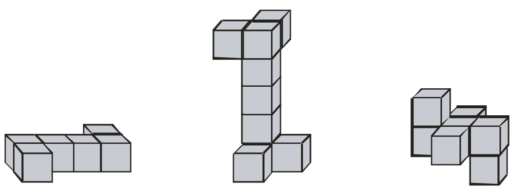
**Problem 12.** Which of the solids below have plane of symetry?



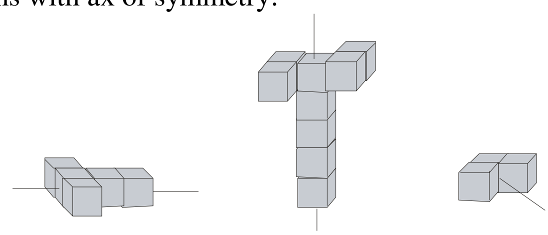
Note that harmony in Nature is realized not only by symmetries. Spirals, fractals, etc. are not less harmonic!

Students should be informed that ax symmetries are not the only symmetries in the space. Center symmetries exist as well. Correspondingly, solids with ax symmetry and solids with center symmetry exist. Of course, the aim here is not to classify 3D congruences, which a subject of the Higher school. It is sufficient to point out several examples which make conscious the idea and the analogy with the plane. For instance:

- Constructions with center of symmetry:



- Constructions with ax of symmetry:



What is suitable in addition is the examination of regular and semi regular solids for the existence of center of symmetry, for the number of the planes or the axes of symmetry, etc.

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