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RISK MEASURES FOR CLASSICAL AND PERTURBED RISK PROCESSES – A SURVEY

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In this review paper we consider several risk measures in actuarial mathematics, such as the ruin probability, the ruin time, the severity of ruin, the surplus immediately before ruin, and the Gerber-Shiu penalty function as a generalization of these measures. We discuss results on these measures for classical and perturbed classical risk processes.

1. Introduction and preliminary facts for the classical risk model

The classical risk model was introduced by Filip Lundberg [30] in 1903 in his doctoral thesis and was further developed and extended by Harald Cramer([5],[6]). In this section we give an introduction to the model and some of its basic properties. For more details see e.g. [22] or [37].

The capital of an insurance company at time $t \geq 0$ is given by

$$U_t = u + ct - \sum_{k=1}^{N_t} X_k.$$

Here $u \geq 0$ is the initial capital, $c \geq 0$ is a constant premium per unit time, X_k is the amount of the kth claim size, and $N(t)$ is a homogeneous Poisson process

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with intensity λ, representing the point process of claim arrivals. Moreover, $\{X_1, X_2, \ldots\}$ is a sequence of i.i.d. random variables, having distribution function F, such that $F(0) = 0$ and with finite mean μ. We suppose that $\{X_n, n = 1, 2, \ldots\}$ is independent of N. The process $U = \{U_t, t \geq 0\}$ is known as the classical risk process.

The safety loading of the company is defined by $\theta = \dfrac{c - \lambda\mu}{\lambda\mu}$. Let us denote by T the ruin time of U:

$$T = \inf\{t \geq 0 : U(t) < 0\}.$$

Then $\psi(u)$, the ruin probability, is defined by

$$\psi(u) = P(T < \infty | U(0) = u),$$

and we shall denote by $\Phi(u) = 1 - \psi(u)$ the survival probability of the risk process U.

Many works are devoted to the study of ruin probabilities as a risk measure. Using random walk theory, it can be proved that if $\theta \leq 0$, then $\psi(u) = 1$ for all $u \geq 0$, while for $\theta > 0$ it holds that $\psi(u) < 1$ for all initial capitals $u \geq 0$. Therefore, in all models in the rest of the paper it is assumed that $\theta > 0$. Other important risk measures are the severity of ruin $|U(T)|$, and the surplus inmediately before ruin $U(T-)$.

Lundberg ([31],[32],[33]) obtained several important properties of the ruin probabilities of the risk process. He proved that under the condition $\theta > 0, \psi(u)$ satisfies the following defective renewal equation:

$$c\psi(u) = \lambda \left(\int_0^u \psi(u - x)\overline{F}(x)\, dx + \int_u^\infty \overline{F}(x)\, dx \right),\ u \geq 0, \tag{1}$$

where $\overline{F} = 1 - F(x)$ is the tail of F. From the above equation it can be deduced that $\psi(0) = \lambda\mu/c$, but in general there are no explicit expressions for $\psi(u)$ for $u > 0$. In order to study further properties of the ruin probability, Lundberg supposed that there exists a positive solution R of the following equation:

$$\lambda(\widehat{m}_X(s) - 1) - cs = 0,$$

where $\widehat{m}_X$ is the moment generating function of F. This equation is called the fundamental Cramer-Lundberg equation and its positive solution R, when it exists, is named the Cramer-Lundsberg adjustment coefficient. It can be proved that when R exists, it is unique, and in this case the claim size distribution F possesses light tails:

$$\overline{F}(x) = 1 - F(x) \leq c\exp\{-ax\},\ x \geq 0,$$

for some fixed constants $a, c > 0$. Lundberg proved the following inequality and asymptotic representation of the ruin probability $\psi(u)$ in terms of the adjustment coefficient R:

$$\psi(u) \leq \exp\{-Ru\},\ u \geq 0,$$

and

$$\psi(u) \sim C\exp\{-Ru\}, u \to \infty.$$

Notice that from the above results, the adjustment coefficient R can be considered as a measure of the insolvency of the company, since maximizing R means minimizing the ruin probability. Several works in risk theory adress this problem (see e.g. [41], [25], [43] and the references therein).

From equation (1) it is possible to deduce the Pollaczek-Khinchine formula:

$$\psi(u) = 1 - \sum_{j=0}^{\infty} \frac{\theta}{1+\theta}\left(\frac{\theta}{1+\theta}\right)^j F_I^{*j}(u),\ u \geq 0, \tag{2}$$

where

$$F_I(x) = (1/\mu)\int_0^x (1 - F(y))\,dy$$

is the integrated tail distribution of F and $*$ denotes convolution of distribution functions. Let $\widehat{G} = \int_0^\infty e^{-sx} dG(x)$ be the Laplace transform of the distribution function G, and let $\widehat{l_f} = \int_0^\infty e^{-sx} f(x)\,dx$ be the Laplace transform of a nonnegative measurable function f.

From (2), it follows that the Laplace transform for the ruin probability of the classical risk process is given by

$$\widehat{l_\psi}(s) = \frac{1}{s} - \frac{c - \lambda\mu}{cs - \lambda(1 - \hat{F}(s))},\ s \geq 0. \tag{3}$$

By inverting the Laplace transform (3) one can calculate explicitly the ruin probability in some important cases, such as the case when the claim distribution is of phase-type or a combination of exponential distributions. From (2) it also follows that for the survival probability $\Phi(u)$ we have

$$\Phi(u) = \frac{\lambda\mu}{c}\sum_{j=0}^{\infty}\left(1 - \frac{\lambda\mu}{c}\right)^j F_I^{*j}(u),$$

hence $\Phi(u)$ can be represented as a compound geometric distribution with parameters $(\lambda\mu/c, F_I)$. This allows to use Panjer's formula in order to obtain numerical approximations of Φ.

Other risk measures such as the joint distribution of the ruin time, the severity of ruin and the surplus just before ruin, for classical and form more general risk process, have been investigated by several authors (see e.g. [1], [7], [10], [11], [17], [19], [27] and the references therein).

When the severity of ruin is not large, it is natural to suppose that the insurance company will not stop its activities, hoping that it will recover in the future. Several risk measures, such as the duration of the negative surplus, distribution of the negative epochs, and recovery costs has been proposed and studied for these cases, see e.g. [4], [8], [24] and [36].

2. Perturbed risk processes and the expected discounted penalty Gerber-Shiu function

In order to model fluctuations in the classical risk model, such as the number of customers, investment of the surplus, or variations of premiums and claims, several risk models with perturbations have been considered in the literature. Gerber [15] introduced the classical risk process perturbed by an independent Brownian motion:

$$X(t) = U(t) + \sqrt{2D}W(t). \tag{4}$$

Here $W(t)$ is a standart Brownian motion which is independent of $U(t)$, $D > 0$ is a constant, and the safety loading coefficient θ is positive. By conditioning on the time of the first claim, Dufresne and Gerber [9] obtained the following result for the ruin probability of this model:

Theorem 1. *For the model* (4) *the ruin probability has the following series expansion:*

$$\psi(u) \quad = \quad 1 - \sum_{j=0}^{\infty} \frac{\theta}{1+\theta}\left(\frac{\theta}{1+\theta}\right)^j P^{*(j+1)} * F_I^{*j}(u), \tag{5}$$

where

$$P(x) = 1 - e^{-(c/D)x}.$$

This formula is a generalization of the Pollaczek-Khinchine formula (2) for ruin probabilities of the classical risk model.

In many insurance models the financial data show large oscillations, hence Gaussian perturbations as above are inadequate. In this cases non-Gaussian stable perturbations were proposed because they allow for large fluctuations, are flexible and admit analytical treatment. Stable processes can be considered also as weak approximations of classical risk processes when the claim variance is infinite (see Theorem 1 in [13]).

We recall some basic definitions and properties of the stable distributions and processes. More information can be found in [3], [38] or [39].

Definition. *A random variable X with distribution function F is called stable if it is stable under convolutions: for all $c_1 > 0, c_2 > 0$ and any independent random variables X_1, X_2 with common distribution F, there exist constants $c_3 > 0$, c_4, and a random variable X_3 with distribution F, such that*

$$c_1X_1 + c_2X_2 \stackrel{D}{=} c_3X_3 + c_4,$$

where $\stackrel{D}{=}$ denotes equality in law. When $c_4 = 0$, the variable X is called strictly stable.

Explicit formulas for the densities of stable distributions exist only for $\alpha = 1/2$, 1, and 2, however their characteristic functions and Laplace transforms are well known:

Lemma 2. *A random variable X has stable distribution if its characteristic function is of the form*

$$E[e^{iXs}] = \exp\left\{i\mu s - \sigma|s|^{\alpha}(1 - i\beta \mathrm{sgn}(s)z(s,\alpha))\right\},$$

where μ is a real constant, $\sigma > 0, \alpha \in (0,2], \beta \in [-1,1]$, and

$$z(s,\alpha) = \begin{cases} \tan(\pi\alpha/2) & \text{if} \quad \alpha \neq 1, \\ -2/\pi \ln|s| & \text{if} \quad \alpha = 1. \end{cases}$$

In terms of the four parameters given above, the Laplace transform of X is given by

$$Ee^{-sX} = \begin{cases} \exp\{(-s\mu - \mathrm{sign}(1-\alpha)\sigma^{\alpha}s^{\alpha}\} & \text{if} \quad \alpha \neq 1, \\ \exp\{\sigma^{\alpha}(-s\mu\sigma^{-\alpha} + s\log s)\} & \text{if} \quad \alpha = 1. \end{cases}$$

We see from the above lemma that stable distributions depend on four parameters, and we shall write $X \overset{D}{=} S_\alpha(\sigma,\beta,\mu)$. When $\beta = 1$, there are no negative jumps of X. The case $\alpha = 2$ gives a normal distribution. For $\alpha < 2$ the absolute moments of X of order r are finite if and only if $r < \alpha$, and the distribution tails decay as a power functions of α, hence they are heavy-tailed.

Definition. *For $0 < \alpha \le 2$, a stochastic process $\{W_\alpha(t), t \ge 0\}$ is called a standard α–stable Lévy process if*

(1) *$W_\alpha(0) = 0$.*

(2) *W_α has independent increments.*

(3) *For $0 \le s < t$, $W_\alpha(t) - W_\alpha(s) \overset{D}{=} S_\alpha((t-s)^{1/\alpha}, \beta, 0)$, where $\beta \in [-1, 1]$.*

Furrer [12] introduced the stable perturbation of the classical risk process:

$$X_\alpha(t) = U(t) - \eta W_\alpha(t), \quad 1 < \alpha < 2, \quad \eta \ge 0, \quad t \ge 0, \tag{6}$$

where U is the classical risk process, W is a standart α-stable process without negative jumps and independent of U, and $\theta > 0$. Due to $\alpha > 1$ the first moments of $X_\alpha(t)$ are finite for all $t \ge 0$, however the variance of $X(t), t \ge 0$ is infinite.

Furrer obtained the following result for the ruin probability of X_α.

Theorem 3. *The ruin probability for the model* (6) *is given by the series expression*

$$\psi(u) \quad = \quad 1 - \sum_{j=1}^{\infty} \frac{\theta}{1+\theta}\left(\frac{\theta}{1+\theta}\right)^j G^{*(j+1)} * F_I^{*j}(u), \tag{7}$$

where

$$G(x) = \sum_{i=0}^{\infty} \frac{(-c/\eta^\alpha)^i}{\Gamma(1+(\alpha-1)i)} x^{(\alpha-1)i}$$

is the Mittag-Leffler function. Moreover, the Laplace transform of ψ is given by the formula

$$\widehat{l}_\psi(s) \quad = \quad \frac{1}{s} - \frac{c - \lambda\mu}{s(c + s^{\alpha-1}\eta^\alpha - \lambda\mu\widehat{F}_I(s)}. \tag{8}$$

Substituting $\alpha = 2$ into (7), the formula (5) of Dufresne and Gerber is obtained. Furrer proved his result by using a result of Zolotarev [45] on the Laplace transform of hitting times of stable processes with drift.

Initially interested in pricing American put options in finance, Gerber and Shiu [20], [21] proposed the following general risk measure for risk processes X :

$$\varphi(u) = E\left[e^{-\delta T}\, w(X(T-), |X(T)|) I_{(T<\infty)} \,\middle|\, X(0) = u\right], \quad u \geq 0.$$

Here T is the ruin time, $\delta \geq 0$ is a constant, and $w(x,y) : R_+ \times R_+ \to R_+$ is a given non-negative function. This risk measure is called the expected discounted penalty Gerber-Shiu function. The parameter δ can be interpreted as a discounting factor at time 0, and w as a penalty function at ruin time. In the particular case of $\delta = 0$, the ruin probability of X is obtained when $w(x,y) \equiv 1$, and when $w(x,y) = 1_{\{x\leq a, y\leq b\}}(x,y)$, one gets the joint distribution of the capital immediately before ruin and the severity at the ruin time.

Gerber and Shiu obtained the following result.

Theorem 4. *For the classical risk process U, the Gerber-Shiu penalty function satisfies the following defective renewal equation:*

$$\begin{aligned} \varphi(u) &= \frac{\lambda}{c}\int_0^u \varphi(u-x)\int_x^\infty e^{-\rho(y-x)} dF(y)\, dx \\ &+ \frac{\lambda}{c} e^{\rho u}\int_u^\infty e^{-\rho x}\int_x^\infty w(x, y-x)\, dF(y)\, dx, \end{aligned} \tag{9}$$

where $\rho = \rho(\delta)$ is the unique non-negative solution of the generalized Lundberg's equation

$$c\rho - \delta = \lambda - \lambda\int_0^\infty e^{-\rho y} dF(y), \quad \rho(0) = 0.$$

Since equation (9) can be written in the convolution form $\phi = \phi * g + h$, where $g(x) = \dfrac{c}{\lambda}\displaystyle\int_0^\infty e^{-\rho z} dF(x+z)dz$, $h(x) = \dfrac{c}{\lambda}\displaystyle\int_x^\infty\int_0^\infty e^{-\rho(u-x)} w(u,y)dydu$, by applying Laplace transforms we obtain the Laplace transform of φ :

$$\widehat{\varphi}(s) = \frac{\widehat{h}(s)}{1-\widehat{g}(s)}. \tag{10}$$

For the classical risk process perturbed by independent Brownian motion, Tsai and Willmot [42] considered a more general penalty function defined by

$$\varphi_D(u) = w_0 \varphi_d(u) + \varphi_c(u),\ u \geq 0,$$

where

$$\varphi_c(u) = E\left[e^{-\delta T} w(X(T-), |X(T)|) I_{(T<\infty, X(T)<0)} \Big| U(0) = u\right],$$

$$\varphi_d(u) = E\left[e^{-\delta T} I_{(T<\infty, X(T)=0)} \Big| X(0) = u\right], \quad w_0 = w(0,0).$$

Here $\varphi_c(u)$ can be interpreted as the penalty function in the case of ruin caused by jump, while $\varphi_d(u)$ is the penalty function when ruin is due to oscillations of the Brownian motion near 0.

Tsai and Willmot proved the following result.

Theorem 5. *For model (4) with $D > 0$, if $\lim_{u\to\infty} e^{-\rho_D u}\varphi_c(u) = 0$ and $\lim_{u\to\infty} e^{-\rho_D u}\varphi_c'(u) = 0$, then the generalized penalty Gerber-Shiu function $\varphi_D(u)$ satisfies the defective renewal equation*

$$\varphi_D(u) = \int_0^u \varphi_D(u-y) g_D(y)\, dy + w_0 e^{-bu} + H_\omega(u),$$

where

$$g_D(y) = \frac{\lambda}{c}\int_0^y e^{-b(y-s)} \int_s^\infty e^{-\rho(x-s)} dF(x)\, ds,$$

$$w(x) = \int_x^\infty w(x, y-x)\, dF(y), \quad b = \rho + c/D,$$

$$H_\omega(u) = \frac{\lambda}{D}\int_0^u e^{-b(u-s)} \int_s^\infty e^{-\rho(x-s)} w(x)\, dx\, ds,$$

and $\rho(\delta)$ is the unique non-negative solution of the equation

$$cs + Ds^2 + \lambda\int_0^\infty e^{-sy} dF(y) = \lambda + \delta. \tag{11}$$

The Laplace transforms of φ_D, g_D and H_ω are given by the expressions

$$\begin{aligned} \widehat{\varphi}_D(s) &= \frac{\hat{H}_\omega(s)}{1-\widehat{g}_D(s)} + \frac{w_0}{(\frac{c}{D}+\rho+s)[1-\widehat{g}_D(s)]}, \quad s \geq 0, \\ \widehat{g}_D(s) &= \frac{\lambda[\widehat{F}(s) - \widehat{F}(\rho)]}{D(\rho - s)(\frac{c}{D}+\rho+s)}, \end{aligned} \tag{12}$$

where

$$\widehat{H}_\omega(s) \quad = \quad \frac{\lambda[\widehat{\omega}(s)-\widehat{\omega}(\rho)]}{D(\rho-s)(\frac{c}{D}+\rho+s)}, \qquad s\geq 0,$$

and

$$\widehat{\omega}(s) \quad = \quad \int_0^\infty e^{-sx}\int_x^\infty w(x,y-x)\,dF(y)\,dx.$$

Moreover, when $D\to 0$ the corresponding Gerber-Shiu function $\varphi_D(u)$ converges to the Gerber-Shiu function $\varphi(u)$ of the classical risk process.

Using the renewal theorem, Tsai and Willmot also obtained the asymptotic formula

$$\phi_D(u)\sim C\exp\{-ku\} \text{ as } u\to\infty,$$

where $C>0$ is a given constant and $-k$ is the unique negative solution of Equation (11). Sarcar and Sen [40] proved Theorem 5 using weak approximations of the perturbed risk process with classical risk processes, under more general conditions on w, namely $|w(x,y)-w_0|\leq a(x+y)^r$, $x,y\geq 0$, for fixed constants $a>0$, $r>1$.

Kolkovska [26] investigated the Laplace transform of the expected discounted Gerber–Shiu penalty function for the classical risk model, perturbed by an independent α–stable process with $1<\alpha\leq 2$.

Kolkovska proved the following theorem.

Theorem 6. *Assume that the claim distribution F is continuous. Under some general conditions on $w(x,y)$ the Laplace transform of the expected discounted Gerber-Shiu penalty function for the perturbed risk process (6), with $1<\alpha<2$ and $\beta=1$, is given by*

$$\widehat{\varphi}(s) \quad = \quad \frac{\lambda(\widehat{\omega}(s)-\widehat{\omega}(\rho))+n(s,\rho)\eta^\alpha}{c(\rho-s)-\lambda(\widehat{F}(s)-\widehat{F}(\rho))-(s^\alpha-\rho^\alpha)\eta^\alpha}, \tag{13}$$

where

$$n(s,\rho)=c_\alpha\int_0^\infty(e^{-su}-e^{-\rho u})\int_u^\infty w(u,z-u)z^{-(\alpha+1)}\,dz\,du,$$

ρ is the unique non-negative solution of the generalized Lundberg equation

$$\lambda+\delta=cs+\lambda\widehat{F}(s)+s^\alpha\eta^\alpha$$

and c_α is a given constant depending on α.

One can verify that the result (7) of Furrer on the ruin probability, the formula (12) of Tsai and Willmott when $\alpha = 2, w_0 = 0$, and the classical result (3) of Gerber and Shiu for the Cramer-Lundberg process can be obtained as particular cases of Theorem 6.

The proof of Theorem 6 is obtained in several steps. The first step is the construction of a sequence of approximating classical risk processes which converges weakly to the perturbed risk process X_α. The next step consists in proving that the corresponding expected discounted penalty Gerber-Shiu functions converge, and the final step is the calculation of the limit. The case $1 < \alpha < 2$ is more complicated than the case $\alpha = 2$ due to the fact that α–stable processes are pure jump processes, with no explicit formulas for their transition densities. Hence, an approach using conditioning on the first claim time cannot be applied directly.

Gerber-Shiu penalty functions for classical risk processes, and for more general risk models with dividends or with barrier and threshold strategies, have been studied by several authors (see e.g. [16], [18], [20], [28], [29] and the references therein).

The importance of the expected Gerber-Shiu penalty function in insurance mathematics has been growing rapidly in the past years. There have been three recent international workshops dedicated to investigations in this field. Ruin probabilities and expected Gerber -Shiu penalty functions for the more general Lévy risk processes, which are processes with independent and stationary increments and no positive jumps, have been studied intensively in the last years (see e.g. [2], [14], [23], [34], [35] and the references therein). In these cases it is difficult to obtain renewal equations for the Gerber-Shiu penalty function, due to the infinite number of jumps in finite time of these processes, and the scale function, which was introduced in risk theory by Zhou [44], is an important tool in the investigations. However, in most cases there are still no explicit results in terms of the parameters of the risk models, and much work remains to be done.

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