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PROGRAMME PACKAGES FOR IMPLEMENTATION OF MODIFICATIONS OF BLACK-SCHOLES MODEL AND WEB APPLICATIONS

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Abstract. In this paper we propose new modules in programming environment MATHEMATICA for the generalizations of Black-Scholes (BS) model taking into account the market price and coefficient of variation. First we derive generalization of Black-Scholes PDE and present its explicit solution. Then we derive the Garman-Kohlhagen's model as generalization of BS model. The proposed modules gives the possibility for visualization and hypersensitive analysis.

1. Introduction

A longstanding problem in finance was valuation of option contracts. An option is a contract that allows the holder to buy or sell financial asset at a fixed price in the future. Is there a relationship between the price of the underlying asset, on one hand, and an option contract written on this asset? This problem was solved by F.Black and M.Scholes in 1973 [1].

Let us consider a stochastic model for the evolution of the price of the underlying asset:

$$\frac{dS_t}{S_t} = \sigma dY_t + \beta dt, \tag{1}$$

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where Y_t is a Brownian motion, S is the price of the underlying asset and σ, β represent respectively the volatility and mean of the returns for investing in the stock. This model is just the "continuous-time" version of the "stochastic returns" model [1].

More generalized form of (1) is the case of coupled stochastic differential equations (SDEs), where the volatility σ is written as the square root of a variance ν:

$$dS = Sbdt + S\sqrt{\nu}dY_1$$

The variance ν is constant in the original Black-Scholes model. Let us now assume that it follows its own SDE in the form

$$d\nu = (\omega - \Theta\nu)dt + \varepsilon\nu^{\gamma}dY_2$$

This representation models mean-reversion in the volatility or variance and is known as Heston model.

We shall be purposely vague about how σ and β are determined for now. Let us denote by r the prevailing short-term interest rate. Let us assume that the value V_t of a call on the stock is given by:

$$V_t = C(S_t, t), \tag{2}$$

where $C(S, t)$ is a smooth function of S and t. Let an investor sells one call option and buys Δ shares of the underlying asset at time t. The change in the value of his holdings over the interval $(t, t + dt)$ is

$$\begin{aligned}(-V_{t+dt} + \Delta S_{t+dt}) - (-V_t + \Delta S_t) = \\ -dV_t + \Delta dS_t.\end{aligned} \tag{3}$$

Combining (1) and (2) and applying Ito's formula [2,3], we can express the variation of the portfolio in terms of the variation of the price of the underlying asset,

$$\begin{aligned} dV_t &= C_S(S_t, t)dS_t + C_t(S_t, t)dt + \\ &+ \frac{1}{2}\sigma^2 S^2 C_{SS}(S_t, t)dt, \end{aligned} \tag{4}$$

to leading order in dt. Substituting this equation into (3) we obtain the following equation for the change in the portfolio value

$$(5) \qquad \begin{aligned}(-C_S(S_t,t)+\Delta)dS_t-(C_t(S_t,t)+ \\ +\frac{1}{2}\sigma^2 S_t^2 C_{SS}(S_t,t))dt.\end{aligned}$$

If the number of shares held in the portfolio was

$$\Delta = C_S(S_t,t)$$

then the dx_t term would vanish in equation (5), rending the return of the portfolio non-volatile over the period of time $(t, t+dt)$ (to leading order in dt).

Since the value of the option-stock portfolio at time t is $-C(S_t,t)+\Delta S_t = -C(S_t,t)+S_t C_S(S_t,t)$, the absence of arbitrage implies that

$$C_t + \frac{\sigma^2}{2} S^2 C_{SS} = r(C - SC_S)$$

or

$$(6) \qquad C_t + \frac{\sigma^2}{2} S^2 C_{SS} + rSC_S - rC = 0.$$

This is the Black-Scholes PDE. To determine the function $C(S,T)$ we must specify boundary conditions. In the case of a call with expiration date T we have:

$$(7) \qquad C(S,T) = (S-X)^+ = max(S - X_0),$$

where X is the strike price (z^+ represents the positive part of z). Indeed, if $S_T \le X$, the option is worthless, and if $S_T > X$, the holder of the call can buy the underlaying asset for X dolars and sell it at market price, making a profit of $S_T - X$.

For a European-style call (which can be exercised only at the date T), $C(S,t)$ is determined by solving the Cauchy problem for the Black-Scholes PDE with condition (7). To value American options, the idea is that we should look for a function $C(S,t)$ that satisfies the Black-Scholes equation in the regions of the (S,t)-plane where option should not be exercised and provide additional boundary conditions along the the region corresponding to price levels where the option should be exercised. One way to arrive at this region is to impose the additional condition on option prices that should hold in the case of American-style options:

$$C(S,t) \geq (S-X)^{+} \ \text{(calls)},$$
$$P(S,t) \geq (X-S)^{+} \ \text{(puts)},$$

since the option is worth as least as much as what would get exercising it immediately. This constraints give rise to an obstacle problem, or differential inequality, for the Black-Scholes equation which can be solved numerically.

The Black-Scholes PDE has a fundamental probabilistic interpretation. The correspondence between PDEs and probabilities via the Fokker-Plank formalism yields

$$C(S_t,t) = \mathbf{E}\{\sum_{i:t<T_i} e^{-r(T_i-t)} F(S_{T_i}|I_t)\},$$

where $\mathbf{E}\{.|I_t\}$ represents the conditional expectation, $T_1 < T_2 < \ldots < T_N$ are different dates for the series of cash-flows represented by $F_i(S_{T_i})$, $i = 1,2,\ldots,N$. S_t is the diffusion process governed by the stochastic differential equation

$$\frac{dS_t}{S_t} = \sigma dY_t + r dt.$$

In the case of call-option, the explicit solution of Black-Scholes equation (6) is:

$$C_0 = S_0 N(d_1) - X e^{-rT} N(d_2), \tag{8}$$

where

$$\begin{aligned} d_1 &= \frac{\ln\left(\frac{S_0}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}, \\ d_2 &= d_1 - \sigma\sqrt{T}, \end{aligned} \tag{9}$$

C_0- value of of a call on the stock; S_0- evolution of the price of the underlying asset; $N(d)$ - standard normal commutative distribution; X - strike price; r - interest rate; T- expiration date (in years); σ - volatility.

In the case of put-option we have the following solution:

$$P_0 = C_0 + X e^{-rT} - S_0. \tag{10}$$

We want to pint out that the original formula (1)-(3)can be used in practice for the "current date" - t in the following way:

$$C_0 = S_0 N(d_1) - Xe^{-r(T-t)} N(d_2), \tag{11}$$

where

$$\begin{aligned} d_1 &= \frac{\ln\left(\frac{S_0}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}, \\ d_2 &= d_1 - \sigma\sqrt{T-t}, \end{aligned} \tag{12}$$

$$\begin{aligned} P_0 &= C_0 + Xe^{-r(T-t)} - S_0 \\ &= S_0 N(d_1) - Xe^{-r(T-t)} N(d_2) + Xe^{-r(T-t)} - S_0 \\ &= Xe^{-r(T-t)}\left(1 - N(d_2)\right) - S_0\left(1 - N(d_1)\right) \\ &= Xe^{-r(T-t)} N(-d_2) - S_0 N(-d_1). \end{aligned} \tag{13}$$

In the field of applied financial mathematics the following coefficients are used in order to estimate market sensitivity of the options or other financial instruments (see [4]):

1. Coefficient *Delta*

$$\begin{aligned} \delta_{C_0} &= N(d_1); \\ \delta_{P_0} &= N(d_1) - 1; \end{aligned} \tag{14}$$

2.Coefficient *Gamma*

$$\begin{aligned} \gamma_{C_0} &= \frac{N'(d_1)}{S_0 \sigma \sqrt{T}}; \\ \gamma_{P_0} &= \frac{N'(d_1)}{S_0 \sigma \sqrt{T}}, \end{aligned} \tag{15}$$

where

$$N'(d_1) = \frac{e^{-\frac{d_1^2}{2}}}{\sqrt{2\pi}};$$

3. Coefficient *Vega*

$$
(16) \qquad \begin{aligned} v_{C_0} &= S_0 N'(d_1)\sqrt{T}; \\ v_{P_0} &= S_0 N'(d_1)\sqrt{T}; \end{aligned}
$$

4. Coefficient *Theta*

$$
(17) \qquad \begin{aligned} \theta_{C_0} &= -\frac{S_0 N'(d_1)\sigma}{2\sqrt{T}} - rXe^{-rT}N(d_2); \\ \theta_{P_0} &= -\frac{S_0 N'(d_1)\sigma}{2\sqrt{T}} + rXe^{-rT}N(-d_2); \end{aligned}
$$

5. Coefficient *Rho*

$$
(18) \qquad \begin{aligned} \rho_{C_0} &= XTe^{-rT}N(d_2); \\ \rho_{P_0} &= -XTe^{-rT}N(-d_2). \end{aligned}
$$

In the case of the "current date" - t coefficients for the market sensitivity are the following:

1'. Coefficient *Delta*

$$
(19) \qquad \begin{aligned} \delta_{C_0} &= N(d_1); \\ \delta_{P_0} &= N(d_1) - 1; \end{aligned}
$$

2'. Coefficient *Gamma*

$$
(20) \qquad \begin{aligned} \gamma_{C_0} &= \frac{N'(d_1)}{S_0\sigma\sqrt{T-t}}; \\ \gamma_{P_0} &= \frac{N'(d_1)}{S_0\sigma\sqrt{T-t}}, \end{aligned}
$$

where

$$
N'(d_1) = \frac{e^{-\frac{d_1^2}{2}}}{\sqrt{2\pi}};
$$

3'. Coefficient *Vega*

$$(21)\qquad \begin{aligned} v_{C_0} &= S_0 N'(d_1)\sqrt{T-t};\\ v_{P_0} &= S_0 N'(d_1)\sqrt{T-t}; \end{aligned}$$

4'. Coefficient *Theta*

$$(22)\qquad \begin{aligned} \theta_{C_0} &= -\frac{S_0 N'(d_1)\sigma}{2\sqrt{T-t}} - rXe^{-r(T-t)}N(d_2);\\ \theta_{P_0} &= -\frac{S_0 N'(d_1)\sigma}{2\sqrt{T-t}} + rXe^{-r(T-t)}N(-d_2); \end{aligned}$$

5'. Coefficient *Rho*

$$(23)\qquad \begin{aligned} \rho_{C_0} &= X(T-t)e^{-r(T-t)}N(d_2);\\ \rho_{P_0} &= -X(T-t)e^{-r(T-t)}N(-d_2). \end{aligned}$$

Remark. Black-Scholes model is approximately accurate due to some suggestions [5]. For this purpose, we are sure only for 4 parameters - C, X, T, r. Recall that the parameters that enter the Black-Scholes formula are (i) the exercise price, or strike price X, (ii) the expiration date T, (iii) the price of the underlying asset C, (iv) the interest rate r, (v) the volatility σ. Of these five parameters, the first four are observable at any given time . In contrast, the volatility of the underlying asset is not directly observable. For each value of the volatility we obtain a different theoretical option value. Consequently, it is easy to show that to each possible option value corresponds a unique volatility parameter. This a consequence of the fact that the Black-Scholes option premium is strictly increasing function of σ. The implied volatility of a traded call is, by definition, the value of σ that solves the equation

$$C(S, t; X, T, r, \sigma) = \text{market price of the call},$$

where the left-hand side represents the Black-Scholes theoretical value, with the same definition applying to puts.

Some modules in programming environment MATHEMATICA can be upgraded with the coefficients mentioned above.

Our aim in this paper is to develop user module in MATHEMATICA as an implication of the Black-Scholes model. This includes:

Parameters' input:

1. Parameter S_0 - current price of the underlying asset - 81;

2. Parameter X - strike price of the option - 80;

3. r - interest rate under the year basis of unrisky asset with maturity equal to the date of option validity - 0.06;

4. parameter T -expiration date (in years) - 0.166;

5. parameter σ - volatility of the underlying asset - 0.3;

Current parameter:

t - for the calculation of call and put options at time t_i, : $i = 1, 2, \ldots$

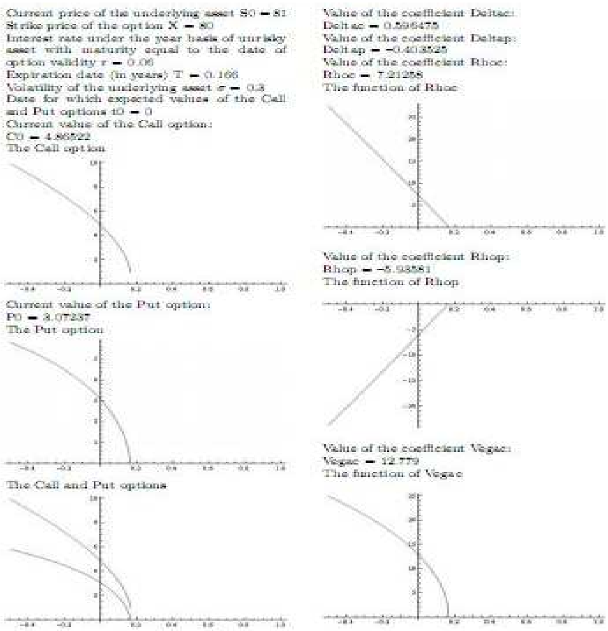
Procedures:

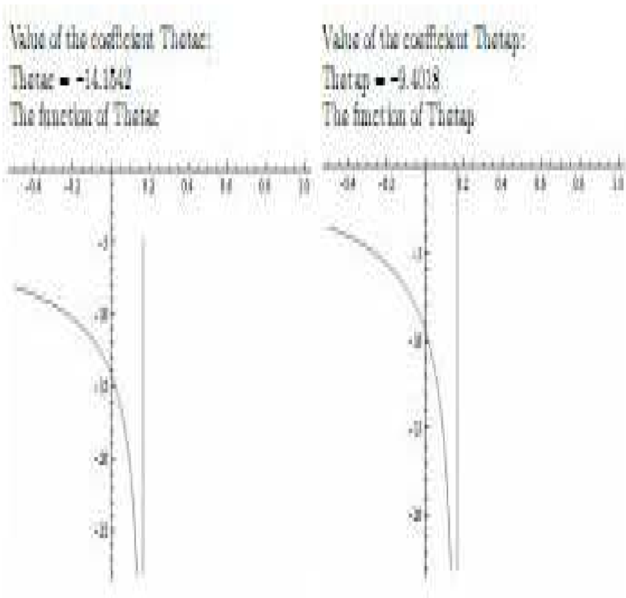
1. Repeatedly addressed the operator of programming environment MATHEMATICA for calculation of normal distribution at points;

2. Calculation of the values of call and put options and visualization;

3. Calculation of the coefficients of sensitivity: Deltac, Deltap, Rhoc, Rhop, Vegas, Thetac, Thetap and visualization.

Below the reader can see the test provided on our control example for Black-Scholes model, realized in software packages of the programs Excel, Matlab, Maple, etc. [10,11,12].





2. Modified Black Scholes equation with coefficient of market price of risk and coefficient of variation

Black Scholes (BS) model can be modified when we take into account the coefficients: λ - market price per standartized unit volatility,or, simply, the market price of risk; α- coefficient of variation.

Then the solution of the so modified BS model in order to find the call-options is the following:

$$C_M = S_M \mathcal{N}(d_1) - Ke^{-r(T-t)}\mathcal{N}(d_2), \tag{24}$$

where

$$\begin{aligned} d_1 &= \frac{ln(\frac{S_M}{K}) + (r + \frac{(\lambda.\alpha)^2}{2})(T-t)}{\lambda.\alpha\sqrt{T-t}}, \\ d_2 &= d_1 - \lambda.\alpha\sqrt{T-t}. \end{aligned} \tag{25}$$

Modulus I. The Black-Scholes model with market-price and ciefficient of variation.

The code is:

```
Print["The Black-Scholes model with market-price and coefficient of variation"];
S0 = Input["Input S0 - current price of the underlying asset"];(* 81 *)
Print["current price of the underlying asset S0=", S0];
X = Input["Input X - strike price of the option"];(* 80 *)
Print["strike price of the option X=", X];
r = Input["Input r - interest rate under the year basis of unrisky asset with maturity
equal to the date of option validity"];(* 0.06 *)
Print["interest rate under the year basis of unrisky asset with maturity equal to the
date of option validity r=", r];
T = Input["Input T - expiration date (in years)"];(* 0.166 *)
Print["expiration date (in years) T=", T];
λ = Input["Input λ - standardized unit volatility (the market price of risk)"];(* 0.9 *)
Print["standardized unit volatility (the market price of risk) λ=", λ];
α = Input["Input α - coefficient of variation"];(* 0.44 *)
Print["coefficient of variation α=", α];
t0 = Input["Input date for which expected value of call option"];(* 0 *)
Print["date for which expected value of call option t0=", t0];
```

$$d1 := \frac{\mathrm{Log}\left[\frac{S0}{X}\right] + \left(r + \frac{(\lambda*\alpha)\wedge 2}{2}\right)(T - t0)}{\lambda*\alpha\sqrt{T - t0}};$$

$$d1t := \frac{\mathrm{Log}\left[\frac{S0}{X}\right] + \left(r + \frac{(\lambda*\alpha)\wedge 2}{2}\right)(T - t)}{\lambda*\alpha\sqrt{T - t}};$$

$$d2 := d1 - \lambda*\alpha\sqrt{T - t0};$$

$$d2t := d1t - \lambda*\alpha\sqrt{T - t};$$

```
Print["current value of the call option: "];
```

$$\mathrm{Print}\left["C0 = ", S0 * \mathrm{CDF}[\mathrm{NormalDistribution}[], d1] - X * e^{-r(T-t0)} * \mathrm{CDF}[\mathrm{NormalDistribution}[], d2]\right];$$

```
Print["The call option"];
```

$$g1 = \mathrm{Plot}\left[S0 * \mathrm{CDF}[\mathrm{NormalDistribution}[], d1t] - X * e^{-r(T-t)} * \mathrm{CDF}[\mathrm{NormalDistribution}[], d2t], \{t, -0.5, 1\}, \mathrm{AxesOrigin} \to \{0, 0\}\right]$$

Remark. The above developed modulus in the program environment MATHEMATICA can be successfully upgraded if we include the mentioned coefficients – λ ГЁ α. Modulus I is modified based on the extended model (12), (13).

3. Another Moduli in Programming Environment MATHEMATICA

3.1. Modulus II. The Garman-Kohlhagen's model.

The Garman - Kohlhagen model: In 1983 Garman and Kohlhagen extended the BlackвЂ"Scholes model to cope with the presence of two interest rates (one for each currency). Let us suppose that r_d - is domestic risk free simple interest rate and r_f - is foreign risk free simple interest rate. Then the domestic currency value of a call option into the foreign currency is:

$$
\begin{aligned}
C_0 &= S_0 e^{-r_f(T-t)} N(d_1) - X e^{-r_d(T-t)} N(d_2) \\
&= e^{-r_d(T-t)} \left(S_0 e^{(r_d - r_f)(T-t)} N(d_1) - X N(d_2) \right)
\end{aligned} \tag{26}
$$

The value of a put option has value:

$$
\begin{aligned}
P_0 &= X e^{-r_d(T-t)} N(-d_2) - S_0 e^{-r_f(T-t)} N(-d_1) \\
&= e^{-r_d(T-t)} \left(X N(-d_2) - S_0 e^{(r_d - r_f)(T-t)} N(-d_1) \right)
\end{aligned} \tag{27}
$$

where

(28)

$$
d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + \left(r_d - r_f + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} = \frac{\ln\left(\frac{S_0 e^{(r_d - r_f)(T-t)}}{X}\right) + \frac{\sigma^2}{2}(T-t)}{\sigma\sqrt{T-t}},
$$

$$
d_2 = d_1 - \sigma\sqrt{T-t}
$$

S_0 - is the current spot rate;

$N(d)$ - is the cumulative normal distribution function;

X - is the strike price;

T - is the time to maturity;

σ - is the volatility.

The code is:

```
(* The Garman-Kohlhagen's Model*)
Print["The Garman-Kohlhagen's Model"];
S0 = Input["Input S0 - current price of the underlying asset"];(* 81 *)
Print["current price of the underlying asset S0=", S0];
X = Input["Input X - strike price of the option"];(* 80 *)
Print["strike price of the option X=", X];
rd = Input["Input rd - domestic risk free simple interest rate"];(* 0.06 *)
Print["domestic risk free simple interest rate rd=", rd];
rf = Input["Input rf - foreign risk free simple interest rate"];(* 0.05 *)
Print["foreign risk free simple interest rate rf=", rf];
T = Input["Input T - expiration date (in years)"];(* 0.166 *)
Print["expiration date (in years) T=", T];
σ = Input["Input σ - volatility of the underlying asset"];(* 0.3 *)
Print["volatility of the underlying asset σ=", σ];
t0 = Input["Input date for which expected values of call and put options"];(* 0.01 *)
Print["date for which expected values of call and put options t0=", t0];
```

$$d1 := \frac{\mathrm{Log}\left[\frac{S0 * e^{(rd-rf)(T-t0)}}{X}\right] + \left(\frac{\sigma^{\wedge}2}{2}\right)(T - t0)}{\sigma\sqrt{T - t0}};$$

$$d1t := \frac{\mathrm{Log}\left[\frac{S0 * e^{(rd-rf)(T-t)}}{X}\right] + \left(\frac{\sigma^{\wedge}2}{2}\right)(T - t)}{\sigma\sqrt{T - t}};$$

$$d2 := d1 - \sigma\sqrt{T - t0};$$

$$d2t := d1t - \sigma\sqrt{T - t};$$

```
Print["current value of the call-option: "];
```

$\mathrm{Print}\left["C0 = ", e^{-rd(T-t0)}\left(S0 * e^{(rd-rf)(T-t0)} * \mathrm{CDF}[\mathrm{NormalDistribution}[], d1] - X * \mathrm{CDF}[\mathrm{NormalDistribution}[], d2]\right)\right];$

```
Print["The call option"];
```

$g1 = \mathrm{Plot}\left[e^{-rd(T-t)}\left(S0 * e^{(rd-rf)(T-t)} * \mathrm{CDF}[\mathrm{NormalDistribution}[], d1t] - X * \mathrm{CDF}[\mathrm{NormalDistribution}[], d2t]\right),\right.$
$\left.\{t, -0.5, 1\}, \mathrm{AxesOrigin} \to \{0, 0\}\right]$

```
Print["current value of the put-option: "];
```

$\mathrm{Print}\left["P0 = ", e^{-rd(T-t0)}\left(X * \mathrm{CDF}[\mathrm{NormalDistribution}[], -d2] - S0 * e^{(rd-rf)(T-t0)} * \mathrm{CDF}[\mathrm{NormalDistribution}[], -d1]\right)\right];$

```
Print["The put option"];
```

$g2 = \mathrm{Plot}\left[e^{-rd(T-t)}\left(X * \mathrm{CDF}[\mathrm{NormalDistribution}[], -d2t] - S0 * e^{(rd-rf)(T-t)} * \mathrm{CDF}[\mathrm{NormalDistribution}[], -d1t]\right),\right.$
$\left.\{t, -0.5, 1\}, \mathrm{AxesOrigin} \to \{0, 0\}\right]$

```
Print["The call and put options"];
Show[g1, g2, AxesOrigin → {0, 0}]
```

3.2. Modulus III. Black-Scholes model with interest of dividents. By investments, the investors are trying to obtain income - divident or increase of the market price of the stocks.

Black-Scholes model can be modified in the case, when we take into consideration the coefficient q – interest of the divident in the period $[t + \Delta t)$. In this case the formulas for the calculation of call-put options can be modified in the following way:

$$C_0 = e^{-r(T-t)} \left(S_0 e^{(r-q)(T-t)} N(d_1) - X N(d_2) \right), \tag{29}$$

$$P_0 = e^{-r(T-t)} \left(X N(-d_2) - S_0 e^{(r-q)(T-t)} N(-d_1) \right), \tag{30}$$

where

$$\begin{aligned} d_1 &= \frac{\ln\left(\frac{S_0 e^{(r-q)(T-t)}}{X}\right) + \frac{\sigma^2}{2}(T-t)}{\sigma\sqrt{T-t}}, \\ d_2 &= d_1 - \sigma\sqrt{T-t}. \end{aligned} \tag{31}$$

The code is:

```
(* Black-Scholes model with interest of dividends*)
Print["Black-Scholes model with interest of dividends"];
S0 = Input["Input S0 - current price of underlying asset (assets)"];(* 81 *)
Print["current price of underlying asset (assets) S0=", S0];
X = Input["Input X - strike price of the option (exercising price)"];(* 80 *)
Print["strike price of the option (exercising price) X=", X];
r = Input["Input r - capitalized interest on the year based riskless asset"];(* 0.06 *)
Print["capitalized interest on the year based riskless asset r=", r];
T = Input["Introduce T - expiration date (in years)"];(* 0.166 *)
Print["date (in years) T=", T];
σ = Input["Input σ - volatility of the capitalized year norm of underlying asset return"];(* 0.3 *)
Print["standard volatility of the capitalized year norm of underlying asset return σ=", σ];
t0 = Input["Input ''todey's date'', for which we shall investigate the value of call and put option"];(* 0.01 *)
Print["''todey's date'', for which we shall investigate the value of call and put option t0=", t0];
q = Input["Input q - expected interest of a dividend"];(* 0.001 *)
Print["expected interest of a dividend q=", q];
```

$$d1 := \frac{\mathrm{Log}\left[\frac{S0 * e^{(r-q)\,(T-t0)}}{X}\right] + \left(\frac{\sigma\char`^2}{2}\right)(T - t0)}{\sigma\sqrt{T - t0}};$$

$$d1t := \frac{\mathrm{Log}\left[\frac{S0 * e^{(r-q)\,(T-t)}}{X}\right] + \left(\frac{\sigma\char`^2}{2}\right)(T - t)}{\sigma\sqrt{T - t}};$$

$$d2 := d1 - \sigma\sqrt{T - t0};$$

$$d2t := d1t - \sigma\sqrt{T - t};$$

```
Print["current value of the call-option: "];
```

$\mathrm{Print}\left["C0 = ", e^{-r\,(T-t0)}\left(S0 * e^{(r-q)\,(T-t0)} * \mathrm{CDF}[\mathrm{NormalDistribution}[], d1] - X * \mathrm{CDF}[\mathrm{NormalDistribution}[], d2]\right)\right];$

```
Print["The call-option"];
```

$g1 = \mathrm{Plot}\left[e^{-r\,(T-t)}\left(S0 * e^{(r-q)\,(T-t)} * \mathrm{CDF}[\mathrm{NormalDistribution}[], d1t] - X * \mathrm{CDF}[\mathrm{NormalDistribution}[], d2t]\right),\right.$
$\left.\{t, -0.5, 1\}, \mathrm{AxesOrigin} \rightarrow \{0, 0\}\right]$

```
Print["current value of the put-option: "];
```

$\mathrm{Print}\left["P0 = ", e^{-r\,(T-t0)}\left(X * \mathrm{CDF}[\mathrm{NormalDistribution}[], -d2] - S0 * e^{(r-q)\,(T-t0)} * \mathrm{CDF}[\mathrm{NormalDistribution}[], -d1]\right)\right];$

```
Print["The put-option"];
```

$g2 = \mathrm{Plot}\left[e^{-r\,(T-t)}\left(X * \mathrm{CDF}[\mathrm{NormalDistribution}[], -d2t] - S0 * e^{(r-q)\,(T-t)} * \mathrm{CDF}[\mathrm{NormalDistribution}[], -d1t]\right), \{t, -0.5, 1\},\right.$
$\left.\mathrm{AxesOrigin} \rightarrow \{0, 0\}\right]$

```
Print["The call and put options"];
Show[g1, g2, AxesOrigin → {0, 0}]
```

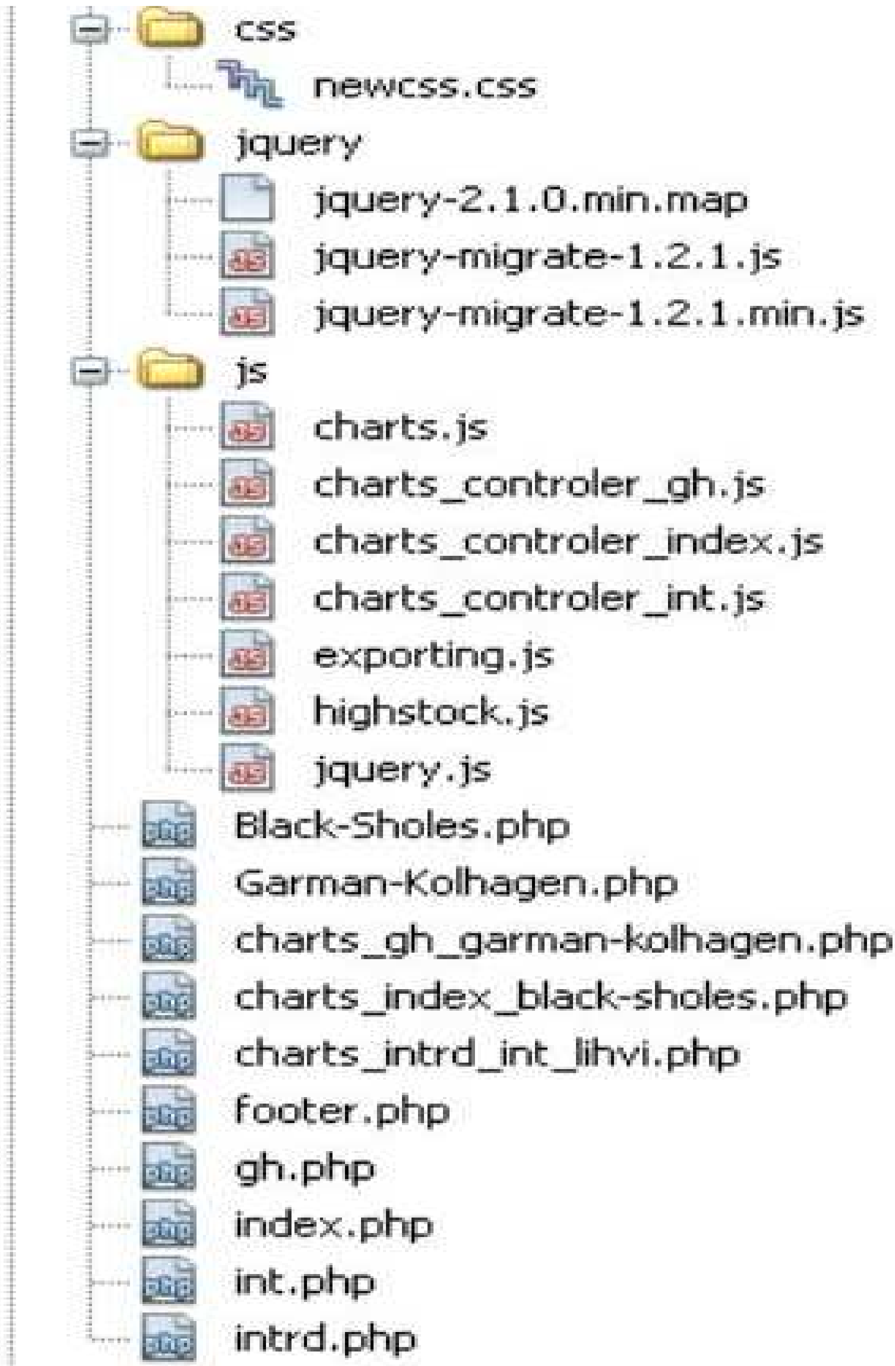
Remark. The same formula is used to price options on foreign rates, expect that now q plays the role of the foreign risk–free interest rate in **Garman–Kohlhagen's model**.

4. Conclusions.

Let us mention that in the field of industrial insurance, the Black Scholes model and its modifications have application in forming insurance bill of exchange, considered as put-option.

In financial crises the problem of imitation modeling, simulation and detailed investigation of the functions of call- and put-options is very modern. The modules presented in this paper (see also [8]) are component of web-based application, realized in the program environmental with central mathematical kernel and in some sense realizes the problem proposed above, and the build software instruments can be used for research investigations, as well as for training.

We want to point out that more general problem connected with the detailed investigation of the studied in the paper models such as models of reporting of return of discrete dividends with discrete taxes and other their modifications, construction of portfolios and minimizing of the risk, as objectives of other ours investigations and usage of software platform with provided access to WEB-based servers for scientific calculations, interval calculations and graphical design. We shall point out that the development of such application is based on the free software NetBeans with open code. Below is given the structure of the application.


css
newcss.css
jquery
jquery-2.1.0.min.map
jquery-migrate-1.2.1.js
jquery-migrate-1.2.1.min.js
js
charts.js
charts_controler_gh.js
charts_controler_index.js
charts_controler_int.js
exporting.js
highstock.js
jquery.js
Black-Sholes.php
Garman-Kolhagen.php
charts_gh_garman-kolhagen.php
charts_index_black-sholes.php
charts_intrd_int_lihvi.php
footer.php
gh.php
index.php
int.php
intrd.php

For other results, see [2–9].

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[11] $http://www.soarcorp.com/black_scholes_calculator.jsp.$

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