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A NEW APPROACH TO IMPROVE THE NUMERICAL PROCEDURE FOR 2D ANTI-PLANE CRACK PROBLEMS IN FUNCTIONALLY GRADED MAGNETO-ELECTRO-ELASTIC MATERIALS*

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Magneto-electro-elastic composite materials have extensive applications in modern smart structures, because they possess good coupling between mechanical, electrical and magnetic fields. This new effect was reported for the first time by Van Suchtelen [1] in 1972. Due to their ceramic structure cracks inevitably exist in these materials. If these cracks extend the material may lose its structural integrity and/or functional properties.

In this study we consider functionally graded magneto-electro-elastic materials (MEEM) subjected to anti-plane time-harmonic load. Our purpose is to evaluate the dependence of the stress concentration near the crack tips on the frequency of the applied external load. We use boundary integral equation method (BIEM) for the numerical solution.

For materials with complex geometry of cracks the numerical procedure becomes too cumbersome. To increase the speed of the computations we derive new fundamental solutions by the Fourier transform. The more simple form of these fundamental solutions leads to decreasing of the number of the numerical computations. Asymptotic for small arguments of the new solutions will be presented. The results can be used to improve the numerical procedure based on the BIEM for complex crack configurations in MEEM.

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1. Introduction

Magneto-electro-elastic composites are advanced materials that possess large magneto-electric effect. This effect doesn't exist in piezoelectric or piezomagnetic phase and has variety of applications in modern smart structures.

In this study we consider functionally graded magneto-electro-elastic materials (MEEM) subjected to anti-plane time-harmonic load. The purpose is to evaluate the dependence of the stress concentration near the crack tips on the frequency of the applied external load. The mathematical model is described by a boundary value problem for a system of partial differential equations. Following Wang and Zhang [2] for the piezoelectric case the boundary value problem is reduced to a system of integro-differential equations along the crack.

Fundamental solutions (or Green's functions) are one of the basic ingredients necessary to solve boundary value problems (BVP) by the boundary integral equation method (BIEM). A limited number of papers consider dynamic fundamental solutions suitable for BIEM implementation (see Dineva et al. [3]). In our work fundamental solutions were initially derived by the Radon transform. For complex configurations of cracks the complex form of the these solutions leads to decreasing of the speed of the numerical computations. We will present a new derivation of the fundamental solution using the Fourier transform and express them as linear combinations of Kelvin functions and natural logarithms. The more simple form of the fundamental solution will significantly improve the numerical realization of the solution of the BVP.

2. Statement of the problem

We consider transversely-isotropic functionally graded magnetoelectroelastic material in the coordinate system $Ox_1x_2x_3$, where Ox_3 is poling direction and symmetry axis and Ox_1x_2 is the isotropic plane. MEEM is subjected to anti-plane mechanical impact on the Ox_3 axis, and in-plane electrical and magnetic impacts in the plane Ox_1x_2. The constitutive equations for this type of medium (see Soh and Liu [4]) are:

$$\sigma_{iQ} = C_{iQJl} u_{J,l}, \tag{1}$$

where $x = (x_1, x_2)$, and $\Gamma = \Gamma^+ \bigcup \Gamma^-$ is an internal crack – an open arc. Here and in what follows comma denotes partial differentiation, it is assumed summation in repeating indexes and small indexes vary $1, 2$, while capital indexes vary $3, 4, 5$; u_J is the generalized displacement vector $u_J = (u_3, \phi, \varphi)$, where u_3 is out of plane elastic displacement, ϕ is the electric potential, φ is the magnetic potential; $\sigma_{iJ} = (\sigma_{i3}, D_i, B_i)$ is the generalized stress tensor, where σ_{iJ}

is the stress, D_i and B_i are the components of the electric and magnetic induction respectively along Ox_i axis; C_{iQJl} is the generalized elasticity tensor defined as: $C_{i33l}(x) = \begin{cases} c_{44}(x), i = l \\ 0, \quad i \neq l \end{cases}$, $C_{i34l}(x) = C_{i43l}(x) = \begin{cases} e_{15}(x), i = l \\ 0, \quad i \neq l \end{cases}$, $C_{i35l}(x) = C_{i53l}(x) = \begin{cases} q_{15}(x), i = l \\ 0, \quad i \neq l \end{cases}$, $C_{i44l}(x) = \begin{cases} -\varepsilon_{11}(x), i = l \\ 0, \qquad i \neq l \end{cases}$, $C_{i45l}(x) = C_{i54l}(x) = \begin{cases} -d_{11}(x), i = l \\ 0, \qquad i \neq l \end{cases}$, $C_{i55l}(x) = \begin{cases} -\mu_{11}(x), i = l \\ 0, \qquad i \neq l \end{cases}$. Functions $c_{44}(x)$, $e_{15}(x)$, $\varepsilon_{11}(x)$ are: elastic stiffness, piezoelectric coefficient and dielectric permittivity, while $q_{15}(x)$, $d_{11}(x)$, $\mu_{11}(x)$ are piezo-magnetic and magneto-elastic coefficients and magnetic permeability correspondingly.

Assuming quasi-static approximation in the Maxwell equation the governing equation in the frequency domain in absence of body force, electric charge and magnetic current is the following:

$$\sigma_{iQ,i} + \rho_{QJ}\omega^2 u_J = 0 \tag{2}$$

where $\rho_{QJ} = \begin{cases} \rho, Q = J = 3 \\ 0, Q, J = 4 \text{or } 5 \end{cases}$, $\rho(x)$ is the mass density, $\omega > 0$ is the frequency.

We assume that all material properties depend on x in one and the same way and describe this by an inhomogeneity function $h(x)$: $c_{44}(x) = h(x)c_{44}$, $e_{15}(x) = h(x)e_{15}$, $\varepsilon_{11}(x) = h(x)\varepsilon_{11}$, $q_{15}(x) = h(x)q_{15}$, $d_{11}(x) = h(x)d_{11}$, $\mu_{11}(x) = h(x)\mu_{11}$.

When the incident SH-wave interacts with the cracks a scattered wave is produced. The total displacement and traction at any point of the plane can be calculated by the superposition principle:

$$u_J = u_J^{in} + u_J^{sc}, t_J = t_J^{in} + t_J^{sc}, \tag{3}$$

where $t_J = \sigma_{iJ}n_i$ and $n_i = (n_1, n_2)$ is the outer normal vector.u_J^{in} and t_J^{in} are displacement and traction of the incident wave fields u_J^{sc} and t_J^{sc} are the scattered by the cracks wave fields.We impose the following boundary conditions: $t_J = 0$, $t_J^{in} = -t_J^{sc}$

$$t_J = 0 \quad \text{or} \quad t_J^{in} = -t_J^{sc}, \quad x \in \Gamma \tag{4}$$

$$u_J^{sc} \to 0 \quad \text{when} \quad (x_1^2 + x_2^2)^{1/2} \to \infty. \tag{5}$$

The boundary condition (4) means that the cracks are free of mechanical traction and also they are magnetoelectrically impermeable. Permeable cracks are also used in literature. Impermeable and permeable crack models are discussed in details in Dineva et al. [3]. We will reduce the boundary value problem (2), (4) and (5) to an equivalent system of integro-differential equations along the cracks and then solved this system numerically.

3. Boundary integral equation method

The fundamental solution u^*_{KM} of (2) is the solution of the equation:

$$\sigma_{iQ,i} + \rho_{QJ}\omega^2 u_J = \delta_{JM}\delta(x,\xi),$$

where δ_{JM} is the Kronecker symbol and $\delta(x,\xi)$ is the Dirak's delta function. Following [3], [5] the fundamental solution can be represented in the following way

$$u^*_{KM} = h^{-1/2} U^*_{KM},$$

where U^*_{KM} is solution of:

$$C_{iJKi}U^*_{KM,ii} + [\rho_{JK}\omega^2 - C_{iJKi}k_i^2]U^*_{KM} = h^{-1/2}(\xi)\delta_{JM}\delta(x,\xi). \tag{6}$$

Equation (6) is with constant coefficients if $p_{JK} = C_{iJKi}h^{-1/2}(h^{1/2})_{,ii} =$ const. When a system with constant coefficients is obtained we find the fundamental solutions in a closed form using direct and inverse Radon transform and calculus with generalized functions.

Following Wang and Zhang [2] and Gross et al. [6]the following representation formulae are valid:

$$\begin{aligned} t^{sc}_J(x,\omega) = C_{iJKl}(x)n_i(x)\int_{Cr}\{[\sigma^*_{\eta PK}(x,y,\omega)\triangle u_{P,\eta}(y,\omega) \\ -\rho_{QP}(y)\omega^2 u^*_{QK}(x,y,\omega)\triangle u_P(y)]\delta_{\lambda l} \\ -\sigma^*_{\lambda PK}(x,y,\omega)\triangle u_{P,l}(y,\omega)]n_\lambda(y)\}d\Gamma(y), \end{aligned} \tag{7}$$

where $\Delta u_J = u_J|_{Cr^+} - u_J|_{Cr^-}$ are jumps of the displacement along the crack or crack opening displacement (COD),$Cr = Cr_1 \bigcup Cr_2$, Cr^+ and Cr^- are the upper and lower bounds of the cracks correspondingly, u^*_{KM} is the fundamental solution and $\sigma^*_{iJK} = C_{iJMl}u^*_{KM,l}$,

Since the fundamental solution and incident wave field are known, using (7) we obtain integro-differential equation along the crack, where unknowns are COD:

$$\begin{aligned} t^{in}_J(x,\omega) = -C_{iJKl}(x)n_i(x)\int_{Cr}\{[\sigma^*_{\eta PK}(x,y,\omega)\triangle u_{P,\eta}(y,\omega) \\ -\rho_{QP}(y)\omega^2 u^*_{QK}(x,y,\omega)\triangle u_P(y)]\delta_{\lambda l} \\ -\sigma^*_{\lambda PK}(x,y,\omega)\triangle u_{P,l}(y,\omega)]n_\lambda(y)\}d\Gamma(y), x \in Cr. \end{aligned} \tag{8}$$

The equation (8) is solved numerically. Once COD are found we can calculate the scattered field at every point on the plane using (7).

4. New derivation of the fundamental solution

Here we will present new fundamental solutions for the case $h(x) = 1$, which corresponds to homogeneous material. The system that we have to solve has the following form:

$$
\begin{aligned}
c_{44}\Delta u^*_{33} + e_{15}\Delta u^*_{43} + q_{15}\Delta u^*_{53} + \rho\omega^2 u^*_{33} &= \delta(x,\xi) \\
c_{44}\Delta u^*_{34} + e_{15}\Delta u^*_{44} + q_{15}\Delta u^*_{54} + \rho\omega^2 u^*_{34} &= 0 \\
c_{44}\Delta u^*_{35} + e_{15}\Delta u^*_{45} + q_{15}\Delta u^*_{55} + \rho\omega^2 u^*_{35} &= 0 \\
e_{15}\Delta u^*_{33} - \varepsilon_{15}\Delta u^*_{43} - d_{11}\Delta u^*_{53} &= 0 \\
e_{15}\Delta u^*_{34} - \varepsilon_{15}\Delta u^*_{44} - d_{11}\Delta u^*_{54} &= \delta(x,\xi) \\
e_{15}\Delta u^*_{35} - \varepsilon_{15}\Delta u^*_{45} - d_{11}\Delta u^*_{55} &= 0 \\
q_{15}\Delta u^*_{33} - d_{11}\Delta u^*_{43} - \mu_{11}\Delta u^*_{53} &= 0 \\
q_{15}\Delta u^*_{34} - d_{11}\Delta u^*_{44} - \mu_{11}\Delta u^*_{54} &= 0 \\
q_{15}\Delta u^*_{35} - d_{11}\Delta u^*_{45} - \mu_{11}\Delta u^*_{55} &= \delta(x,\xi)
\end{aligned}
$$

where $\Delta = \dfrac{\partial^2}{\partial x_1^2} + \dfrac{\partial^2}{\partial x_2^2}$.

After linear transformation and introducing the coefficients:
$\tilde{c}_{44} = c_{44} + \dfrac{q_{15}^2}{\mu_{11}}, \tilde{e}_{15} = e_{15} - \dfrac{d_{11}q_{15}}{\mu_{11}}, \tilde{\varepsilon}_{11} = \varepsilon_{11} - \dfrac{d_{11}^2}{\mu_{11}}$ we obtain:

$$
\begin{aligned}
\tilde{c}_{44}\Delta u^*_{33} + \tilde{e}_{15}\Delta u^*_{43} + \rho\omega^2 u^*_{33} &= \delta(x,\xi) \\
\tilde{c}_{44}\Delta u^*_{34} + \tilde{e}_{15}\Delta u^*_{44} + \rho\omega^2 u^*_{34} &= 0 \\
\tilde{c}_{44}\Delta u^*_{35} + \tilde{e}_{15}\Delta u^*_{45} + \rho\omega^2 u^*_{35} &= \frac{q_{15}}{\mu_{11}}\delta(x,\xi) \\
\tilde{e}_{15}\Delta u^*_{33} - \tilde{\varepsilon}_{11}\Delta u^*_{43} &= 0 \\
\tilde{e}_{15}\Delta u^*_{34} - \tilde{\varepsilon}_{11}\Delta u^*_{44} &= \delta(x,\xi) \\
\tilde{e}_{15}\Delta u^*_{35} - \tilde{\varepsilon}_{11}\Delta u^*_{45} &= \frac{d_{11}}{\mu_{11}}\delta(x,\xi) \\
q_{15}\Delta u^*_{33} - d_{11}\Delta u^*_{43} - \mu_{11}\Delta u^*_{53} &= 0
\end{aligned}
$$

$$\begin{aligned} q_{15}\Delta u^*_{34} - d_{11}\Delta u^*_{44} - \mu_{11}\Delta u^*_{54} &= 0 \\ q_{15}\Delta u^*_{35} - d_{11}\Delta u^*_{45} - \mu_{11}\Delta u^*_{55} &= \delta(x,\xi) \end{aligned}$$

In a similar way the following system is obtained:

$$\begin{aligned} \tilde{a}\Delta u^*_{33} + \rho\omega^2 u^*_{33} &= \delta(x,\xi) \\ \tilde{a}\Delta u^*_{34} + \rho\omega^2 u^*_{34} &= \frac{\tilde{e}_{15}}{\tilde{\varepsilon}_{11}}\delta(x,\xi) \\ \tilde{a}\Delta u^*_{35} + \rho\omega^2 u^*_{35} &= \left(\frac{q_{15}}{\mu_{11}} - \frac{d_{11}}{\mu_{11}}\frac{\tilde{e}_{15}}{\tilde{\varepsilon}_{11}}\right)\delta(x,\xi) \\ \tilde{e}_{15}\Delta u^*_{33} - \tilde{\varepsilon}_{11}\Delta u^*_{43} &= 0 \\ \tilde{e}_{15}\Delta u^*_{34} - \tilde{\varepsilon}_{11}\Delta u^*_{44} &= \delta(x,\xi) \\ \tilde{e}_{15}\Delta u^*_{35} - \tilde{\varepsilon}_{11}\Delta u^*_{45} &= \frac{d_{11}}{\mu_{11}}\delta(x,\xi) \\ q_{15}\Delta u^*_{33} - d_{11}\Delta u^*_{43} - \mu_{11}\Delta u^*_{53} &= 0 \\ q_{15}\Delta u^*_{34} - d_{11}\Delta u^*_{44} - \mu_{11}\Delta u^*_{54} &= 0 \\ q_{15}\Delta u^*_{35} - d_{11}\Delta u^*_{45} - \mu_{11}\Delta u^*_{55} &= \delta(x,\xi) \end{aligned}$$

where: $\tilde{a} = \tilde{c}_{44} + \dfrac{\tilde{e}^2_{15}}{\tilde{\varepsilon}_{11}}$.

We will apply Fourier transform to solve the above system. The Fourier transform is defined as:

$$F(g)(\eta) = \frac{1}{2\pi}\int_{R^2} g(x)e^{-i<x,\eta>}dx,$$

where $g \in \Im(R^2)$.
For generalized functions $f \in \Im'$ we have:

$$(F(f), g) = (f, F(g)), g \in \Im$$

Some basic properties of the Fourier transform are the following:

$$\begin{aligned} F(af + bg) &= aF(f) + bF(g) \\ F(D^\alpha f) &= (-i\eta)^\alpha F(f) \\ F(f(x-\xi)) &= e^{-i<x,\xi>}F(f) \end{aligned}$$

$$F(\delta(x,\xi)) = \frac{1}{2\pi}e^{-i\eta\xi}.$$

We apply the Fourier transform to the equation:

$$\tilde{a}\Delta u_{33}^* + \rho\omega^2 u_{33}^* = \delta(x,\xi)$$

and obtain:

$$(-|\eta|^2 + k^2)F(u_{33}^*) = \frac{1}{\tilde{a}}\frac{1}{2\pi}e^{-i\eta\xi}$$

where $k = \omega\sqrt{\dfrac{\rho}{\tilde{a}}}$ or $F(u_{33}^*) = \dfrac{1}{-|\eta|^2 + k^2}\dfrac{1}{\tilde{a}}\dfrac{1}{2\pi}e^{-i\eta\xi}$ in a similar way we obtain:

$$\begin{aligned}
F(u_{34}^*) &= \frac{1}{-|\eta|^2 + k^2}\frac{1}{\tilde{a}}\frac{1}{2\pi}e^{-i\eta\xi}\frac{\tilde{e}_{15}}{\tilde{\varepsilon}_{11}} \\
F(u_{35}^*) &= \frac{1}{-|\eta|^2 + k^2}\frac{1}{\tilde{a}}\frac{1}{2\pi}e^{-i\eta\xi}\left(\frac{q_{15}}{\mu_{11}} - \frac{d_{11}}{\mu_{11}}\frac{\tilde{e}_{15}}{\tilde{\varepsilon}_{11}}\right) \\
F(u_{43}^*) &= \frac{\tilde{e}_{15}}{\tilde{\varepsilon}_{11}}F(u_{33}^*) = F(u_{34}^*) \\
F(u_{44}^*) &= \frac{\tilde{e}_{15}}{\tilde{\varepsilon}_{11}}\frac{1}{-|\eta|^2 + k^2}\frac{1}{\tilde{a}}\frac{1}{2\pi}e^{-i\eta\xi}\frac{\tilde{e}_{15}}{\tilde{\varepsilon}_{11}} + \frac{1}{|\eta|^2}\frac{1}{\tilde{\varepsilon}_{11}}\frac{1}{2\pi}e^{-i\eta\xi} \\
F(u_{45}^*) &= \frac{\tilde{e}_{15}}{\tilde{\varepsilon}_{11}}\frac{1}{-|\eta|^2 + k^2}\frac{1}{\tilde{a}}\frac{1}{2\pi}e^{-i\eta\xi}\left(\frac{q_{15}}{\mu_{11}} - \frac{d_{11}}{\mu_{11}}\frac{\tilde{e}_{15}}{\tilde{\varepsilon}_{11}}\right) - \frac{1}{|\eta|^2}\frac{1}{\tilde{\varepsilon}_{11}}\frac{d_{11}}{\mu_{11}}\frac{1}{2\pi}e^{-i\eta\xi} \\
F(u_{53}^*) &= F(u_{35}^*), \\
F(u_{54}^*) &= F(u_{45}^*) \\
F(u_{55}^*) &= \frac{1}{-|\eta|^2 + k^2}\frac{1}{\tilde{a}}\frac{1}{2\pi}e^{-i\eta\xi}\left(\frac{q_{15}}{\mu_{11}} - \frac{d_{11}}{\mu_{11}}\frac{\tilde{e}_{15}}{\tilde{\varepsilon}_{11}}\right)^2 + \frac{1}{|\eta|^2}\frac{1}{\tilde{\varepsilon}_{11}}\frac{d_{11}}{\mu_{11}}\frac{1}{2\pi}e^{-i\eta\xi} \\
&\quad -\frac{1}{\mu_{11}}\frac{1}{|\eta|^2}\frac{1}{2\pi}e^{-i\eta\xi}
\end{aligned}$$

The inverse Fourier transform is defined as:

$$\begin{aligned}
F^{-1}(f) &= F(f(-x)),\ f \in \Im' \\
F^{-1}(F(f)) &= f,\ F(F^{-1}(f)) = f,\ f \in \Im'
\end{aligned}$$

F and F^{-1} are one-to-one on $\Im'$.

To obtain the fundamental solution we need the following formulae:

$$\begin{aligned} F^{-1}\left(\frac{1}{|\eta|^2-k^2}e^{-i\eta\xi}\right) &= -\frac{1}{2\pi}K_0(ik|x-\xi|) \\ F^{-1}\left(\frac{1}{|\eta|^2}e^{-i\eta\xi}\right) &= -\ln|x-\xi| \end{aligned}$$

(see H. Bateman, A. Erdelai [7], I. S. Gradshteyn and I. M. Ryzhik [8]), where $K_\nu(z)$ is the Kelvin function of order ν.

Applying the inverse Fourier transform we have:

$$\begin{aligned} u^*_{33} &= \left(\frac{1}{2\pi}\right)^2 \frac{1}{\tilde{a}}K_0(ik|x-\xi|) \\ u^*_{34} &= \frac{1}{\tilde{a}}(\frac{1}{2\pi})^2\frac{\tilde{e}_{15}}{\tilde{\varepsilon}_{11}}K_0(ik|x-\xi|) \\ u^*_{35} &= \frac{1}{\tilde{a}}\left(\frac{1}{2\pi}\right)^2\left(\frac{q_{15}}{\mu_{11}}-\frac{d_{11}}{\mu_{11}}\frac{\tilde{e}_{15}}{\tilde{\varepsilon}_{11}}\right)K_0(ik|x-\xi|) \\ u^*_{43} &= u^*_{34} \\ u^*_{44} &= \frac{\tilde{e}_{15}}{\tilde{\varepsilon}_{11}}\frac{1}{\tilde{a}}\left(\frac{1}{2\pi}\right)^2\frac{\tilde{e}_{15}}{\tilde{\varepsilon}_{11}}K_0(ik|x-\xi|)+\frac{1}{\tilde{\varepsilon}_{11}}\frac{1}{2\pi}\ln|x-\xi| \\ u^*_{45} &= \frac{1}{\tilde{a}}\left(\frac{1}{2\pi}\right)^2\left(\frac{q_{15}}{\mu_{11}}-\frac{d_{11}}{\mu_{11}}\frac{\tilde{e}_{15}}{\tilde{\varepsilon}_{11}}\right)\frac{\tilde{e}_{15}}{\tilde{\varepsilon}_{11}}K_0(ik|x-\xi|)+\frac{1}{\tilde{\varepsilon}_{11}}\frac{d_{11}}{\mu_{11}}\frac{1}{2\pi}\ln|x-\xi| \\ u^*_{53} &= u^*_{35}, \\ u^*_{54} &= u^*_{45} \\ u^*_{55} &= \frac{1}{\tilde{a}}\left(\frac{1}{2\pi}\right)^2\left(\frac{q_{15}}{\mu_{11}}-\frac{d_{11}}{\mu_{11}}\frac{\tilde{e}_{15}}{\tilde{\varepsilon}_{11}}\right)^2K_0(ik|x-\xi|) \\ &\quad+\left(\frac{1}{\tilde{\varepsilon}_{11}}\frac{d_{11}}{\mu_{11}}-\frac{1}{\mu_{11}}\right)\frac{1}{2\pi}\ln|x-\xi| \end{aligned}$$

To implement fundamental solutions in BIEM we need also the asymptotic of the fundamental solutions for small arguments. If we use the asymptotic formula ($|z|$ is small):

$$K_0(z) \sim -\ln\left(\frac{z}{2}\right)-\gamma$$

we conclude that we have: $u^*_{KJ} \sim A_{KJ}\ln(r)$, where $K,J=3,4,5$ and $r=|x-\xi|$ and A_{KJ} depend only on the material constants as was shown above.
For the derivatives of the fundamental solutions we have to use the formula:

$$K_0'(z) = -K_1(z),$$

(see I. S. Gradshteyn and I. M. Ryzhik [8]) and the asymptotic formula for small arguments:

$$K_\alpha(z) \sim \frac{\Gamma(\alpha)}{\alpha}\left(\frac{2}{z}\right)^\alpha, \quad \alpha > 0.$$

The asymptotic for the derivatives has the following form:

$$u^*_{KJ,j} \sim B_{KJ}\frac{1}{r}r_{,j},$$

where B_{KJ} depend on the material constants.

5. Conclusion

We considered functionally graded magneto-electro-elastic materials under anti-plane wave.

A new derivation of the fundamental solution is presented which simplifies the numerical procedure.

The results can be used to find stress concentration near crack tips for magneto-electro-elastic materials with complex crack geometry.

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