МАТЕМАТИКА И МАТЕМАТИЧЕСКО ОБРАЗОВАНИЕ, 2001
MATHEMATICS AND EDUCATION IN MATHEMATICS, 2001
Proceedings of Thirtieth Spring Conference of
the Union of Bulgarian Mathematicians
Borovets, April 8–11, 2001

THINKING ABOUT CRITICAL THINKING IN THE MATHEMATICS CLASSROOM

Wilfred E. Boykin

We mathematics educators agree that all students can think and think critically. Further, we believe that the primary responsibility of mathematics educators is to teach our students to think-about-thinking. Most mathematics educators are constructionists who believe that it is possible to engage students in the actual construction of knowledge. Acquisition of mathematical concepts is the result of thinking reflectively, critically, creatively, imaginatively, and analytically. It is, then, the role of educators to model the learning of mathematics in the form of reasoning – reasoning analytically about quantitative and spatial phenomena. Further, we educators teach by guiding our students through the learning process using critical thinking strategies. This papers is an attempt to explore the role of critical thinking in the acquisition of mathematical concepts in the classroom.

For the purpose of discussion, I have considered critical thinking in the acquisition of mathematical concepts to be characterized by:

a. Inductive thinking; Deductive thinking: (Beginning with selected data and drawing conclusions; Considering the whole picture)

b. Comparing and Contrasting: (Beginning with a statement, compare it to the intended form; See how the beginning statement is different from the form.)

c. Metaphorical referencing; (Consider a mathematical problem. Translate the problem in terms of a real-life experience, e. g. Factors and ingredients.)

d. Mystery and Inquiry. (Problem situation. What questions does one ask in order to solve the mystery? What are questions that may lead to solving the mystery?

Too often it is the case that many of our students leave educational institutions unprepared to cope successfully with demands of the workplace. Many blame the education establishment for not keeping abreast of changing needs of students. Lightening fast developments in technology are effecting changes in content, pedagogy, and applicability to real world problems. Many students enjoy more comprehensive computer capability in their homes than in some classrooms. What was once characterized as book-and-table oriented instruction is now hand-held calculator instruction. The need to equip students to cope with these changes is international in scope and not limited to any one segment of our society.

Changing societal demands have been important in making changes in content and pedagogy. The curriculum of earlier years driven by rote memorization and algorithmic methods has been supplanted by a curriculum based on 'thinking about skills' and 'thinking-about-thinking' activities. Basic skills important for the first part of the 20th century are not the basic skills of today. Traditional methods which were used in the delivery of instruction of years ago is not appropriate for instruction in our schools today. Replacing traditional methods and out-moded content with calculators in the mathematics classroom have forced students to communicate mathematically with precision and fluency. Education of the child is no longer relegated only to the classroom; this process continues through the use of the laptop computer, the personal computer at home as well as at school and with electronic miracles that fit conveniently in the palm of the hand.

What, then, should drive the way in which we educators teach mathematics in order to maximize understanding of concepts? We look first to futuristic thinkers who have faced similar questions and who have influenced underlying ideas of curriculums in mathematics.

One of the most influential educational philosophers who changed the shape of modern education was John Dewey (1910), who, in his book, How We Think [1], envisioned a plan for the future of education. At the core of his plan was "reflective thought". Dewey described reflective thinking as suspending judgment, maintaining a healthy skepticism, and exercising an open mind. The activities which mirrored reflective thinking called for active, persistent, and careful consideration of any belief in light of the ground that supports it. More specifically, Dewey's description contained an important component called critical thinking which had both an intellectual and an emotional component. Herein, students were expected to examine, poke, question, and reflect on what they had learned, critically. Skepticism, questioning, and reflection were essential. Furthermore, a person who was engaged in critical thinking would be able to examine a problem, find a solution, think about why one was or was not successful, and learn from these successes and/or failures. Critical thinking was not negative.

About sixty years later newer and different content and pedagogy became the *new math*. Sparking discussions at all levels of curriculum reform, many philosophers frantically looked for plausible justifications for inclusion of new and different mathematics and pedagogy in the curriculum. The ideas of functions and sets were introduced early in the curriculum. The idea of form and structure became a part of early grade mathematics. SMSG and UICSM mathematics books actually had more reading about mathematics than notations and symbols. In order to understand this new mathematics, one had to *read about* and *think* about mathematics. Mathematics educators were paying more and more attention to thinking skills in the acquisition of mathematical concepts than to memorization of skills and facts. Robert Ennis [2], writing in Harvard Educational Review made a plea for a clear definition of critical thinking in the process of educating a child. He stated that"more cognitive stuff was to be acquired in school than elementary reading, writing, and arithmetic and banks of memorized and soon-to-be-forgotten facts". Moreover, Ennis questioned the relevance of critical thinking in a child's education - not as a subset of reflective thinking but as a kind of thinking of great importance in the acquisition of knowledge.

Many found dissatisfaction with "new math" and sought solace in "Back to Basics" only to find that their hopes met with a lesser degree of success than they wanted. At-

tempts to improve problem solving abilities caused changes in the way problem solving was being taught. These procedures continue today to follow Polya's Problem Solving approaches in some form of another. All of these approaches had as their underlying principle the need to have students *doing* and *understanding* as they **did** mathematics. Overcoming the loss of local and regional control of what students were to learn in favor of a national curriculum in the United States, NCTM published the 1989 STANDARDS and later STANDARDS 2000. The STANDARDS documents were guidelines for all Americans to follow in the design and implementation of content in mathematics, P-12. We are at this point in the U.S. today. The NCTM(National Council of Teachers of Mathematics, USA) identifies as the central goal of teaching mathematics as that of the development of mathematical power for all students. Power includes the ability to explore, conjecture and reason logically; to solve non-routine problems; to communicate about and through mathematics; and to connect ideas within mathematics and between mathematics and other activities. In doing so, students gain self-confidence and a disposition to seek, evaluate and use quantitative and spatial information in solving problems and in making decisions [3].

Mathematics educators continue to bemoan the fact that many American schools paint a dismal picture of the ideal of acquisition of concepts. Very successful students who are quick to memorize can master algorithmic techniques and thus succeed only because they can master an impressive number of these techniques. Many spend twelve or so years in mathematics classes that do nothing more than prepare human robots who can do computations but have few clues about what they have learned. It is obvious that many students have encounters with the study of mathematics that leave them unchanged, have made no impact on their 'thinking' strategies, and have not given the mathematical understandings which are crucial to even the basic needs of the changing societal demands of today's workplace. Results of the most recent TIMMS study evince the fact that students with very high scores can not make acceptable scores on entrance tests which require application of simple algebraic concepts in securing a job. Teaching toward test-taking is common-place in American schools where high scores have little reliability. High school diplomas mean practically nothing in many cases.

Changes in technology and in content continue to influence curriculums. Usiskin, mathematics professor/educator at University of Chicago, maintains that subject lines are no longer as well defined as they once were. He maintains that there is no distinct dichotomy between algebra and calculus today. Many of the calculus concepts are being integrated into the curriculum earlier and earlier. Logical thinking is more than working with Euler circles or Venn Diagrams. Many educators hypothesize that schools as well as textbooks must change their presentation of content in order to keep abreast of needs of society and understand the mathematics that is being taught.

To attempt to bring together the myriad aspects of thinking is no small task. It is important, however, to attempt to describe the kind of 'thinking' we need to use in our classrooms with the hopes that our students will be able to cope with 'thinking-about-thinking' in manageable ways. For the purpose of this paper, the author proposes that mathematics educators engage in concerted efforts in the presentation of mathematical content with critical thinking strategies embed into the mathematics curriculum. More specifically, instruction should model critical thinking strategies in the acquisition of concepts and students should be made aware of these strategies that are being used.

General comments of authors about critical thinking tend to cover a broad range of topics. Lipman [4] maintains that critical thinking improves the quality and quantity of meaning and, in essence, it is the process of making a claim or opinion and backing it up with reasons. Phi Delta Kappan, Chuska [5] infers that critical thinking is analogous to analytical thinking. In its rudimentary form, critical thinking begins with a claim followed by some type of judgment or its authenticity – the kind of thinking in reading, social studies, and science. Further, he says, there are four conditions for thinking to occur: We must have something to think ABOUT, there must be something to think WITH, there must be some WAYS in which to think, and there must be something to think FOR. Benjamin Bloom did not place critical thinking as a level of cognitive thinking in his Taxonomy of Educational Objectives, however a logical place for this activity encompasses Bloom's Analysis, Synthesis, and Evaluation. Critical thinking is like a tune-up with refinement, readjustment, and re-evaluation. Others contend that critical thinking is NOT problem solving, where problem solving is a process or product in finding a meaning solution and critical thinking is judgment with no regard to an outcome. Beyer [6] states that critical thinking is a set of attitudes and dispositions as well as a number of cognitive operations.

An Englishman, de Bono [7], believes that thinking can be taught and he has developed workshops to do this. He claims that "thinkers are made, not born. "Beyer echos this idea and says that children should be taught to think. Fraser and West [8] note that children who exhibit the following characteristics are good critical thinkers: (1) Have an alertness to the need to evaluate information, (2) Have a need to make assertions and evaluate sources, (3) Have a willingness to test opinions, and (4) Have desire to consider all ingredients in a problem. One with low aptitude for critical thinking (1) exercises mental carelessness and is passive in working through a problem, (2) Chooses answers on the basis of few clues and with gut feelings, and (3) Fails to take into account a clearly defined objective.

Although some attention has been given to the role of assessing outcomes in critical thinking, there is need for improvement. The Watson-Glazer Critical Thinking Appraisal, an often-used assessment instrument, defines five key skills: Drawing inferences, Recognizing assumptions, Drawing conclusions, Interpreting data, and Evaluation arguments.

Good teachers, who provide learning environments conducive to critical thinking, have a vision of the kind of mathematics teaching and learning they want in their classrooms. They say that the main barrier to implementing their visions is that students come into their classes believing in and practicing a kind of mathematics that is devoid of questioning, of creativity, or sense-making (Flores) [9]. In order for critical thinking to take place, students must be active learners in the learning process, not passive recipients of information. They should identify and solve unstructured problems that require use of multiple information sources. Learning by doing should be emphasized. Working in groups should be encouraged. Creative use of technology is essential. General principles in achieving critical thinking skills include (1) learning meaningfully and strive for fluency in the subject, (2) economize and generalize knowledge building upon existing knowledge structures, (3) encourage use of more concrete experiences, and (4) explore underlying first principles in what has been learned and help students to make connections among higher levels of thought. Teachers who promote critical thinking strategies in the classroom model and have their children engaged in doing mathematics. Doing is

interpreted as: recognizing and describing patterns, constructing physical or conceptual models, creating symbols to help us represent information in different forms, forming and justifying judgements, and drawing inferences about our conceptions. Teachers must help students make sense of what they have learned. Most of us agree that in order to think critically, first of all we must think!

Characteristics of critical thinking lead to exemplary reflective thinking. Teachers should design their lectures and classroom discussions with these four characteristics or critical thinking imbedded into their teaching strategies. Textbook authors should design meaningful problems which utilize critical thinking processes based on the above four strategies. Mathematics instructors are encouraged to call attention to specific instances of the use of the author's definition of critical thinking activities as they model problem solving activities and when they are lecturing.

State-of-the-art technology has made it possible to revamp content in school mathematics. NCTM has provided effective broad curriculum changes in listing topics that should be emphasized and those that should not. Proof, number sense, geometry, etc. are being stressed at all grades. By and large, the textbook is the curriculum in most American schools. Although concerted efforts to improve teacher knowledge of content and methodology are improving, many teachers are ill-prepared to teach the mathematics they are assigned to teach. Within the next few years, it is expected that preparation of teachers will receive a great deal of attention not only at the local level but also at the state and national level. Mathematics teachers will, out of necessity, compel textbook authors to integrate strategies of critical thinking in mathematics texts.

REFERENCES

[1] J. Dewey. How We Think, Southern Illinois Publishers, Vol 8, 346.

[2] R. Ennis. A Concept of Critical Thinking, Harvard Educational Review, Winter, 1962, 81–111.

[3] National Council of Teachers of Mathematics, STANDARDS 2000, 6.

[4] M. Lipman. Critical Thinking, What can it be?, Educational Leadership, September, 1988, 40.

[5] K. R. Chuska. Phi Delta Fastback, #244. Phi Delta Kappa, Bloomington, IN, 1986. 25.

[6] B. Beyer. Practical Strategies for the teaching of Thinking, Allyn and Bacon, Inc., Boston, 1987, 273.

[7] E de Bono. de Bono's Thinking Course. Facts on File Pub., NY, 1985. 192.

[8] D. Fraser, E. West. Social Studies in Secondary Schools, New York Roland Press, 1961, 222.

[9] J. T. Sowder, B. Shappelle. Providing a Foundation for Teaching. Mathematics in the Middle Grades, State University of NY Press, 1995, 336.

Wilfred E. Boykin
Georgia College & State University
Milledgeville, GA 31061
USA

МИСЛИ ЗА КРИТИЧЕСКОТО МИСЛЕНЕ В ЧАСОВЕТЕ ПО МАТЕМАТИКА

Уилфред Бойкин

Ние, специалистите по математическо образование, смятаме, че всички ученици могат да мислят и то – критично. Освен това, вярваме, че основната отговорност на преподавателите по математика е да учат учениците си да мислят за мисловния процес. Мнозина сред преподавателите са привърженици на конструкционизма и вярват, че е възможно учениците да станат строители на собствените си знания. Усвояването на математически знания е резултат от това, да се разсъждава, да се мисли критично, творчески, аналитично и с въображение. Следователно ролята на преподавателите по математика е да моделират учебния процес във формата на аналитични разсъждения за качествени и пространствени явления. Нещо повече, ние обучаваме учениците си, като ги ръководим по време на учебния процес с помощта на стратегии, основани на критичното мислене. Този доклад е опит да се изследва ролята на критичното мислене за усвояване на математически понятия в клас.