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ON THE FINITE DEFORMATIONS OF TWO DROPS DUE TO ELECTRIC FIELD

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In this paper we investigate the finite deformations of two drops due to electric field. The fluids are homogenous, incompressible and Newtonian.

Reynolds' number is assumed small enough to investigate the problem in quasisteady Stokes' approximation. It is also supposed that the initial form of the drops are spherical, but could be with different radii and different fluid phases.

The electric and hydrodynamic problems are separated and the electric one has influence on the hydrodynamic with the boundary conditions. The Maxwell's equations are turned to Laplace's equations, and together with Stokes' equations are solved by semianalytical-seminumerical method. We use boundary-integral type of these equations to solve them by the method of boundary elements. The kinematics condition gives the new form to the particles.

The results obtained indicate that interactions between two and three fluid phases due to electric field lead to deformations of the drops. Parametrical analysis of the deformations of some dimensionless parameters of the problem has been given graphically.

1. Introduction. Basic ideas for investigation of the matter of deformations of fluid particles had been presented for first time by Taylor, G. I. [15]. Taylor, D. T. & Acrivos, A. [18] prove that in uniform flow in Stokes approximation, an initially spherical particle remains spherical without any deformations.

Chervenivanova, E. & Zapryanov, Z. [4] obtain small deformations of drop moving with a uniform velocity in spherical container, full with viscous fluid. Although the flow is uniform, there are deformations of the drop because it is in a container which causes the deformations.

The problems of single drop subjected in viscous flow, are in the basis for solving problems of compound drops (drop in drop), drop near to a plane wall or two separated drops.

Small deformations of two fluid drops are presented for first time in [3]. They obtain deformations of two fluid droplets, drop and bubble and drop and rigid particle in uniform flow. A parametric analysis of the small deformations relative to the distance between drops and the ration of viscosities of the different phases. "Dimple" formation is on of the basic results of the paper.

The influence of the electric field on a water drop has been investigated experimentally in [8] and [20]. The critical value of dimensionless parameter (E^*) after which the drop breaks up was found. In [1] and [17] was observed drop's break up with conical tips. Ramos & Castellanos [12] present theoretical result for the influence of the coefficients of permittivity and conductivity on the conical tips formation. Torza, Cox & Mason [19] by experiment find another model of breaking up drop, which is divided into two spherical parts connected with thin "throat". Sherwood [13] using the method of boundary elements solves numerically the problem of fluid particle deformation under the influence of electric field.

The form that two equal fluid drops achieved in presence of electric field give by experiment O'Konski & Thacker [9]. Taylor G. I. shows that due to the same electric field but different parameters of the fluid phases (conductivities, permittivities) there are deformations of the interfaces based on electrostatic charge. Taylor, G. I. [16] Brazier-Smith [2], investigate deformations and stability of the couple of water drops with equal radii due to uniform electric field. Sozou [14] using bipolar co-ordinate system presents semianalytical decision for velocities in and out of the drops, presuming keeping of the spherical form.

2. Formulation of the problem. The problem for defining the finite deformations of two fluid drops, due to the electric field, is separated into two problems — electrostatic and hydrodynamic.

The drops on Figure 2.1 are compounded of fluid 2 with viscosity μ_2, conductivity σ_2, permittivity ε_2 and fluid 3 with viscosity μ_3, conductivity σ_3, permittivity ε_3. The electric field that acts on the axis connecting the centres of the drops is with intensity E_0. Under its influence the interfaces of the drops deform. The initial form of fluid drops is spherical with undistorted radius R_1 of the first sphere and undistorted radius R_2 of the second one. With S_2 is marked the interface between phase 1 and phase 3, and with S_1 – the interface between phase 1 and phase 2. The interfacial tensions over S_1 and S_2 are γ_1 and γ_2 respectively. The fluids 1, 2 and 3 are situated in Ω_1, Ω_2 and Ω_3 respectively, while Ω_1 is infinite area outside the drop (Fig. 2.1).

We solve the problem in quasisteady approximation, by using the short form of Maxwell's equations (Laplace's equations) and Stoke's equations. In each point the electric potential and the velocity of the flow in each moment is governed by the following

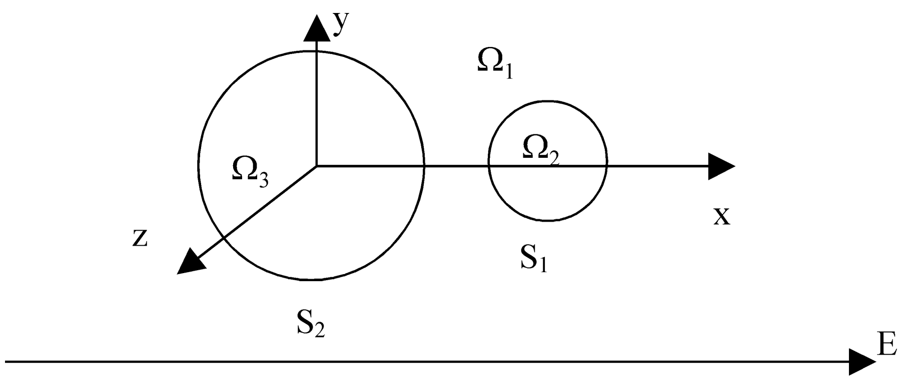

Fig. 2.1. Scheme of two drops

equations:

— Laplace's equations:

$$\Delta\varphi^k = 0 \quad (k = 1, 2, 3) \tag{2.1}$$

— discontinuity equations:

$$\frac{\partial u_i^k}{\partial x_i} = 0 \quad (i, k = 1, 2, 3) \tag{2.2}$$

— Stokes' equations:

$$\frac{\partial \sigma_{ij}^k}{\partial x_j} = 0 \tag{2.3}$$

where σ_{ij}^k is stress tensor:

$$\sigma_{ij}^k = -p^k \delta_{ij} + \mu_k \left(\frac{\partial u_i^k}{\partial x_j} + \frac{\partial u_j^k}{\partial x_i} \right).$$

The index $k = 1$ when $\mathbf{x} \in \Omega_1$, k=2 for $\mathbf{x} \in \Omega_2$ and $k = 3$ $\mathbf{x} \in \Omega_3$, while p^k is the hydrodynamic pressure of the respective fluid. The electric potential in the three phases must satisfy the following boundary conditions:

$$\varphi^1(\mathbf{x}_0) \to E_0.\mathbf{x}_0^1 \qquad |\mathbf{x}_0| \to \infty \tag{2.1.a}$$

$$\varphi^1(\mathbf{x}_0) = \varphi^3(\mathbf{x}_0) \qquad \mathbf{x}_0 \in S_2 \tag{2.1.b}$$

$$\varphi^1(\mathbf{x}_0) = \varphi^2(\mathbf{x}_0) \qquad \mathbf{x}_0 \in S_1 \tag{2.1.c}$$

$$\sigma_1 \frac{\partial \varphi^1}{\partial n}(\mathbf{x}_0) = \sigma_3 \frac{\partial \varphi^3}{\partial n}(\mathbf{x}_0) \qquad \mathbf{x}_0 \in S_2 \tag{2.1.d}$$

$$\sigma_1 \frac{\partial \varphi^1}{\partial n}(x_0) = \sigma_2 \frac{\partial \varphi^2}{\partial n}(x_0) \quad \mathbf{x}_0 \in S_1 \tag{2.1.e}$$

where E_0 is the intensity of the electric filed, $\mathbf{x}_0^1$ is x-component in Decart co-ordinate system $Oxyz$ of the vector $\mathbf{x}_0$ and σ_1, σ_2, σ_3 are the electric conductivity of the respective fluids, and $\dfrac{\partial}{\partial n}$ is normal derivative to the surface, pointed out of the respective domain.

The flow field must be governed by the following boundary conditions:

$$u_i^1(\mathbf{x}_0) \to 0 \quad |\mathbf{x}_0| \to \infty \tag{2.3.a}$$

$$u_i^1(\mathbf{x}_0) = u_i^2(\mathbf{x}_0) \quad \mathbf{x}_0 \in S_1 \tag{2.3.b}$$

$$\begin{aligned} &\sigma_{ij}^1(\mathbf{x}_0)\, n_j(\mathbf{x}_0) - \sigma_{ij}^2(\mathbf{x}_0)\, n_j(\mathbf{x}_0) = \\ &= \gamma_1 n_i \frac{\partial n_j}{\partial x_j} - \left(\tau_{ij}^1(\mathbf{x}_0)\, n_j(\mathbf{x}_0) - \tau_{ij}^2(\mathbf{x}_0)\, n_j(\mathbf{x}_0) \right) \quad \mathbf{x}_0 \in S_1 \end{aligned} \tag{2.3.c}$$

$$u_i^1(\mathbf{x}_0) = u_i^3(\mathbf{x}_0) \quad \mathbf{x}_0 \in S_2 \tag{2.3.d}$$

$$\begin{aligned} &\sigma_{ij}^1(\mathbf{x}_0)\, n_j(\mathbf{x}_0) - \sigma_{ij}^3(\mathbf{x}_0)\, n_j(\mathbf{x}_0) = \\ &= \gamma_2 n_i \frac{\partial n_j}{\partial x_j} - \left(\tau_{ij}^1(\mathbf{x}_0)\, n_j(\mathbf{x}_0) - \tau_{ij}^3(\mathbf{x}_0)\, n_j(\mathbf{x}_0) \right) \quad \mathbf{x}_0 \in S_2 \end{aligned} \tag{2.3.e}$$

Here $\mathbf{n}$ is the single outer normal to the interface S_1 or S_2, $\tau_{ij}^k = -\dfrac{\varepsilon_k}{4\pi} \left(\dfrac{\left(E^k\right)^2}{2} \delta_{ij} - E_i^k E_j^k \right)$

is the Maxwell's electric stress tensor for the respective phases ($k = 1, 2, 3$). Its values are determined by the results of the electric problem (2.5) with boundary conditions (2.1.a–e) and equations $E^k = -\nabla\varphi^k$. Let us assume that S_1 and S_2 are Lyapunov's surfaces. The decision of (2.3) with boundary conditions (2.3.a–e) gives us the velocity at each moment in every point of S_1 and S_2. The deformation of the interfaces is determined in each moment by the normal component of the velocity and the kinematic condition:

$$\frac{d\mathbf{x}_s}{dt} = n_i\,(u_i.n_i) = n_i.u_n \tag{2.4}$$

Here $\mathbf{x}_s$ is a point of the respective surface S_1 or S_2, while u_n is the normal component of the velocity in this point.

Following [6] and [10] we turn the system (2.1)–(2.3) with boundary conditions (2.1.a)–(2.1.e) and (2.3.a)–(2.3.e) to integral system and the following dimensionless parameters are included:

$E_\gamma = \dfrac{\varepsilon_1 E_0^2 R_1}{\gamma_2}$ – the relation between the electric and capillary forces;

$\mu_{12} = \dfrac{\mu_1}{\mu_2}, \mu_{13} = \dfrac{\mu_1}{\mu_3}$ – the relation of the viscousities of the different neighbouring phases;

$\varepsilon_{21} = \dfrac{\varepsilon_2}{\varepsilon_1}, \varepsilon_{31} = \dfrac{\varepsilon_3}{\varepsilon_1}$ – t he relation of the electric permittivity of the different neighbouring phases;

$\sigma_{12} = \dfrac{\sigma_1}{\sigma_2}, \sigma_{13} = \dfrac{\sigma_1}{\sigma_3}$ – the relation of the electric conductivity of the different neighbouring phases.

$\gamma_{12} = \dfrac{\gamma_1}{\gamma_2}$ – the relation of the coefficient of interface tension of the two surfaces of the drops;

$R_{12} = \dfrac{R_1}{R_2}$ – the relation of the radii of the two drops.

3. Algorithm for determination of deformations of two drops due to electric field. On each time step of the algorithm, first we solve the electrostatic problem which has influence on the hydrodynamic one by the way of Maxwell's electric stress tensor. On its turn, the solving of the hydrodynamic problem gives us velocities of the fluids in the different phases. Using the kinematic condition for the normal velocity components on the fluid surfaces, we get their deformation. With the new form (changed boundary conditions) we solve once again the electrostatic problem and after that the hydrodynamic one, as the number of time steps determines how many times this procedure will be done. We assume that the form reach the equilibrium when the normal component of the velocity becomes less than the preliminary set minimum at every point of the interfaces. Another criteria for end of the procedure is when the normal component becomes more than the initially set number; then we consider drop's break up.

The main steps of the algorithm followed are:

— change of the co-ordinate system from Decart's to cylindrical, in order to transform the boundary integrals to one-dimensional;

— introduction of boundary elements over the boundaries of the domains – arcs of circles;

— introduction of local polar co-ordinate system on each boundary element;

— calculation of the integrals of the single- and double-layer over each boundary element;

— subtraction of the integrals singularities;

— calculation of the velocity on the interfaces;

— determination of the drops form from the kinematic condition.

For determination of the drops form in each time-step, we use the kinematic condition of the following type:

$x_s^{new} = x_s + n_i \left(u_i . n_i \right) .dt$ where dt is a preliminary set time-step.

For the calculations a project in Code Warrior C has been conducted, as the main results have been obtained through Power Mac 200/6400 in the Laboratory to the Department of Mechanics of Continua at the FMI of the Sofia University "St. Kl. Ohridski".

4. Result. The algorithm for obtaining the finite deformations of two drops due to electric field was tested for a single drop in presence of electric field and had a good agreement with the results of Sherwood [13].

The deformation of fluid interfaces when they are with equal radii does not depend essentially from the distance between drops in low intensities $E_\gamma = 0.4$ of the electric field. The initial distance between the centers of the drops on Fig. 1 is 2.1 R1, on Fig. 2 is 2.5 R1, on Fig. 3 is 3.5R1 with $\mu_{12} = 0.50$, $\mu_{13} = 1.50$, $E_\gamma = 0.4$, $\varepsilon_{21} = 2.0$, $R_{12} = 1.0$, $\varepsilon_{31} = 3.0$, $\sigma_{12} = 10.0$, $\sigma_{13} = 15.0$, $dt = 0.01$, $\gamma_{12} = 1.0$.

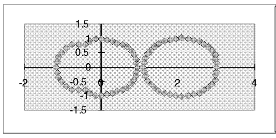

Fig. 1

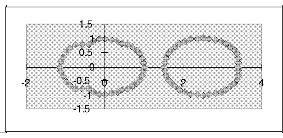

Fig. 2

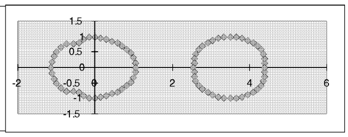

Fig. 3

When we increase the intensity of the electric field, the form reached by the first drop is elongating, till the second one retains almost spherical. On Fig. 4 $E_\gamma = 1.0$, on Fig. 5 $E_\gamma = 2.0$, on Fig. 6 $E_\gamma = 5.0$ with $\mu_{12} = 0.50$, $\varepsilon_{21} = 2.0$, $\mu_{13} = 1.5$, $R_{12} = 1.0$, $\varepsilon_{31} = 3.0$, $\sigma_{12} = 10.0$, $\sigma_{13} = 15.0$, $dt = 0.01$, $\gamma_{12} = 1.0$.

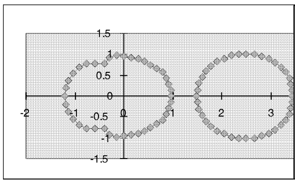

Fig. 4

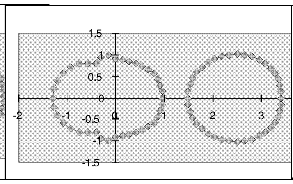

Fig. 5

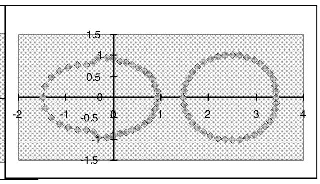

Fig. 6

The ratio between the radii of the two drops causes different pictures of deformation as shown on Figs 7, 8, 9. On Fig. 7 $R_{12} = 0.5$, on Fig. 8 $R_{12} = 1.25$, on Fig. 9 $R_{12} = 1.49$ with $\mu_{12} = 0.5$, $E_\gamma = 0.5$, $\mu_{13} = 2.0$, $\varepsilon_{21} = 2.0$, $\varepsilon_{31} = 3.0$, $\sigma_{12} = 10.0$,

$\sigma_{13} = 15.0$, $dt = 0.01$, $\gamma_{12} = 1.0$. The change of ratio causes increase of influence of the initially bigger drop to the smaller.

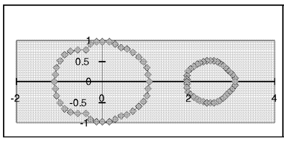

Fig. 7

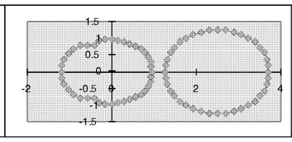

Fig. 8

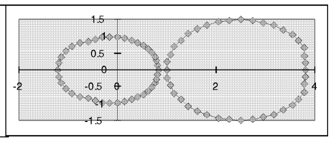

Fig. 9

The problem for the finite deformations of two drops due to electric field has ten dimensionless parameters each of which has an influence on the process, so further results should be present in recent papers.

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ВЪРХУ КРАЙНИТЕ ДЕФОРМАЦИИ НА ДВЕ КАПКИ ПОРОДЕНИ ОТ ЕЛЕКТРИЧНО ПОЛЕ

Явор Христов

В настоящата работа се разглеждат крайните деформации на две капки, породени от електрично поле. Флуидите се предполагат хомогенни, несвиваеми и Нютонови. Числото на Рейнолдс се предполага достатъчно малко, за да се разглежда задачата в Стоксово приближение. Първоначалната форма на частиците е сферична. Електрическата и хидродинамическата задачи са разделени като електрическата влияе на хидродинамичната чрез граничните условия и по-точно чрез тензора на Максуел. Уравненията на Максуел са сведени до Лапласови и заедно с уравненията на Стокс са решени полуаналитично-получислено, използвайки метода на граничните елементи. Кинематичното условие определя новата форма на частиците.

Резултатите показват, че взаимодействието между две или три фази под въздействието на електрично поле води до деформации в капките. Параметричен анализ по някои от безразмерните параметри е представен графично.