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ON THE FINITE DEFORMATIONS OF COMPOUND DROP DUE TO ELECTRIC FIELD

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The finite deformations of compound drop containing another drop due to electric field are obtained. The fluids are homogenous, incompressible and Newtonian. The cases of concentric and eccentric particles are investigated together.

The problem is investigated in quasisteady Stokes' approximation and the initial form of the drops is spherical.

The electric and hydrodynamic problems are separated and the electric one has influence on the hydrodynamic with the Maxwell's stress tensor in the boundary conditions. The Maxwell's equations are turned to Laplace's equations and together with Stokes' equations are solved with semianalytical-seminumerical method including boundary elements. The kinematic condition is used to obtain the new form to the particles.

The results obtained show that due to electric field there are deformations of the inner and the outer drop. Parametrical analysis of the deformations of some dimensionless parameters of the problem is given graphically.

1. Introduction. The first investigations of compound drops had been made by Chambers & Kopac (1937) and had to do with the processes of coalescence of live cells and drops from different oils.

The compound multiple drops according to [5] are divided into three main types: A, B and C. A type contains one large internal drop (bubble), type B contains several small internal drops, and type C globules entrap large number of internal drops.

Brunn, P. & Roden, I. [2] come to the conclusion that compound concentric drop set in uniform flow in Stokes' approximation, as in the case of single drop, has no deformation.

Chervenivanova, E. & Zapryanov, Z. [4] observe that eccentric compound drop in uniform flow in Stokes' approximation shows deformations. The deformations of inner and outer interfaces are obtained to different values of capillary number and different initial position of the internal interface related to external. In [14] using the method of boundary elements investigate analytically and numerically the concentric compound drop in different shear flows. They come to the conclusion that in that case, despite the concentricity, the two interfaces are deforming.

Using the Navier-Stokes' equations and the method of finite elements, Bazlekov, I., Shopov P. and Zapryanov, Z. [1] derive the finite deformations of eccentric compound drop in uniform flow with middle values of Reynold's number.

The influence of the electric field on a water drop has been investigated experimentally by Wilson & Taylor [18] They find the critical value of dimensionless parameter (E^*) after which the drop breaks up. Taylor [15] improves theoretically this value supposing that the drop preserves its spherical form until the break up. Taylor & McEwan [16], in the respective paper observe drop's break up with conical tips. Ramos & Castellanos [12] present theoretical result for the influence of the coefficients of permittivity and conductivity on the conical tips formation. In [17] experimentally was found another model of breaking up drop, which is divided into two spherical parts connected with thin "throat". Sherwood [13] using the method of boundary elements solves numerically the problem of fluid particle deformation under the influence of electric field.

2. Formulation of the problem. To solve the problem of the finite deformations of a compound fluid drop, due to the electric field, we divide it into two problems – electrostatic and hydrodynamic.

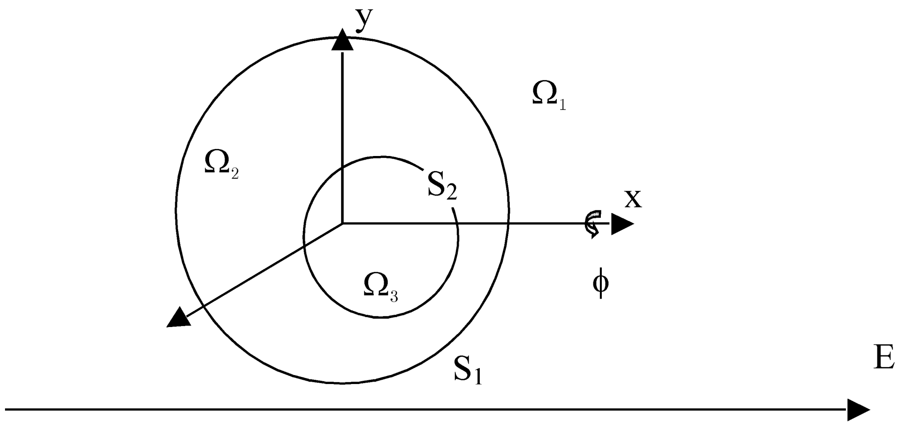

Fig. 2.1 Scheme of eccentric compound drop

The drop on Figure 2.1 is two-component, compound of fluid 2 with viscosity μ_2, conductivity σ_2, permittivity ε_2 and fluid 3 with viscosity μ_3, conductivity σ_3, permittivity ε_3. The electric field that acts on the axis connecting the centres of the drops is with intensity E_0. Under its influence the two interfaces of compound drop deform. The initial form of compound fluid drop is spherical with undistorted radius R_1 of the external sphere and undistorted radius R_2 of the internal one. With S_1 is marked the interface between phase 1 and phase 2, and with S_2 – the interface between phase 2 and phase 3. The interfacial tensions over S_1 and S_2 are γ_1 and γ_2, respectively. The fluids 1, 2 and 3 are situated in Ω_1, Ω_2 and Ω_3, respectively, while Ω_1 is infinite area outside the drop (Fig. 2.1).

We solve the problem in quasisteady approximation, by using the short form of Maxwell's equations (Laplace's equations) and Stoke's equations. In each point the electric potential and the velocity of the flow in each moment is governed by the following equations:

— Laplace's equations:

$$\Delta\varphi^k = 0 \ (\Omega = 1, 2, 3) \tag{2.1}$$

— discontinuity equations:

$$\frac{\partial u_i^k}{\partial x_i} = 0 \ (i, k = 1, 2, 3) \tag{2.2}$$

— Stoke's equations:

$$\frac{\partial \sigma_{ij}^k}{\partial x_j} = 0 \tag{2.3}$$

where σ_{ij}^k is stress tensor:

$$\sigma_{ij}^k = -p^k \delta_{ij} + \mu_k \left(\frac{\partial u_i^k}{\partial x_j} + \frac{\partial u_j^k}{\partial x_i} \right).$$

The index $k = 1$ when $x \in \Omega_1$, $k = 2$ for $x \in \Omega_2$ and $k = 3$ for $x \in \Omega_3$, while $p^k$is the hydrodynamic pressure of the respective fluid. The electric potential in the three phases must satisfy the following boundary conditions:

$$\varphi^1 (\mathbf{x}_0) \to E_0 . x_0^1 \quad |\mathbf{x}_0| \to \infty \tag{2.1.a}$$

$$\varphi^2 (\mathbf{x}_0) = \varphi^3 (\mathbf{x}_0) \quad \mathbf{x}_0 \in S_2 \tag{2.1.b}$$

$$\varphi^2 (\mathbf{x}_0) = \varphi^1 (\mathbf{x}_0) \quad \mathbf{x}_0 \in S_1 \tag{2.1.c}$$

$$\sigma_2 \frac{\partial \varphi^2}{\partial n} (x_0) = \sigma_3 \frac{\partial \varphi^3}{\partial n} (x_0) \quad \mathbf{x}_0 \in S_2 \tag{2.1.d}$$

$$\sigma_2 \frac{\partial \varphi^2}{\partial n} (x_0) = \sigma_1 \frac{\partial \varphi^1}{\partial n} (x_0) \quad \mathbf{x}_0 \in S_1 \tag{2.1.e}$$

where E_0 is the intensity of the electric filed, x_0^1 is x-component in Decart co-ordinate system $Oxyz$ of the vector x_0 and σ_1, σ_2, σ_3 are the electric conductivity of the respective fluids, and $\frac{\partial}{\partial n}$ is normal derivation n to the surface, pointed out of the respective domain.

The flow field must be governed by the following boundary conditions:

$$u_i^1 (\mathbf{x}_0) \to 0 \quad |\mathbf{x}_0| \to \infty \tag{2.3.a}$$

$$u_i^1 (\mathbf{x}_0) = u_i^2 (\mathbf{x}_0) \quad \mathbf{x}_0 \in S_1 \tag{2.3.b}$$

$$\begin{gathered} \sigma_{ij}^1 (\mathbf{x}_0) n_j (\mathbf{x}_0) - \sigma_{ij}^2 (\mathbf{x}_0) n_j (\mathbf{x}_0) = \\ = \gamma_1 n_i \frac{\partial n_j}{\partial x_j} - \left(\tau_{ij}^1 (\mathbf{x}_0) n_j (\mathbf{x}_0) - \tau_{ij}^2 (\mathbf{x}_0) n_j (\mathbf{x}_0) \right) \quad \mathbf{x}_0 \in S_1 \end{gathered} \tag{2.3.c}$$

$$u_i^2 (x_0) = u_i^3 (\mathbf{x}_0) \quad \mathbf{x}_0 \in S_2 \tag{2.3.d}$$

$$\begin{gathered} \sigma_{ij}^2 (\mathbf{x}_0) n_j (\mathbf{x}_0) - \sigma_{ij}^3 (\mathbf{x}_0) n_j (\mathbf{x}_0) = \\ = \gamma_2 n_i \frac{\partial n_j}{\partial x_j} - \left(\tau_{ij}^2 (\mathbf{x}_0) n_j (\mathbf{x}_0) - \tau_{ij}^3 (\mathbf{x}_0) n_j (\mathbf{x}_0) \right) \quad \mathbf{x}_0 \in S_2 \end{gathered} \tag{2.3.e}$$

Here n is the single outer normal to the interface S_1 or S_2, $\tau_{ij}^k = -\frac{\varepsilon_k}{4\pi} \left(\frac{\left(E^k\right)^2}{2} \delta_{ij} - E_i^k E_j^k \right)$ is the Maxwell's electric stress tensor for the respective phases ($k = 1, 2, 3$). Its values are determined by the results of the electric problem (2.5) with boundary conditions $(2.1.a - e)$ and equations $E^k = -\nabla \varphi^k$. Let us assume that S_1 and S_2 are Lyapunov's surfaces. The decision of (2.3) with boundary conditions $(2.3.a - e)$ gives us the velocity at each moment in every point of S_1 and S_2. The deformation of the interfaces is determined in each moment by the normal component of the velocity and the kinematic condition:

$$\frac{d\mathbf{x}_s}{dt} = n_i\left(u_i.n_i\right) = n_i.u_n \tag{2.4}$$

Here $\mathbf{x}_s$ is a point of the respective surface S_1 or S_2, while u_n is the normal component of the velocity in this point.

Following Greengard [7] and [10] we get a system of integral equations and the following dimensionless parameters are included:

$E_\gamma = \dfrac{\varepsilon_2 E_0^2 R_1}{\gamma_2}-$ the relation between the electric and capillary forces;

$\mu_{21} = \dfrac{\mu_2}{\mu_1}, \mu_{32} = \dfrac{\mu_3}{\mu_2}-$ the relation of the viscousities of the different neighbouring phases;

$\varepsilon_{21} = \dfrac{\varepsilon_2}{\varepsilon_1}, \varepsilon_{32} = \dfrac{\varepsilon_3}{\varepsilon_2}-$ the relation of the electric permittivity of the different neighbouring phases;

$\sigma_{21} = \dfrac{\sigma_2}{\sigma_1}, \sigma_{32} = \dfrac{\sigma_3}{\sigma_2}-$ the relation of the electric conductivity of the different neighbouring phases.

$\gamma_{12} = \dfrac{\gamma_1}{\gamma_2}-$ the relation of the coefficient of interface tension of the two surfaces of the drops;

$R_{21} = \dfrac{R_2}{R_1}-$ the relation of the radii of the two drops.

3. Algorithm for determination of deformations of compound drop due to electric field. On each time step of the algorithm, first we solve the electrostatic problem which has influence on the hydrodynamic one by the way of Maxwell's electric stress tensor. On its turn, the solving of the hydrodynamic problem gives us velocities of the fluids in the different phases. Using the kinematic condition for the normal velocity components on the fluid surfaces, we get their deformation. With the new form (changed boundary conditions) we solve once again the electrostatic problem and after that the hydrodynamic one, as the number of time steps determines how many times this procedure will be done. The criteria for the end of procedure is reaching equilibrium form of the drops or "break up".

The main steps of the algorithm followed are:

— change of the co-ordinate system from Decart's to cylindrical, in order to transform the boundary integrals to one-dimensional;

— introduction of boundary elements over the boundaries of the domains - arcs of circles;

— introduction of local polar co-ordinate system on each boundary element;

— calculation of the integrals of the single- and double-layer over each boundary element;

— subtraction of the integrals' singularities;
— calculation of the velocity on the interfaces;
— determination of the drops' form from the kinematic condition.

We use the kinematic condition of the following type:

$\mathbf{x}_s^{new} = x_s + n_i\,(u_i.n_i)\,.dt$ where dt is a preliminary set time-step.

We assume that the form reaches the equilibrium when the normal component of the velocity becomes less than the preliminary set minimum at every point of the interfaces. Another criteria for end of the procedure is when the normal component becomes more than the initially set number; then we consider drop's break up.

For the calculations a project in Code Warrior C has been conducted, as the main results have been obtained through Power Mac 200/6400 in the Laboratory to the Department of Mechanics of Continua at the FMI of the Sofia University "St. Kl. Ohridski".

4. Results. The algorithm for obtaining the finite deformations of compound drop due to electric field was tested for a single drop in presence of electric field and had a good agreement with the results of Sherwood [13].

The deformation of fluid interfaces depends essentially on the position of the inner drop within the outer drop. When the distance between them is not small the deformation is very weak compared to the case when they are situated closer.

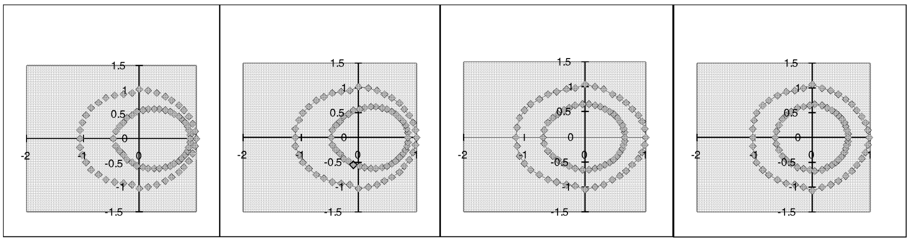

Fig. 1 Fig. 2 Fig. 3 Fig. 4

The distance between the centers of the drops on Fig. 1 is 0.35 R1, on Fig. 2 is 0.3 R1, on Fig. 3 is 0.15 R1 and on Fig. 4 is 0.14 R1 with $\mu_{21} = 1.00$, $\mu_{32} = 2.00$, $E_\gamma = 2.5$, $\varepsilon_{21} = 2.5$, $R_{21} = 0.6$, $\varepsilon_{32} = 1.5$, $\sigma_{21} = 20.0$, $\sigma_{32} = 25.0$, $dt = 0.01$, $\gamma_{12} = 1.0$

When we increase the intensity of the electric field, the equilibrium form reached of the two drops is with growing up deformation, but after the critical value of $E_\gamma = 3.294$ break up appears. On Fig. 5 $E_\gamma = 1.5$, on Fig. 6 $E_\gamma = 3.20$, on Fig. 7 $E_\gamma = 3.294$ with $\mu_{21} = 1$, $\varepsilon_{21} = 2.5$, $\mu_{32} = 2.0$, $R_{21} = 0.6$, $\varepsilon_{32} = 1.5$, $\sigma_{21} = 20$, $\sigma_{32} = 25$, $dt = 0.01$, $\gamma_{12} = 1.0$.

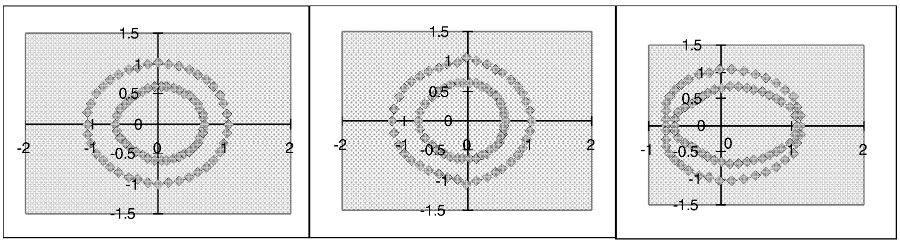

Fig. 5 Fig. 6 Fig. 7

The ratio between the permittivities ε_{21}causes different pictures of deformation of concentric drops as shown on Fig. 8, 9, 10, 11. On fig. 8 $\varepsilon_{21} = 0.5$, on Fig. 9 $\varepsilon_{21} = 1.0$,

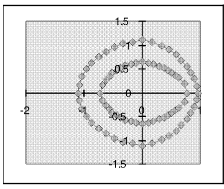
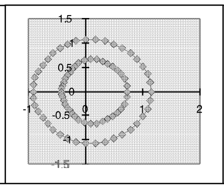
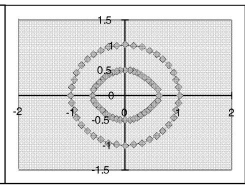
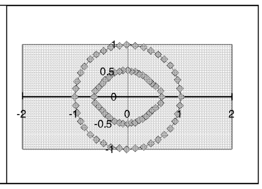

Fig. 8 Fig. 9 Fig. 10 Fig. 11

on Fig. 10 $\varepsilon_{21} = 1.5$, on Fig. 11 $\varepsilon_{21} = 2.5$ with $\mu_{21} = 1.5$, $E_\gamma = 5.5$, $\mu_{32} = 2.0$, $R_{21} = 0.5$, $\varepsilon_{32} = 2.0$, $\sigma_{21} = 20.0$, $\sigma_{32} = 25.0$, $dt = 0.01$, $\gamma_{12} = 1.0$. The change of ratio causes deformation on the outer interface S_1 but also on the inner drop. When the ratio become bigger than 2.5 the numerical simulation breaks down as predicted by Sherwood [13].

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ВЪРХУ КРАЙНИТЕ ДЕФОРМАЦИИ НА СЪСТАВНА КАПКА ПОРОДЕНИ ОТ ЕЛЕКТРИЧНО ПОЛЕ

Явор Христов, Запрян Запрянов

В настоящата работа се разглеждат крайните деформации на съставна капка, породени от електрично поле. Флуидите се предполагат хомогенни, несвиваеми и Нютонови. При достатъчно малко число на Рейнолдс в Стоксово приближение първоначалната форма на частиците се предполага сферична. Електрическата и хидродинамическата задачи са разделени като електрическата влияе на хидродинамичната чрез граничните условия и по-точно чрез тензора на Максуел. Уравненията на Максуел са сведени до Лапласови и заедно с уравненията на Стокс са решени полуаналитично-получислено, използвайки метода на граничните елементи. Кинематичното условие определя новата форма на частиците.

Резултатите показват, че под въздействието на електрично поле съставната капка се деформира. Влиянието на някои от безразмерните параметри е представено графично.