МАТЕМАТИКА И МАТЕМАТИЧЕСКО ОБРАЗОВАНИЕ, 2001
MATHEMATICS AND EDUCATION IN MATHEMATICS, 2001
Proceedings of Thirtieth Spring Conference of
the Union of Bulgarian Mathematicians
Borovets, April 8–11, 2001

AN ASSESSMENT OF GREEK STUDENTS' KNOWLEDGE ON PLACE VALUE CONCEPTS AND WHOLE NUMBER OPERATIONS

Sonia Kafoussi, Evagelos Ntziahristos

In recent years, mathematics educators have given a new perspective for mathematics teaching and learning, based on constructivist epistemology (cf. von Glasersfeld [19]). So, they accept that mathematical learning is a process in which children reorganize their mental activity as they try to resolve situations that are problematic for them (Cobb, Wood and Yackel [3]). This reorganization is mainly promoted as children reflect on their own activity as well as they interact with the other members of the classroom.

In our country, we are in the process of rewriting mathematics school textbooks, based on a new mathematics curriculum. Towards this purpose, we have started an effort to assess students' current knowledge on different topics existing in the curriculum in primary grades, so that we can locate the difficulties they have had. In this article, we present and compare the results on second and third grade students' knowledge, concerning place value concepts and whole number operations.

Theoretical background. In our country, the main topics in mathematics curriculum on second and third grade are place value concepts and the four whole-number operations with two and three digit numbers.

Place value concepts—addition and subtraction. In other countries, many researchers have observed that children have a lot of difficulties in understanding place value concepts as well as addition and subtraction algorithms (Kamii [9], Kouba et al. [10], Labinowicz [11]). As Fuson [7] has commented children interpret and treat multidigit numbers as single digit numbers placed adjacent to each other, rather than using multiunit meanings for the digits in different positions.

Moreover, in recent years, researchers have tried to construct models which describe the conceptual structures children must built for place value understanding. Steffe and Cobb [15] have emphasized that students have to construct initially the concept of ten as a unit for understanding place value. They have distinguished between three different concepts of ten: the numerical, the abstract and the iterable unit which students construct. According to their findings, the construction of the iterable unit is indispensable for the understanding of positional principle of the numeration system. Fuson [7] has observed that children in United States seem to use two different kinds of conceptual multiunit items for multiunit quantities, collected multiunits and sequence multiunits. Collected multiunits are based on collections of objects, where sequence multiunits are multiunits chunks within the number-word sequence. Jones et al. [8] have mentioned that research in mathematics education have shown that counting, partitioning, grouping and number relationships seem to constitute the key constructs for the development of multidigit number sense. The understanding of place-value concepts is a prerequisite for the learning of addition and subtraction algorithms.

Multiplication and division. Research on multiplication and division has been more limited than research on addition and subtraction. However, it is accepted that multiplication (or division) is initially conceived as a repeated addition (or as a repeated subtraction). Steffe [16] has emphasized that children's understanding of multiplication and division is based on their ability to develop and elaborate their counting scheme so that it can include composite units and has analyzed the kind of composite units that children construct. Moreover, Hans ter Heege [17] has given evidence that the students invent their own informal thinking strategies in the computation of multiplication results which can facilitate the memorization of the multiplication tables. On the other hand, many researchers have emphasized that the learning of multidigit multiplication and division algorithms demands the understanding of place value concepts and a proportional reasoning (cf. Lampert [12]).

Word problems. There is an extensive research work on arithmetic word problems (Carpenter and Moser [1], Carpenter, Hilbert and Moser [2], De Corte, Verschaffel and Verschueren [4], Fischbein et al. [5], Nesher [13], Riley, Greeno and Heller [14], Vergnaud [18]). Researchers have examined this subject from different perspectives. One of the most important conclusion of these studies is that the semantic structure of the problem influence children's solutions. According to their semantic nature, additive and subtraction word problems have been classified into three main categories: combine problems, change problems and compare problems. Moreover, multiplication and division word problems have also been classified in the following categories: Problems describing a mapping rule, multiplicative compare problems and cartesian multiplication problems.

These findings influenced the tasks we gave to the students and the interpretation of our results.

Methodology. 82 second-grade students and 151 third-grade students coming from all socio-economic levels of the population participated in our research program and the sample can be considered cross-section. Second graders were interviewed and a questionnaire was used for the third graders. The research took place at the end of the school year in 1996–97 and 1997–98.

In the presentation of our results, the percentages given correspond to the proportion of students responding correctly to an item.

Results. Place value. 68% of the second graders could identify the number of tens in a two digit number, but only 45% could write a two digit number given the number of tens and ones. Moreover, only 38% of the students could identify the number of tens between two given two digit numbers.

Third graders performance was even worse in analogous items on three digit numbers. 83% of the students failed to answer the questions correctly which were concerned the construction of a three-digit number, having given the number of the units of different ranges. Many students wrote four digit numbers without having taken into consideration the range of the unit as well as the number of the units. Other students added the three numbers which represented the number of the units of every range or omitted one of the numbers. For example, when we asked children to construct the number which was supposed to contain 25 simple units, 6 hundreds and 3 tens, many of them responded 2563, 628, 385 or 625.

47% of the third-grade students could identify the number of hundreds between two given three digit numbers, but only 14% could make a multiple partitioning of a three-digit number (e.g. using coins of a hundred drachma each, a ten drachma and one drachma, make 845 drachma in three different ways).

Addition and subtraction. The percentages of children who successfully completed each of the tasks are shown in Table 1 for the second graders and in Table 2 for the third graders.

Table 1

Item	Percent correct
$\begin{array}{r}35\\+\underline{38}\end{array}$	76%
$\begin{array}{r}54\\+\underline{27}\end{array}$	79%
$\begin{array}{r}75\\-\underline{26}\end{array}$	48%
$\begin{array}{r}84\\-\underline{56}\end{array}$	54%
$46 + 26$	55%
$72 - 59$	26%
$34 - \_ = 16$	18%
$28 + \_ = 62$	24%

Table 2

Item	Percent correct
$\begin{array}{r}345\\+\underline{428}\end{array}$	84%
$\begin{array}{r}492\\+\underline{375}\end{array}$	85%
$\begin{array}{r}689\\+\underline{267}\end{array}$	69%
$\begin{array}{r}853\\-\underline{427}\end{array}$	71%
$\begin{array}{r}936\\-\underline{263}\end{array}$	75%
$\begin{array}{r}704\\-\underline{587}\end{array}$	60%
$\_ + 250 = 960$	68%
$\_ - 320 = 110$	47%

Most second graders successfully performed addition algorithm involving regrouping, in contrast with their performance on subtraction algorithm which was much lower. The third- graders performance was much better on addition algorithms as well as on subtraction algorithms. These results show that perhaps the involvement of the students with three digit numbers facilitates their learning on these topics. The difficulties for the third graders appeared when addition and subtraction algorithms involved two regroupings.

Moreover, many problems were presented when we asked children to calculate sums and differences which were written horizontally. The performance of our pupils on these tasks was very poor. It is interesting to mention that the majority of the second graders who solved successfully these tasks wrote the numbers in a vertical form and executed the typical algorithm. When we asked them to solve these problems in a different way, they could not provide us an alternative strategy. This suggests that even the students who responded correctly had not the ability to use other strategies to solve these problems.

Multiplication and division. The following tables show second (cf. Table 3) and third grade students (cf. Table 4) performance on multiplication and division facts as well as on multiplication and division algorithms.

Table 3

Item	Percent correct
6×7	45%
7×7	44%
4×9	46%
8×9	35%
25:5	26%
30:5	23%
42:6	17%
42:7	16%

Table 4

Item	Percent correct
68 ×7	62%
139 ×6	62%
48 ×19	44%
376:5	60%
689:3	54%

About 10% of the second grade students who responded correctly on the multiplication facts used the model of repeated addition (They wrote them as a sum) The majority tried to recall the results by memory. The behavior of the children who responded correctly on the division facts was similar. However, as it is shown in Table 3, the students' performance on the division items was lower than their performance on the multiplication items. Many students could not give any answer, because they could not "remember" the result immediately. Unfortunately, they could not find an alternative strategy for these calculations. They have rather adopted the belief that the memorization of these facts was indispensable for the production of the correct answers.

For the third graders, we observed that their errors were due either to an inadequate knowledge of the multiplication tables and division facts or to a misunderstanding of multiplication and division algorithms (e.g. on the multiplication of a two-digit number with another two digit number they multiplied independently units by units and tens by tens or they added the digit to be carried over in a wrong way).

Word problems on addition and subtraction. Both second-grade (cf. Table 5) and third-grade (cf. Table 6) students had many difficulties to solve word problems.

We considered that students answers were correct, only if they had chosen the right operation and they had executed the calculations correctly. We found out that about half of the wrong answers were due to a wrong choice of the operation and the others to a wrong result. The first problem was a simple combine problem and was considered the easiest of all. However, it appears that the students in both classrooms seem to have a lot of problems with change problems and compare word problems.

Multiplication and division word problems. The results of the second and the third graders' performance on multiplication and division word problems are shown on the Tables 7 and 8 respectively.

The performance of the students in solving multiplication and division problems was very poor. In contrast with the results we have already mentioned on addition and subtraction word problems, in this case errors due to the selection of a wrong operation were dominant. More specifically, the operation of addition was used by the majority of the students.

Conclusions. The results of our research program indicate that both the second and the third grade students have a superficial understanding of the mathematical items we assessed.

Table 5

Item	Percent correct
George has 38 books in his library. Mary has 45 books in hers. How many books have they both got?	72%
Mary has 67 dr. She want to buy a copybook which costs 100 dr. How many dr. does she need?	28%
Ann has collected 54 stamps. She gave 18 stamps to her sister. How many stamps does she have now?	28%
Tasos has 37 marbles. Nick has 35 marbles more than Tasos. How many marbles does Nick have?	49%
Helen has 56 dr. in her pocket. Mary has 17 dr. less than Helen. How many dr. does Mary have?	34%

Table 6

Item	Percent correct
Last week the students of two schools watched a theatrical performance. The first school had 387 students and the second school had 268 students. How many students did they watch the performance?	76%
A year has 365 days. The summer, the Christmas and the east vacations last 109 days. How many days do children go to school?	50%
John and Nick collect stamps. John has 138 stamps and Nick has 84 stamps more than John. How many stamps does Nick have?	61%
The students of a school participated in the reforestation of a mountain. Third grade students planted 235 trees. Fourth grade students planted 76 trees less than three graders. How many trees do fourth grade students plant?	46%
Nick bought a copybook which cost 480 dr. and a pencil which cost 218 dr. He gave 1000 dr. to the bookseller. What change did he take?	44%

Table 7

Item	Percent correct
Helen bought 8 boxes of chocolates. Every box contains 7 chocolates. How many chocolates does Helen have?	32%
Four friends want to share fairly 36 marbles. How many marbles will everyone take?	20%
Mary bought 42 sweets. She wants to put them in sachets. Every sachet contains 6 sweets. How many sachets will she need?	17%

Table 8

Item	Percent correct
The bus which carries out the itinerary Kiceli-Pagrati runs 12 itineraries every day. In every itinerary it carries 38 passengers. How many passengers does it carry every day?	46%
Helen bought 5 roses and she paid 165 dr. The next day, she bought 7 daisies and she paid 623 dr. How many dr. does a rose cost? How many dr. does a daisy cost?	29%
A block of flats has three floors. Every floor has 5 apartments and every apartment has 4 rooms. How many rooms does the block of flats have?	42%

Firstly, the great failure of our students to items concerning place value concepts shows that they have not become conscious of the structure of numeration system. In our opinion, this also means that their better performance on the execution of algorithms (especially for addition and subtraction) can not be interpreted as an indication of the understanding of the logic underlying these processes. They, rather, execute mechanically them, by applying the rule “add or subtract the units of the same ranges” or by imitating the steps of these processes that their teacher have taught them.

Secondly, students have not developed a variety of thinking strategies to solve an addition or subtraction arithmetical sentence. Even though they had been taught different strategies on these topics at school, they could not utilize them.

Furthermore, the memorization seems to be the only pathway for obtaining the correct results on multiplication and division.

Finally, problem solving seems to generate many difficulties to students. This means that students had not the opportunity to develop their cognitive schemata in such a way that could allow them to solve correctly different kinds of word problems. Many of them manipulate the numbers accidentally, without taking into account the structure of the problem.

The failure of our students can be interpreted as a result of the way that school textbooks present these mathematical topics. According to the existing textbooks, children are obliged to follow the predetermined rules presenting at the top of the page and to apply them in many exercises. They have not the opportunity to construct the conceptual structures underlying these mathematical topics (cf. Theoretical background). Furthermore, word problems are presented as an application of these rules and not as the basis for the teaching of the mathematical concepts (cf. Freudenthal [6]).

In our country, we are in the process to organize primary mathematics education according to recent recommendations of mathematics educators. We hope that this effort can positively influence students' beliefs and their knowledge about mathematics.

REFERENCES

[1] T. P. Carpenter and J. M. Moser. The development of addition and subtraction problem solving skills. In: Addition and subtraction: A cognitive perspective (Eds T. P. Carpenter, J. M. Moser, T. A. Romberg) Hillsdale, N. J., LEA, 1982, 9–24.

[2] T. P. Carpenter, J. Hiebert and J. M. Moser. The effect of problem structure on first graders initial solution processes for simple addition and subtraction problems. *Journal for Research in mathematics education*, **12**, No 1 (1981), 27–39.

[3] P. Cobb, T. Wood, E. Yackel. A constructivist approach to second grade mathematics. In: Radical Constructivism in Mathematics Education (Ed. E. von Glasersfeld) Kluwer Academic Publishers, 1991, 157–176.

[4] E. De Corte, L. Verschaffel, J. Verschueren. First graders solution processes in elementary word problems. In: Proceedings of the Sixth Conference of the International Group for the PME (Ed. A. Vermandel) Antwerpen, 1982, 91–96.

[5] E. Fischbein et al. The role of implicit models in solving verbal problems in multiplication and division. *Journal for Research in Mathematics Education*, **16**, No 1 (1985), 3–17.

[6] H. Freudenthal. Didactic phenomenology of mathematical structures. Dordrecht, Reidel, 1983.

[7] K. C. Fuson. Research on whole number. Addition and subtraction. In: Handbook of research on Mathematics Teaching and Learning (Ed. D. Grouws) NY, Macmillan, 1992, 243–275.

[8] G. Jones et al. Multidigit number sense: A framework for instruction and assessment. *Journal for Research in Mathematics Education*, **27**, No 3, 1996, 310–336.

[9] C. Kamii. Place value: An explanation of its difficulty and educational implications for the primary grades. *Journal of Research in Childhood Education*, **1**, No 2 (1986), 75–86.

[10] Kouba et al. Results of the fourth NAEP assessment of Mathematics: Number, operations and word problems. *Arithmetic Teacher*, **35**, No 8 (1988), 14–19.

[11] E. Labinowicz. Learning from children: New beginnings for teaching numerical thinking. Menlo Park, CA, Addison-Wesley, 1985.

[12] M. Lampert. Teaching and learning long division for understanding in school In: Analysis of arithmetic for Mathematics teaching (Eds G. Leinhardt, R. Putnam, R. Hattrup) LEA, 1992, 221–282.

[13] P. Nesher. Solving multiplication word problems. In: Analysis of arithmetic for Mathematics teaching (Eds G. Leinhardt, R. Putnam, R. Hattrups) LEA, 1992, 189–219.

[14] M. S. Riley, J. G. Greeno and J. I. Heller. Development of children's problem solving ability in arithmetic. In: The development of mathematical thinking (Ed. H. P. Ginsburg) NY, Academic Press, 1983, 153–196.

[15] L. P. Steffe, P. Cobb. Construction of arithmetical meanings and strategies. NY, Springer-Verlag, 1988.

[16] L. P. Steffe. Children's multiplying schemes. In: The development of multiplicative reasoning in the learning of mathematics (Eds G. Harel, J. Confrey) NY, State University of New York Press, 1994, 3–40.

[17] H. ter Heege. The acquisition of basic multiplication skills. *Educational Studies in Mathematics*, **16** (1985), 375–388.
[18] G. Vergnaud. Multiplicative structures. In: Number concepts and operations in the middle grades (Eds J. Hiebert, M. Behr) LEA, 1988, 141–161.
[19] E. von Glasersfeld. An exposition of constructivism: Why some like it Radical. *Journal for Research in Mathematics Education*, Monograph number 4, 1990, 19–29.

Sonia Kafoussi
Evagelos Ntziahristos
University of Athens
Athens, Greece

ОЦЕНЯВАНЕ НА ЗНАНИЯТА НА ГРЪЦКИТЕ УЧЕНИЦИ ВЪРХУ ПОНАТИЯ, СВЪРЗАНИ С ПОЗИЦИОННА БРОЙНА СИСТЕМА И С ОПЕРАЦИИ С ЦЕЛИ ЧИСЛА

С. Кафуси, Е. Циахристос

В последните години дейците в областта на математическото образование дават нова перспектива на обучението по математика, която се основава на конструктивната епистемология (вж. [19]). Те приемат схващането, че математическото обучение е процес, при който децата реорганизират умствената си дейност при опитите си да решат проблемни за тях ситуации [3]. Тази реорганизация се извършва най-вече, когато децата размишляват върху собствената си дейност и когато общуват с други деца от класа.

В Гърция понастоящем се пишат нови учебници по математика въз основа на ново учебно съдържание. Във връзка с това започнахме да оценяваме знанието на учениците по различни теми от учебното съдържание в началните класове, за да идентифицираме трудностите, които те срещат. В настоящата статия представяме и сравняваме резултатите от оценяване на знанието на ученици от втори и трети клас върху понятия, свързани с позиционна бройна система и с операции с цели числа.