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CONFORMAL TRANSFORMATION OF SPECIAL COMPOSITIONS IN A THREE-DIMENSIONAL WEYL SPACE*

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Special compositions, generated by a net in a space with a symmetric linear connection are considered in [6]. In [6] it is also introduced the prolonged covariant differentiation of satellites of the metric tensor of a Weyl space. In this paper, special compositions generated by a net in a 3-dimensional Weyl space are studied. Conformal geometry of special compositions in a 3-dimensional Weyl space is considered. It is proven, that an orthogonal Cartesian composition exists only in a 3-dimensional Riemannian space, where the form of the curvature tensor is found.

1. Preliminaries. Let $W_3\left(g_{ij},\omega_k\right)$ be a 3-dimensional Weyl space with a metric tensor g_{ij} and a complementary vector ω_k. The coefficients of the Weyl connection ∇ are determined by the equation: $\Gamma^k_{ij}=\left\{ {k \atop ij} \right\}-\left(\omega_i\delta^k_j+\omega_j\delta^k_i-g_{ij}g^{ks}\omega_s\right)$, where $\left\{ {k \atop ij} \right\}$ are the Cristoffel symbols, determined by g_{ij}, $\det(g_{ij})\neq 0$. The following equations are valid: $\nabla_k g_{ij}=2\omega_k g_{ij}$, $\nabla_k g^{ij}=-2\omega_k g^{ij}$ [9]. Following [7], the prolonged covariant differentiation $\overset{\circ}{\nabla}$ of the satellite A with weight $\{p\}$ in the Weyl space is defined by $\overset{\circ}{\nabla}_i A=\nabla_i A-p\omega_i A$.

Let $(\underset{1}{v},\underset{2}{v},\underset{3}{v})$ be a net in W_3, defined by the independent tangent vector fields $\underset{k}{v}^i$ of the curves of the net $(k=1,2,3)$. We determine the inverse covectors $\overset{k}{v}_i$ of $\underset{k}{v}^i$ $(k=1,2,3)$, respectively, by the equations:

$$\underset{i}{v}^k\overset{s}{v}_k=\delta^s_i \quad\Leftrightarrow\quad \underset{i}{v}^k\overset{i}{v}_s=\delta^k_s. \tag{1.1}$$

In the paper [7] there are found the derivative equations:

$$\overset{\circ}{\nabla}_i\underset{k}{v}^s=\underset{k}{\overset{m}{T}}_i\underset{m}{v}^s,\quad \overset{\circ}{\nabla}_i\overset{k}{v}_s=-\underset{m}{\overset{k}{T}}_i\overset{m}{v}_s,\qquad k=1,2,3. \tag{1.2}$$

Later we will consider a net $(\underset{1}{v},\underset{2}{v},\underset{3}{v})\in W_3$, for which the independent tangent vector fields $\underset{k}{v}^i$ are normalized by the terms [9]:

$$g_{ij}\underset{1}{v}^i\underset{1}{v}^j=g_{ij}\underset{2}{v}^i\underset{2}{v}^j=g_{ij}\underset{3}{v}^i\underset{3}{v}^j=1,\quad \cos\underset{sk}{\omega}=g_{ij}\underset{s}{v}^i\underset{k}{v}^j, \tag{1.3}$$

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where $\underset{sk}{\omega} = \underset{ks}{\omega}$ are the angles defined by $\underset{s}{v}$ and $\underset{k}{v}$, $s, k = 1, 2, 3$, $s \neq k$. In the paper [4] the following relations are given:

$$g_{ik}\underset{s}{v}^k = \cos\underset{s1}{\omega}\overset{1}{v}_i + \cos\underset{s2}{\omega}\overset{2}{v}_i + \cos\underset{s3}{\omega}\overset{3}{v}_i, \qquad s = 1, 2, 3. \tag{1.4}$$

The net $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v}) \in W_3$, for which conditions (1.3) and (1.4) are valid, will be called *normalized*. Let us remark that normalized nets $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v}) \in V_3$ are studied in the paper [1]. According to [4, Lemma 1.1] for the coefficients of equations (1.2) the following equations are valid:

$$\cos\underset{s1}{\omega}\underset{s}{\overset{1}{T}}_k + \cos\underset{s2}{\omega}\underset{s}{\overset{2}{T}}_k + \cos\underset{s3}{\omega}\underset{s}{\overset{3}{T}}_k = 0, \ \cos\underset{km}{\omega}\underset{s}{\overset{m}{T}}_i + \cos\underset{sm}{\omega}\underset{k}{\overset{m}{T}}_i = \partial_i\cos\underset{ks}{\omega}, \ k, s = 1, 2, 3. \tag{1.5}$$

Let us take a given composition $X_2 \times X_1$ in W_3, where X_2 ($\dim X_2 = 2$) and X_1 ($\dim X_1 = 1$) are the fundamental manifolds of the composition. Then through each point $p \in W_3$ there exists exactly one position $P(X_2)$ and $P(X_1)$, from X_2 and X_1 respectively. Following [5], W_3 is a space of composition $W_3(X_2 \times X_1)$, provided there exists a tensor field a_i^j of type $(1, 1)$ for which are valid the following equations:

$$a_i^j a_j^k = \delta_i^k, \tag{1.6}$$

and the condition for integration of the structure a_i^j. According to [5], the Nijenhuis tensor N_{ij}^k for a_i^j.is annulled, i.e. $N_{ij}^k = a_i^s\nabla_s a_j^k - a_j^s\nabla_s a_i^k - a_s^k\left(\nabla_i a_j^s - \nabla_j a_i^s\right) = 0$.

In [6] is defined the affinor a_i^k of the composition in the Weyl space W_n. In W_3 for affinor a_i^k, determined uniquely by the net $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v})$, are realized the following conditions:

$$a_i^k = \underset{1}{v}^k\overset{1}{v}_i + \underset{2}{v}^k\overset{2}{v}_i - \underset{3}{v}^k\overset{3}{v}_i = \delta_i^k - \overset{3}{v}_i\underset{3}{v}^k, \quad a_k^s\underset{1}{v}^k = \underset{1}{v}^s, \quad a_k^s\underset{2}{v}^k = \underset{2}{v}^s, \quad a_k^s\underset{3}{v}^k = -\underset{3}{v}^s. \tag{1.7}$$

Let τ be a conformal transformation of $W_n\,(g_{ij}, \omega_k)$ into $\overline{W}_n\left(\overline{g}_{ij}, \overline{\omega}_k\right)$. Then following [9], in the corresponding points of these spaces we have: $\overline{g}_{ij} = g_{ij}$, $\overline{\omega}_i = \omega_i - p_i$, where the covector p_i is called *the vector of the conformal transformation* τ.

Let $\overline{\Gamma}_{ij}^k$ and Γ_{ij}^k be the coefficients of the Weyl connections of $\overline{W}_3$ and W_3, respectively. Then we have [9]: $\overline{\Gamma}_{ij}^k = \Gamma_{ij}^k + \delta_i^k p_j + \delta_j^k p_i - g_{ij}g^{ks}p_s$.

Let W_3 and $\overline{W}_3$ be conformally equivalent Weyl spaces. Then with respect of the connection ∇ of W_3 the derivative equations give the expression of (1.2), while with respect of the connection $\overline{\nabla}$ of $\overline{W}_3$ they have the following form:

$$\overset{\circ}{\overline{\nabla}}_i\underset{k}{v}^s = \underset{k}{\overset{m}{P}}_i\underset{m}{v}^s, \quad \overset{\circ}{\overline{\nabla}}_i\overset{k}{v}_s = -\underset{m}{\overset{k}{P}}_i\overset{m}{v}_s, \quad k = 1, 2, 3. \tag{1.8}$$

The relation between the coefficients in (1.2) and (1.8) in the case of conformal transformation τ is found in [8], i.e.

$$\underset{s}{\overset{l}{P}}_k = \underset{s}{\overset{l}{T}}_k + p_m\underset{s}{v}^m\overset{l}{v}_k - p\overset{l}{v}g_{km}\underset{s}{v}^m, \tag{1.9}$$

where $p\overset{l}{v} = g^{nk}p_n\overset{l}{v}_k$, $s = 1, 2, 3$, $l = 1, 2, 3$.

The vector of the conformal transformation p_k has the form:

$$p_j = \left(p_m\underset{1}{v}^m\right)\overset{1}{v}_j + \left(p_m\underset{2}{v}^m\right)\overset{2}{v}_j + \left(p_m\underset{3}{v}^m\right)\overset{3}{v}_j. \tag{1.10}$$

We can assert that when we have a conformal transformation τ of compositions $X_2 \times$

$X_1 \in W_3$ and $X_2 \times X_1 \in \overline{W}_3$, associated with the normalized net $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v})$, conditions (1.5) are valid about $\underset{s}{\overset{l}{T}}_k$ and their analogous equations about $\underset{s}{\overset{l}{P}}_k$.

2. Conformal transformation of a composition in $\boldsymbol{W_3}$. Following [10], the composition $X_2 \times X_1 \in W_3$ is called *geodesic-Chebyshevian*, if the tangent section of $P(X_2)$ and the tangent vector of the curve $P(X_1)$ can be translated parallelly in the direction of every curve of $P(X_2)$.

The composition $X_2 \times X_1 \in W_3$ is called *Chebyshevian-geodesic*, if the tangent section of $P(X_2)$ is translated parallelly in the curve $P(X_1)$, and the tangent vector of $P(X_1)$ is translated parallelly in the curve $P(X_1)$, i.e. the curve $P(X_1)$ is geodesic.

Definition 2.1. *A composition $X_2 \times X_1 \in W_3$ is called conformally geodesic-Chebyshevian (respectively conformally Chebyshevian-geodesic) when it can be transformed into a geodesic-Chebyshevian (respectively Chebyshevian-geodesic) composition $X_2 \times X_1 \in \overline{W}_3$ by the transformation τ.*

In [3, Theorem 1, Theorem 3, Theorem 4] are found geometrical characteristics and conditions for geodesic-Chebyshevian and Chebyshevian-geodesic compositions, i.e.

1) If $X_2 \times X_1 \in \overline{W}_3$ is a geodesic-Chebyshevian composition, then according to [3, Theorem 1], we obtain:

$$\underset{3}{\overset{1}{P}}_k \underset{1}{v}^k = \underset{3}{\overset{1}{P}}_k \underset{2}{v}^k = \underset{3}{\overset{2}{P}}_k \underset{1}{v}^k = \underset{3}{\overset{2}{P}}_k \underset{2}{v}^k = \underset{1}{\overset{3}{P}}_k \underset{1}{v}^k = \underset{1}{\overset{3}{P}}_k \underset{2}{v}^k = \underset{2}{\overset{3}{P}}_k \underset{1}{v}^k = \underset{2}{\overset{3}{P}}_k \underset{2}{v}^k = 0. \tag{2.1}$$

2) If $X_2 \times X_1 \in \overline{W}_3$ is a Chebyshevian-geodesic composition, then according to [3, Theorem 3, Theorem 4], we obtain:

$$\underset{3}{\overset{1}{P}}_k \underset{3}{v}^k = \underset{3}{\overset{2}{P}}_k \underset{3}{v}^k = \underset{1}{\overset{3}{P}}_k \underset{3}{v}^k = \underset{2}{\overset{3}{P}}_k \underset{3}{v}^k = 0, \qquad \underset{1}{\overset{2}{P}}_k \underset{3}{v}^k = \underset{2}{\overset{1}{P}}_k \underset{3}{v}^k = \left(\underset{s}{\overset{s}{P}}_k - \varpi_k \right) \underset{3}{v}^k = 0, \qquad s = 1, 2, 3. \tag{2.2}$$

Theorem 2.1. *A composition $X_2 \times X_1 \in W_3$, determined by the normalized net $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v})$, is a conformally geodesic-Chebyshevian if and only if the following conditions are valid:*

$$\left| \begin{array}{ll} \underset{3}{\overset{1}{T}}_k \underset{2}{v}^k = \cos \underset{23}{\omega} p \overset{1}{v}, & \underset{3}{\overset{2}{T}}_k \underset{1}{v}^k = \cos \underset{13}{\omega} p \overset{2}{v}, \\ \underset{1}{\overset{3}{T}}_k \underset{1}{v}^k = \underset{2}{\overset{3}{T}}_k \underset{2}{v}^k = p \overset{3}{v}, & \underset{3}{\overset{3}{T}}_k \underset{1}{v}^k = \cos \underset{13}{\omega} p \overset{3}{v}, \\ \underset{3}{\overset{3}{T}}_k \underset{2}{v}^k = \cos \underset{23}{\omega} p \overset{3}{v}, & \underset{3}{\overset{1}{T}}_k \underset{1}{v}^k = \cos \underset{13}{\omega} p \overset{1}{v} - p_m \underset{3}{v}^m, \\ \underset{3}{\overset{2}{T}}_k \underset{2}{v}^k = \cos \underset{23}{\omega} p \overset{2}{v} - p_m \underset{3}{v}^m, & \underset{1}{\overset{3}{T}}_k \underset{2}{v}^k = \underset{2}{\overset{3}{T}}_k \underset{1}{v}^k = \cos \underset{12}{\omega} p \overset{3}{v}. \end{array} \right. \tag{2.3}$$

The vector p_k of the conformal transformation τ satisfies the following condition:

$$\begin{gathered} 2 p_m \underset{1}{v}^m = p \overset{1}{v} + \cos \underset{12}{\omega} p \overset{2}{v} + \underset{3}{\overset{3}{T}}_k \underset{1}{v}^k, \quad p_m \underset{2}{v}^m = \cos \underset{12}{\omega} p \overset{1}{v} + p \overset{2}{v} + \underset{3}{\overset{3}{T}}_k \underset{2}{v}^k, \\ p_m \underset{3}{v}^m = - \left(\underset{1}{\overset{3}{T}}_k \underset{1}{v}^k + \underset{3}{\overset{1}{T}}_k \underset{1}{v}^k + \underset{3}{\overset{2}{T}}_k \underset{2}{v}^k \right). \end{gathered} \tag{2.4}$$

Proof. Let $X_2 \times X_1 \in W_3$ be a conformally geodesic-Chebyshevian composition. Then for $X_2 \times X_1 \in \overline{W}_3$ equations (2.1) are valid. After contracting equation (1.9) to the vectors $\underset{1}{v}^k$ and $\underset{2}{v}^k$, and having in mind (1.5) and (2.1), we obtain (2.3).

Conversely, if equations (2.3) are valid, then from (1.9) it follows (2.1). In order to determine the vector of the conformal transformation p_k from (1.10), we use the functions $\overset{1}{pv}$, $\overset{2}{pv}$ and $\overset{3}{pv}$ from (2.3). Equations (2.4) are obtained through a suitable transformation of (2.3). $\square$

Analogously, using (1.5), (1.9) and (2.2), it follows that:

Theorem 2.2 *A composition $X_2 \times X_1 \in W_3$, determined by the normalized net $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v})$, is conformally Chebyshevian-geodesic if and only if the following conditions are valid:*

$$
(2.5) \qquad \left|
\begin{array}{l}
\overset{1}{\underset{3}{T}}_k \underset{3}{v}^k = \overset{1}{pv}, \quad \overset{2}{\underset{3}{T}}_k \underset{3}{v}^k = \overset{2}{pv}, \quad \overset{1}{\underset{1}{T}}_k \underset{3}{v}^k = \cos \underset{13}{\omega} \overset{1}{pv}, \quad \overset{2}{\underset{2}{T}}_k \underset{3}{v}^k = \cos \underset{23}{\omega} \overset{2}{pv}, \\
p_m \underset{1}{v}^m = \left(\cos \underset{13}{\omega} \omega_k + \overset{3}{\underset{3}{T}}_k - \overset{3}{\underset{1}{T}}_k \right) \underset{3}{v}^k, \quad p_m \underset{2}{v}^m = \left(\cos \underset{23}{\omega} \omega_k + \overset{3}{\underset{3}{T}}_k - \overset{3}{\underset{2}{T}}_k \right) \underset{3}{v}^k, \\
p_m \underset{3}{v}^m = \omega_k \underset{3}{v}^k = \overset{3}{pv} - \overset{3}{\underset{3}{T}}_k \underset{3}{v}^k, \quad \overset{2}{\underset{1}{T}}_k \underset{3}{v}^k = \cos \underset{13}{\omega} \overset{2}{pv}, \quad \overset{1}{\underset{2}{T}}_k \underset{3}{v}^k = \cos \underset{23}{\omega} \overset{1}{pv}.
\end{array}
\right.
$$

The vector p_k of the conformal transformation τ has the form (1.10)*, where the coefficients $p_m \underset{s}{v}^m$, $s = 1, 2, 3$, are defined in* (2.5)*.*

3. Orthogonal compositions in W_3.

Definition 3.1 [11]. *A composition $X_2 \times X_1 \in W_3$ is orthogonal, when the vectors $\underset{1}{v}^k$, $\underset{3}{v}^k$ and $\underset{2}{v}^k$, $\underset{3}{v}^k$ are orthogonal, i.e.*

$$
(3.1) \qquad g_{ij} \underset{1}{v}^i \underset{3}{v}^j = g_{ij} \underset{2}{v}^i \underset{3}{v}^j = 0 \quad \Longleftrightarrow \quad \cos \underset{13}{\omega} = \cos \underset{23}{\omega} = 0.
$$

In the paper [11] it is introduced a tensor of type $(0, 2)$:

$$
(3.2) \qquad a_{ij} = a_i^k g_{kj}.
$$

which is called *the tensor on the composition* in W_n. It is proved, that a composition in W_n is orthogonal if and only if $a_{ij} = a_{ji}$. In this case, the tensor a_{ij} is called *associated* with the metric tensor g_{ij}. From (3.2) it follows, that a_{ij} is nondegenerate, i.e. $\det(a_{ij}) \neq 0$.

Let the composition $X_2 \times X_1 \in W_3$, determined by the normalized net $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v})$ be orthogonal. Using (1.6), (3.2) and $a_{ij} = a_{ji}$, for the metric tensor g_{ij} we have the form:

$$
(3.3) \qquad a_i^s a_j^k g_{sk} = g_{ij}.
$$

According to [5], the condition (3.3) for the metric tensor means, that we can consider W_3 as a Riemannian space with the structure of an almost product a_i^j about the connection $\widetilde{\nabla}$, determined by g_{ij}. Having in mind (1.1), (1.3), (1.7), (3.1), (3.2) and (3.3) we get:

Lemma 3.1. *The Weyl space W_3 is a space of orthogonal composition $X_2 \times X_1$, determined by the normalized net $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v})$ if and only if the metric tensor g_{ij} and the tensor a_{ij} associated with it, have the form:*

$$
(3.4) \qquad g_{ij} = \overset{1}{v}_i \overset{1}{v}_j + \overset{2}{v}_i \overset{2}{v}_j + \overset{3}{v}_i \overset{3}{v}_j + \cos \underset{12}{\omega} \left(\overset{1}{v}_i \overset{2}{v}_j + \overset{2}{v}_i \overset{1}{v}_j \right), \quad a_{ij} = g_{ij} - 2 \overset{3}{v}_i \overset{3}{v}_j.
$$

Taking into account (1.3), (3.1), (3.3) and (3.4), the tensor g_{ij} in W_3 determines a *positive definite metric* (Riemannian), and the tensor a_{ij} – an *associated* metric, which is indefinite with signature $(2,1)$.

Theorem 3.1. *Let the conformally geodesic-Chebyshevian composition $X_2 \times X_1 \in W_3$ be orthogonal. Then the following relations are valid:*

$$\underset{3}{\overset{3}{T}}_k = 0,\ \underset{3}{\overset{1}{T}}_k \underset{2}{v}^k = \underset{3}{\overset{2}{T}}_k \underset{1}{v}^k = 0,\ \underset{1}{\overset{3}{T}}_k \underset{1}{v}^k = \underset{2}{\overset{3}{T}}_k \underset{2}{v}^k = p_m \underset{3}{v}^m \quad . \tag{3.5}$$

The metric tensor on W_3 has the form (3.4).

Proof. The equations (3.5) follow from (3.1), (3.4) and Theorem 2.1. □

Having in mind Theorem 2.2, (3.1) and (3.4), we get the following:

Theorem 3.2. *Let the conformally Chebyshevian-geodesic composition $X_2 \times X_1 \in W_3$ be orthogonal. Then the following conditions are valid:*

$$\underset{3}{\overset{3}{T}}_k = 0,\ \underset{1}{\overset{1}{T}}_k \underset{3}{v}^k = \underset{2}{\overset{1}{T}}_k \underset{3}{v}^k = \underset{2}{\overset{2}{T}}_k \underset{3}{v}^k = \underset{1}{\overset{2}{T}}_k \underset{3}{v}^k = 0. \tag{3.6}$$

The metric tensor of W_3 has the form (3.4), *and the vector of the conformal transformation τ has the form:*

$$p_k = -\underset{1}{\overset{3}{T}}_s \underset{3}{v}^s \overset{1}{v}_k - \underset{2}{\overset{3}{T}}_s \underset{3}{v}^s \overset{2}{v}_k + \omega_s \underset{3}{v}^s \overset{3}{v}_k. \tag{3.7}$$

Let the composition $X_2 \times X_1 \in W_3$ be orthogonal and Cartesian. Then, according to [2], for the affinor a_k^s with respect to the Weyl connection ∇, we have: $\nabla_j a_k^s = 0$. According to [9], the integrability condition for the last equation has the form:

$$R_{ijk.}{}^{l} a_l^s = R_{ijl.}{}^{s} a_k^l. \tag{3.8}$$

where $R_{ijk}{}^{s}$ is the curvature tensor for connection ∇. In [2, Theorem 5] it is proven, that the curvature tensor of an arbitrary Weyl space W_3 has the following form:

$$R_{ijk.}{}^{s} = \frac{1}{3}\left\{\left(g_{jk}S_{im} - g_{ik}S_{jm}\right)g^{ms} + S_{jk}\delta_i^s - S_{ik}\delta_j^s + \left(S_{ji} - S_{ij}\right)\delta_k^s\right\}, \tag{3.9}$$

where $S_{jk} = 2R_{jk} + R_{kj} - \frac{3R}{4}g_{jk}$, R_{jk} is the Ricci tensor, and $R = g^{jk}R_{jk}$ - the scalar curvature. Using (1.1) and (1.7), we can prove that the equation (3.8) is equivalent to:

$$R_{ijk.}{}^{l} \overset{3}{v}_l \underset{3}{v}^s = R_{ijl.}{}^{s} \underset{3}{v}^l \overset{3}{v}_k. \tag{3.10}$$

Substituting (3.9) in (3.10) and using a series of transformations, in view of (1.7), (3.3) and (3.4), we obtain:

$$S_{kj} = S_{jk} = \frac{3R}{4}\left(g_{jk} - 2\overset{3}{v}_j\overset{3}{v}_k\right) = \frac{3R}{4}a_{jk}, \quad R_{jk} = \frac{R}{2}\left(g_{jk} - \overset{3}{v}_j\overset{3}{v}_k\right) = \frac{R}{4}\left(g_{jk} + a_{jk}\right).$$

According to (3.9) and the last equation, because of the symmetry of Ricci tensor, it follows that:

Theorem 3.3. *Every W_3, containing an orthogonal Cartesian composition $X_2 \times X_1$, is a Riemannian space V_3. The curvature tensor of V_3 of type $(0,4)$ has the form:*

$$R_{ijkl} = \frac{1}{3}\left(g_{jk}g_{il} - g_{ik}g_{jl} - \overset{3}{v}_j\overset{3}{v}_k g_{il} + \overset{3}{v}_i\overset{3}{v}_k g_{jl} - \overset{3}{v}_i\overset{3}{v}_l g_{jk} + \overset{3}{v}_j\overset{3}{v}_l g_{ik}\right),$$

where $R_{ijkl} = g_{ls}R_{ijk.}{}^{s}$ is determined by the Riemannian connection $\widetilde{\nabla}$ with components *the Cristoffel symbols* $\left\{ {k \atop ij} \right\}$.

Using (1.3), we immediately obtain the following:

Corollary 3.1. *Let* V_3 *be a Riemannian space, containing the orthogonal Cartesian composition* $X_2 \times X_1$*. Then, for the Ricci curvatures in the direction of the net vectors* $(\underset{1}{v}, \underset{2}{v}, \underset{3}{v})$*, the following equalities hold:*

$$R_{jk}\underset{1}{v}^j \underset{1}{v}^k = R_{jk}\underset{2}{v}^j \underset{2}{v}^k = \frac{R}{2}, \quad R_{jk}\underset{3}{v}^j \underset{3}{v}^k = 0.$$

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КОНФОРМНА ТРАНСФОРМАЦИЯ НА СПЕЦИАЛНИ КОМПОЗИЦИИ В ТРИМЕРНО ВАЙЛОВО ПРОСТРАНСТВО

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Специални композиции, породени от мрежа в пространство със симетрична линейна свързаност са изучавани в [6]. В [6] е въведено продължено ковариантно диференциране на сателитите на метричния тензор във Вайлово пространство. В тази статия изучаваме специални композиции, породени от мрежа в тримерно Вайлово пространство. Разгледана е конформна геометрия на специални композиции в тримерно Вайлово пространство. Доказано е, че ортогонално декартова композиция съществува само в тримерно Римановo пространство, където е намерен вида на тензора на кривина.