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AN APPLICATION OF COMPUTER METHODS TO THE TRIANGLE GEOMETRY

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In the paper we want to demonstrate the considerable possibilities of the computer methods by solving geometrical problems. All calculations here are made by computer algebra. We use computer graphic for visualization of some curves and surfaces. In our opinion some of them are very nice and can be used in the applied art.

We consider an arbitrary triangle $\triangle ABC$ (Fig. 1), so that $A(a,0)$, $B(b,0)$, $C(0,c)$. The unique restrictions are $b-a\neq 0$, $c\neq 0$, which are the conditions for existence of triangle.

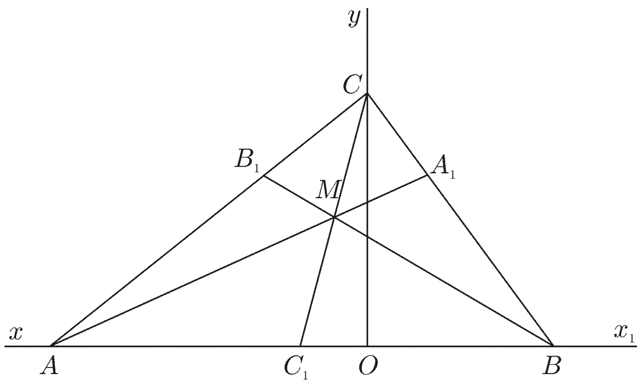

Fig. 1

If $M(x,y)$ (Fig. 1), then for the coordinates of the points A_1, B_1, C_1 we find:

$$x(A_1)=\frac{b(-ay-cx+ca)}{-cx+ca-yb},\quad y(A_1)=\frac{cy(a-b)}{-cx+ca-yb};\quad x(B_1)=\frac{a(-by-cx+cb)}{-cx+cb-ya},$$

$$y(B_1)=\frac{cy(b-a)}{-cx+cb-ya};\qquad x(C_1)=\frac{cx}{c-y},y(C_1)=0$$

We introduce the following functions:

$$L_1=(AA_1)^2,L_2=(BB_1)^2,L_3=(CC_1)^2.$$

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Evidently, the function L_1 is constant on the straight line AM, L_2 – on the straight line BM, L_3 – on the straight line CM. We find:

$$L_1 = (\frac{b(-ay - cx + ca)}{-cx + ca - yb} - a)^2 + \frac{c^2y^2(a-b)^2}{(-cx + ca - yb)^2};$$

$$L_2 = (\frac{a(-by - cx + cb)}{-cx + cb - ya} - b)^2 + \frac{c^2y^2(a-b)^2}{(-cx + cb - ya)^2};$$

$$L_3 = \frac{c^2x^2}{(c-y)^2} + c^2$$

Solving the systems:

$$\frac{\partial(L_2 - L_3)}{\partial x} = 0, \qquad \frac{\partial(L_2 - L_3)}{\partial y} = 0;$$

$$\frac{\partial(L_1 - L_3)}{\partial x} = 0, \qquad \frac{\partial(L_1 - L_3)}{\partial y} = 0;$$

$$\frac{\partial(L_1 - L_2)}{\partial x} = 0, \qquad \frac{\partial(L_1 - L2)}{\partial y} = 0,$$

we find the points P, Q, R with coordinates:

$$x(P) = \frac{b^2(b-a)}{c^2 + b^2}, \qquad y(P) = \frac{(ab + c^2)c}{c^2 + b^2};$$

$$x(Q) = \frac{a^2(a-b)}{c^2 + a^2}, \qquad y(Q) = \frac{(ab + c^2)c}{c^2 + a^2};$$

$$x(R) = a + b, \qquad y(R) = 0.$$

Now we introduce **the triangle transformation**: $\triangle ABC \to \triangle PQR$.

The areas of the both triangles (if $a < b$) are

$$s(\triangle ABC) = \frac{(b-a)c}{2}, \quad s(\triangle PQR) = \frac{2(ab + c^2)cab(-b + a)}{(c^2 + b^2)(a^2 + c^2)}.$$

The following assertions hold true:

Proposition 1. *$q = 0$ if and only if when the given triangle $\triangle ABC$ is rectangular. More precisely:*

1. *$a = 0$ iff $\measuredangle A$ is a right angle,*
2. *$b = 0$ iff $\measuredangle B$ is a right angle,*
3. *$c^2 + ab = 0$, iff $\measuredangle C$ is a right angle.*

Proposition 2. *a. $c^2 + ab > 0$ iff $\measuredangle C$ is acute, b. $c^2 + ab < 0$ iff $\measuredangle C$ is obtuse.*

We give a picture (Fig. 2) in the case when $\measuredangle C$ is an acute angle. In the picture (Fig. 3) $\measuredangle C$ is a right angle. In the picture (Fig. 4) $\measuredangle C$ is an obtuse angle.

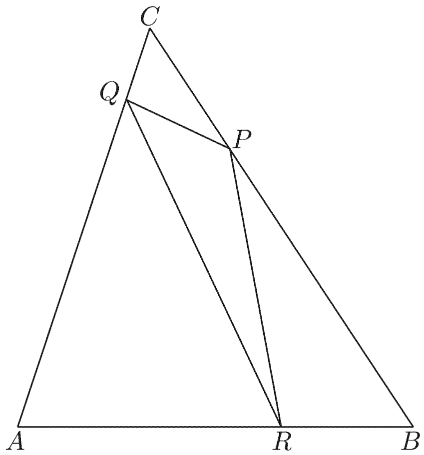

Fig. 2

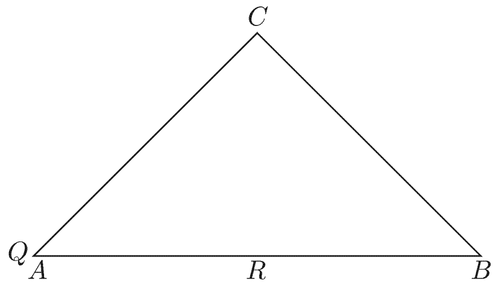

Fig. 3

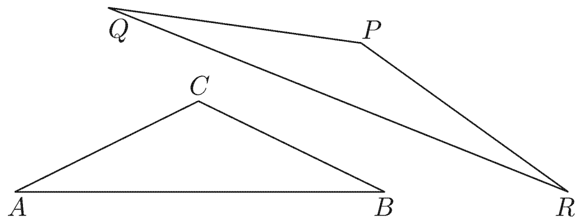

Fig. 4

We denote the function $s(\triangle PQR)$ by

$$d := \frac{2(ab+c^2)cab(-b+a)}{(c^2+b^2)(a^2+c^2)}.$$

If $a=-2$, $b=3$, then we get the function

$$d_1 = \frac{60c(c^2-6)}{(c^2+9)(c^2+4)}.$$

The graph of this function is given by Fig. 5

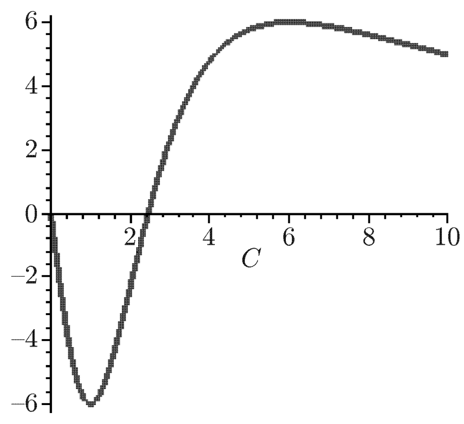

Fig. 5

Fig. 6

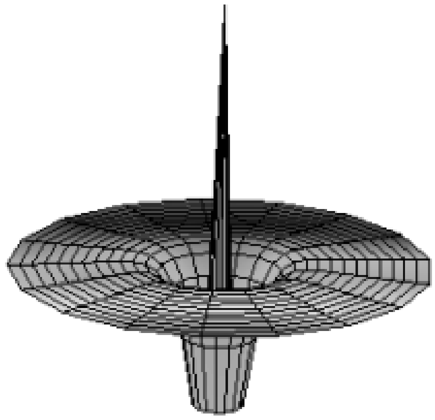

Fig. 7

By rotation of this curve we get the following very nice cognac glass (Fig. 6). By another rotation we get Fig. 7

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ПРИЛОЖЕНИЕ НА КОМПЮТЪРНИ МЕТОДИ В ГЕОМЕТРИЯТА НА ТРИЪГЪЛНИКА

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Целта на работата е да се демонстрира ефективноста на компютърните методи в геометрията на триъгълника. По същество всичко в работата е извършено със средства на компютърната алгебра и компютърната графика. За произволна точка от вътрешноста на даден триъгълник ABC, се въвеждат дължините на трите върхови трансверзали. Като особени точки на разликите на тези дължини се дефинират точките P, Q, R. Дефинира се триъгълната трансформация $\triangle ABC \to \triangle PQR$, за която се установяват някои свойства. Правоъгълните триъгълници се отличават с това, че за тях индуцираният триъгълник е изроден. С помоща на функцията лице на дефинирания триъгълник се чертаят графики на функции и визуализират ротационни повърхнини, някои от които са любопитни и могат да представляват интерес за изкуството, включително и приложното изкуство.