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GENERALIZED NET MODEL OF THE PROCESS OF LEARNING*

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In this article we introduce a Generalized Net (GN) model of the process of learning. The presented GN is also useful as a model of the process of resolving scientific problems – considering the attendant trials and mistakes. The so constructed GN-model can be used for modelling, simulation (with the aim to make prognoses) and for tracing of the modelled learning process.

1. Introduction. Our aim is to present a Generalized Net model of the process of learning. [1]. The following aspects of the machine learning process are considered [3-8]:

- Analysis of the variety of properties, which the object of teaching ought to attain through the teaching procedure;
- Specifying the learning units that are required by the learning - knowledge, questions, problems, advices and explanations;
- Selecting a set of teaching methods and techniques;
- Observing the knowledge that is achieved during the teaching and the necessity of additional work at the next step.

Each training topic has an impact on a subset of properties of the object of teaching. This subset and the training topic define a set of evaluation criterions for the objects of teaching. Any model of the process of learning ought to consider the four given aspects as a complex interacting system in order to achieve accuracy.

2. A Generalized Net model of the learning process. There are a number of GN models of the learning process in [3] that consider too abstract, general cases. In [8] we have presented a GN model of learning process rather for humans, than for abstract objects. For abstract objects [7] suggests several mechanisms for knowledge and skills evaluation, such as the level of assimilation of the training material and some other individual properties.

Figure 1 illustrates a GN model of the process of teaching an object by solving specific problems. The model allows monitoring the changes of the properties of the object of teaching and monitoring the condition of previously achieved knowledge.

* **Key words:** artificial intelligence, generalized net, atabase, intelligent tutoring environment, user model

There are two kinds of places in the GN model – P-places and S-places. The P-places present the teaching problems while the S-places present the status of the objects of training. More precisely, the teaching problems (tasks) are presented by the α tokens which are in the P-places, the objects of teaching are presented by the β tokens which are in the S-places, and the obtained additional knowledge is presented by the γ tokens that are also in the P-places. We denote the α tokens by $\alpha_1, \alpha_2, \ldots$, or if there is no risk of ambiguity – only by α; for β and γ tokens the same notation is used.

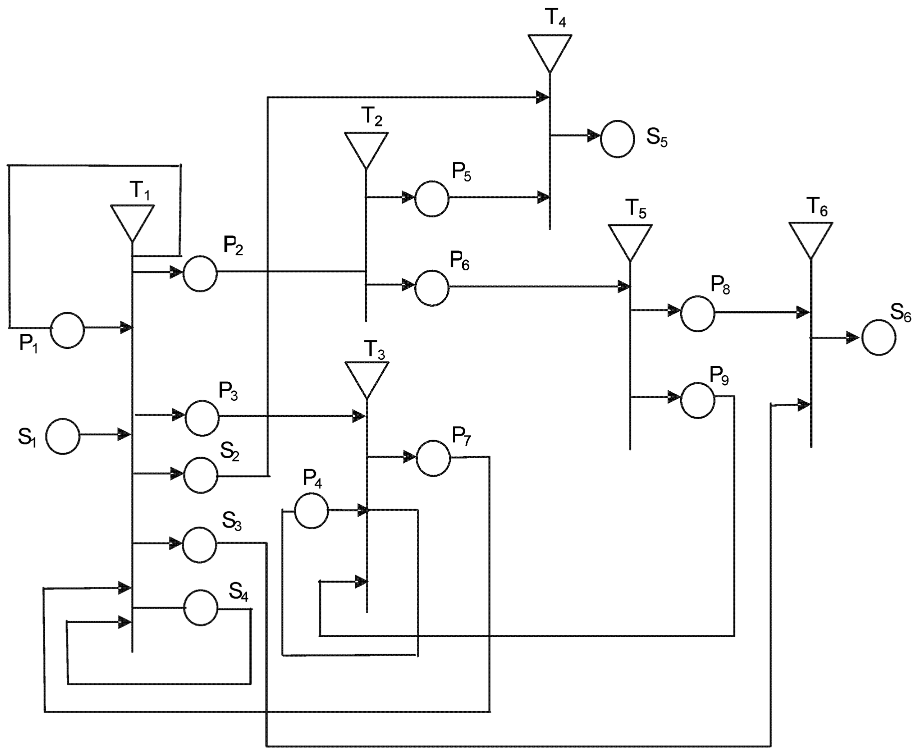

Fig. 1. A GN model of process of learning

In the initial time moment of the GN operation all the α tokens are in the P_1 place and have the characteristic:

$$x_0^{\alpha} = \text{“educational problem (task)”}.$$

At the beginning there are one or more β tokens in the S_1place with the following characteristic:

$$x_0^{\beta} = \text{“identifier of the object of teaching, level of initial knowledge”}.$$

The GN comprises of six transitions. The first transition is defined as:

$T_1 =< \{P_1, S_1, P_7, S_4\}, \{P_1, P_2, P_3, S_2, S_3, S_4\}, r_1, M_1, \wedge(\vee(P_1, P_7), \vee(S_1, S_4)) >$,

where its predicate matrix is:

$$r_1 = \begin{array}{c|cccccc} & P_1 & P_2 & P_3 & S_2 & S_3 & S_4 \\ \hline P_1 & \text{true} & W_1 & W_2 & \text{false} & \text{false} & \text{false} \\ S_1 & \text{false} & \text{false} & \text{false} & \text{false} & \text{false} & \text{true} \\ P_7 & \text{false} & W_1 & W_2 & \text{false} & \text{false} & \text{false} \\ S_4 & \text{false} & \text{false} & \text{false} & W_3 & W_4 & W_5 \end{array}$$

and the capacity matrix is:

$$M_1 = \begin{array}{c|cccccc} & P_1 & P_2 & P_3 & S_2 & S_3 & S_4 \\ \hline P_1 & m_1 & m_2 & m_2 & 0 & 0 & 0 \\ S_1 & 0 & 0 & 0 & 0 & 0 & m_3 \\ P_7 & 0 & m_2 & m_2 & 0 & 0 & 0 \\ S_4 & 0 & 0 & 0 & m_4 & m_5 & m_6 \end{array}$$

where

$W_1 =$ "the educational problem (task) has been solved",
$W_2 =$ "the educational problem (task) has not been solved",
$W_3 =$ "there is a token in the P_5 place",
$W_4 =$ "there is a token in the P_8 place' ',
$W_5 =$ "there is no token neither in the P_5 place, nor in the P_8 place".

Following the physical limitations, m_i denotes the capacities of the arcs, corresponding to the number of the objects of teaching and the number of topics of the educational process.

Whenever the value of W_1 becomes **true**, the α token splits in two. The first of the new tokens, which we denote again with α, remains in P_1. The second one, which we denote with α_i moves to P_2 place, where i corresponds to the number of of the new α token that has the characteristic: $x_1^{\alpha_i} =$ *"the object β_i has given an answer to the problem"*.

If the value of the W_2 predicate is **true**, then the α_i token moves into the P_3 place and obtains the characteristics:

$x_1^{\alpha_i} =$ "impossibility of retrieving an answer to the problem from the object β_i''.

The β tokens move in the S_4place where they obtain the characteristic: $x_1^{\beta}=$ *"identifier of the object of teaching, acquired skills and knowledge by the current moment"*.

Each one of the β tokens moves from the S_4 place either to the S_2 place or to the S_3 place according to the values of the W_3 and W_4 predicates. The β token moves to the S$_2$ place if the value of the W_3 predicate is **true** – that is if there is a token in the P$_5$ place. The β token moves to the S_3 place if the value of the W_4 predicate is **true** – that is if there is a token in the P_8 place. The β token remains in the S_3 place is the values of both W_3 and W_4 are false.

The next transition of the GN is defined as:

$$T_2 =< \{P_2\}, \{P_5, P_6\}, r_2, M_2, \vee(P_2), >,$$

where the predicate matrix and the capacity matrix have the form:

$$r_2 = \begin{array}{c|cc} & P_5 & P_6 \\ \hline P_2 & W_6 & W_7 \end{array} \quad \text{and} \quad M_2 = \begin{array}{c|cc} & P_5 & P_6 \\ \hline P_2 & m_7 & m_8 \end{array}$$

where:

$W_6 =$ "The solution of the problem is correct",

$W_7 =$ "The solution of the problem is incorrect, imprecise or incomplete".

Upon entering the P_5 place the tokens obtain the characteristic:

textit"the given solution of the problem is correct",

And if the W_6 predicate is satisfied then the tokens move to the P_6 place and obtain the characteristic: *"the given solution is incorrect, imprecise or incomplete".*

The T_3 transition models the process of teaching the object. A detailed model of this process is given by the E'GN in [3]. The relation between the E and E' GNs may be described by the hierarchical operator H_1(see. [1]), which is applied to the GN E. H_1 replaces the T_3 transition by a sub GN E'. The complete implementation of the GN E' may be found in [3].

During the GN operation the γ token remains in the P_4 place. Its initial characteristic is: "*auxiliary information*". The T_3 transition is defined as:

$$T_3 =< \{P_3, P_4, P_9\}, \{P_4, P_7\}, r_3^{,} M_3^{,} \wedge (P_4, \vee(P_3, P_9)) >,$$

where

$$r_3 = \begin{array}{c|cc} & P_4 & P_7 \\ \hline P_3 & \text{false} & \text{true} \\ P_4 & \text{true} & \text{true} \\ P_9 & \text{false} & \text{frue} \end{array} \quad \text{and} \quad M_3 = \begin{array}{c|cc} & P_4 & P_7 \\ \hline P_3 & 0 & m_9 \\ P_4 & m_{10} & m_{11} \\ P_9 & 0 & m_{12} \end{array}$$

The γ token that is in the P_4 position splits in two new tokens. The first of them, which is again denoted by γ, remains in the P$_4$ place, while the second one – γ_i moves to the P_7 place. In the P_7 place the token γ_i merges with the α token which moves either from the P_3 place or from the P_9 places, obtaining the characteristic:

"considering the auxiliary information and performing a new attempt to solve the problem".

The T_4 transition is defined as:

$$T_4 =< \{P_5, S_2\}, \{S_5\}, r_4^{,} M_4^{,} \wedge (P_5, S_2) >,$$

where

$$r_4 = \begin{array}{c|c} & S_5 \\ \hline P_5 & \text{true} \\ S_2 & \text{true} \end{array} \quad \text{and} \quad M_4 = \begin{array}{c|c} & S_5 \\ \hline P_5 & m_{13} \\ S_2 & m_{14} \end{array}$$

The α-token in place P$_5$ and the β-token in place S$_2$ move to place S$_5$ and merge to become a β-token, that obtains a final characteristic:

"an object of teaching that has positive experience".

The T_5 transition is defined as:

$$_5 =< \{P_6\}, \{P_8, P_9\}, r_5, M_5, \wedge(P_6) >,$$

where

$$r_5 = \begin{array}{c|cc} & P_8 & P_9 \\ \hline P_6 & W_8 & W_9 \end{array} \quad \text{and} \quad M_5 = \begin{array}{c|cc} & P_8 & P_9 \\ \hline P_6 & m_{15} & m_{16} \end{array}$$

$W_8 =$ "The object fails to solve the problem",
$W_9 =$ "The object requires additional information".

The α token that is in the P_6 place may move to the P_8 place in if there is an incorrect, imprecise or incomplete answer and if the W_8 predicate is satisfied. The α token then obtains the characteristic: *"solving attempts abandoned".*

If the W_9 predicate is satisfied, then the α token will move from the P_6 place to the P_9place and will obtain the characteristic:

"necessity of additional information",

Having obtained this characteristic, the α token moves to the T_3 transition and continues the educational process by acquiring auxiliary information and assimilating it.

The last transition – T_6 is defined as:

$$T_6 =< \{P_8, S_3\}, \{S_6\}, r_6, M_6, \wedge (P_8, S_3) >,$$

where

$$r_6 = \begin{array}{c|c} & S_6 \\ \hline P_8 & \text{true} \\ S_3 & \text{true} \end{array} \quad \text{and} \quad M_6 = \begin{array}{c|c} & S_6 \\ \hline P_8 & m_{17} \\ S_3 & m_{18} \end{array}$$

The α-token in place P_8 and the β-token in place S_3 move to place S_6 and merge to become a β-token which obtains its final characteristic: *"an object of teaching wth negative experience".* We have used the expressions "positive experience" and "negative experience" to refer to the number of successful or unsuccessful attempts that the object of teaching has made to solve the problem (task).

4. Conclusion and future research. The described GN is a model of the process of learning. Furthermore, the GN also describes the real process of resolving scientific problems – considering the attendant trials and mistakes. In future, the presented GN may be worked out in detail using hierarchical operators which replace a given transition or a place by a complete GN that has the same, but detailed behaviour. The presence of the time component in the characteristics of the tokens enables various analysis and statistical techniques to be applied to the process of learning. The purpose of such analysis would be to find ideas for improving the process.

The GNs are used in the modelling of paralleled processes. By means of complete GN models such processes can be simulated. The advantages of the GN model in comparison to the other sets models are:

- Control and real time regulation of the modelling process if this process is slow enough;
- Investigation of the optimal ways for training of real processes;
- Proposing ideas for improvement of the real processes.

The GN model is more compact in comparison to model by used of the other kind Petri net.

Using temporal intuitionistic fuzzy estimations [2] in the characteristic of the token, it is possible to make various analysis and statistics in the process of learning, considering the level of acquisition and forgetfulness of knowledge in the different time intervals. This technique allows to investigate the dynamics during the time in a variety of problems which will be presented in the future.

Already there are program tools that can realize part of the above described procedures (modelling, simulation, optimizing procedures related to the training processes). We will use these existing program tools to develop software application which realizes in practice models of processes in a University including processes of machine learning of the students. The described GN model will be one component of this software application.

REFERENCES

[1] K. Atanassov. Generalized Nets, World Scientific, Singapore, 1991.

[2] K. Atanassov. Intuitionistic Fuzzy Sets. Springer, Heidelberg, 1999.

[3] K. Atanassov, H. Aladjov. Generalized Nets in Artificial Intelligence. Vol. 2: Generalized Nets and Machine Learning . Prof. M. Drinov Academic Publishing House, Sofia, 2001.

[4] R. Atkinson, G. Bower, E. Crothers. An Introduction to Mathematical Learning Theory. New York, John Wiley & Sons, 1965.

[5] R. Banerji. Theory of Problem Solving and Approach to Artificial Intelligence. New York, American Elsevier Publ. Co., 1969.

[6] E. Charniak, McDermott. Introduction to Artificial Intelligence. Reading MA, Addison-Wesley, 1987.

[7] D. Langova, K. Paskalev. Cognitive modeling in intelligent tutoring systems, Contemporary trends in the development of fundamental and applied sciences – VI national conference of the Bulgarian Scientists union, Stara Zagora, 1995, 279–284.

[8] D. Langova-Orozova, H. Aladjov, K. Atanassov. Generalized Net For Machine Learning With Current Estimations, *Advanced Studies in Contemporary Mathematics*, **3**, 2001, No 2, 61–76.

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ОБОБЩЕНО МРЕЖОВ МОДЕЛ НА ПРОЦЕСА НА ОБУЧЕНИЕ

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Представя се обобщена мрежа, моделираща процес на обучение. Тази мрежа може да се разглежда и като модел на реалния процес на търсене на решение на научен проблем с пробите и грешките, които реално го съпровождат. Така конструиранията обобщено мрежов модел може да бъде използван за симулация (с цел подобряване) и проследяване на моделирания обучаващ процес.