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# WEBLABS: A CONTEXT FOR EXPLORING MATHEMATICS, INFORMATICS AND SCIENCE THROUGH VIRTUAL COLLABORATION*

**Evgenia Y. Sendova, Iliana H. Nikolova, Georgi St. Gachev, Liliana Iv. Moneva**

The paper presents research carried out within *WebLabs*, a European project, where researchers, teachers and young students learn collaboratively in a technology enhanced environment. Some exploratory activities in mathematics and informatics carried out by the Bulgarian team are illustrated and discussed.

**The WebLabs Project.** Recent educational research emphasizes on "knowledge put into functional use in a multitude of different situations and contexts" [1]. Many educators are working in the filed of "new cultures of assignments", "project-oriented work" and are very interested in constructing environments in which the learners can actively work on a conceptual level [2]. This is especially true in the case of mathematics taught at junior high school level where the mathematical concepts are often "shadowed" by the necessities of calculating, symbol manipulating by hand, drawing, etc. Restricting the math education to these technical skills creates a very twisted image of mathematics and the young learners can hardly experience the real spirit of mathematics as a research field.

To cultivate abilities of young learners to generate, test, play with ideas and to build a culture of communicating them to their peers from different countries is one of the fundamental goals of the *WebLabs* project [3]. The project aims at creating a virtual laboratory and a community of young learners, teachers and researchers exploring collaboratively mathematical and science phenomena.

**1.1. The focus of WebLabs.** This is a 3-year EU-funded educational research project carried out by scientific and educational institutions from Bulgaria, Cyprus, Italy, Portugal, Sweden and UK. It creates a new representational infrastructure for e-learning. The focus is on collaborative construction, description and interpretation of important mathematical and science concepts by means of modern ICTs. A web-based environment, *wplone*, is used to mediate collaboration so that the participants can share ideas and constructively criticize each other's evolving knowledge and working models [4]. The project embraces exploratory activities in the following domains: *mathematics, kinematics and dynamics, complex biological and ecological systems, robotics.*

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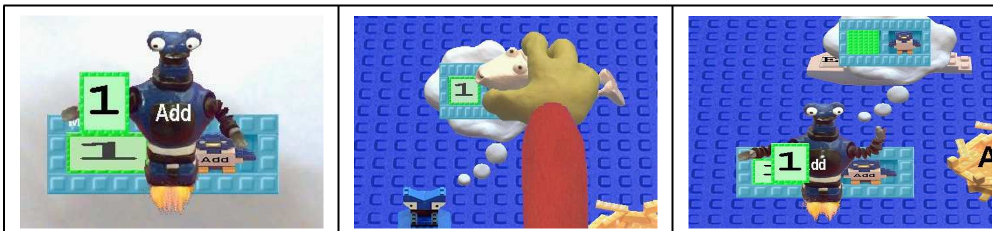
**1.2. The computer environment Toon Talk.** *WebLabs* uses an environment for visual programming called *Toon Talk* in which the source code is animated thus allowing for abstract computational concepts to be represented by concrete analogues, instantiated in cartoon-like characters [5]. *ToonTalk* has some unique features suitable for visualizing and exploring mathematics concepts and ideas when working with junior-high school students. The mathematical activities are integrated in a natural way with cultivating some programming skills. The programs in *ToonTalk* take the form of animated robots, which can be named, picked up and trained to perform a certain sequence of elementary steps (Fig. 1). A bird is the metaphor for the output of a procedure. After the training, the robots run *forever* if the initial conditions are satisfied. Such type of programming is especially appealing to young programmers since it is amusing, requires creativity and stimulates the acquiring of new skills for solving problems.

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|---|---|---|
| The robot is first trained to add 1 to 1 | A concrete number in the robot's thought bubble is then erased to generalize the process | Now the robot can add 1 to any current number in the box and then pass the result to the bird (*forever*) |

Fig. 1. Training a robot to count

**Developing a collaborative community.** The students in *WebLabs* have the chance of working together not only with their peers from other countries, but also with distinguished researchers. Rather than being just *reality check for researchers* they are real partners in a research process, in which new learning methods and computer technologies are used and experimented with. When using *ToonTalk* as a means for modeling the students learn how to work with contemporary computer environments in a natural way. In the context of carefully designed educational activities they gain knowledge about important processes and phenomena from mathematics, physics, biology, and compare their understanding with the rest of the participants. The communication is carried out by the so-called *Webreports* – a specially designed concept enabling the young learners to share and discuss the problems they have solved, and even more interesting – the problems they have formulated and implemented by means of *ToonTalk* robots.

**1.4. WebLabs e-learning elements.** The *WebLabs* project embraces various elements of e-learning. The specific means through which they are implemented are:

- *long distance birds:* a *ToonTalk* concept for information exchange among different computers;

- *Web-reports*: a mechanism for the participants to describe and share their ideas and programming constructs; a visual on-line editor is available to compose web-reports on individual or group work, to add comments and publish working *ToonTalk* models of students' ideas as they develop;
- *Plone*: a web-based environment used to mediate the collaborative learning activities over distance.

What follows, are excerpts from web-reports reflecting the activities of 12-13-year-old students from Sofia on number sequences in the frames of *Weblabs*.

**2. WebLabetisc – children's endeavor for a better communication in an international context.** The language, in which the students are expected to publish their web-reports is English but the native tongue is also accepted for local communications. The latter sometimes creates confusion and frustration. Let us illustrate this phenomenon with excerpts from a group web-report [6]:

*When browsing through plone in search for interesting sequences we had very unexpected experience. We moved step-by-step through the sequences suggested by Nikmous, Kiriakos, Irakli – all in Greek. The sequences are very clear but when the comments are in a language, which we don't understand, it is very annoying. So our teacher, asked us: Can you think of a way to express ToonTalk ideas so that anyone could understand them?* Yana suggested to use pictures for representing the ToonTalk characters and drew some on the board (Fig. 2):

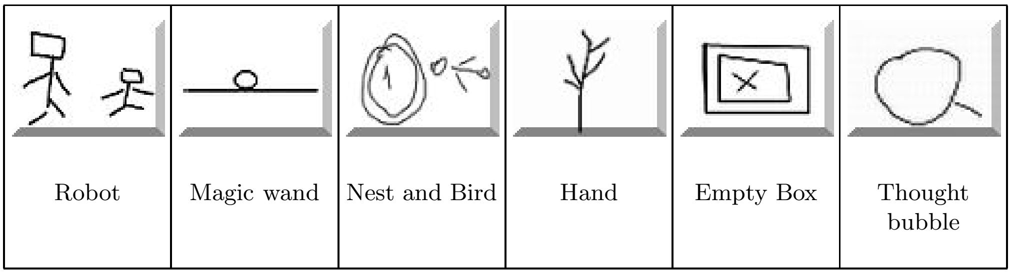


Fig. 2. Pictures representing ToonTalk tools

*The teacher challenged us to translate our Counting Robot in the new language. We all thought that this was easy but soon realized that we didn't have symbols for actions in our alphabet (or rather – weblabetics). So we added arrows for "puts"*

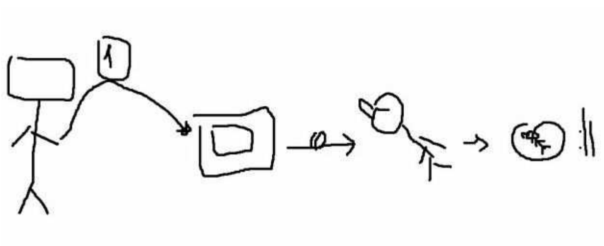

Fig. 3. Program code (Counting Robot) in *weblabetics*

*Isn't this clear for everybody? Well, just in case you lack the experience:*

*A robot puts 1 in a box, then copies the content, gives it to a bird, which puts it in its nest. Afterwards everything is repeated. Do you see the "**:||**" sign at the end – this is the music symbol for a repetition –Peter thought of it! In short, this is our old friend – the Counting Robot (in new clothes...)*

*We hope that now it would be easier to talk about ToonTalk and our ideas to everyone in the WebLabs project.*

*Our teacher told us the story of the Babylonian tower – a common language for everyone is more effective than many languages for a few.*

Children were faced with a typical e-learning problem while trying to learn collaboratively over distance – the language problem. In an attempt to overcome it, they reached the idea of designing a graphical scripting language for visual programming.

**3. Bugging and debugging – constructing the factorial function by editing an "adding-up" robot.** The idea of editing a program not only to *debug* it but also to modify its performance is very powerful from educational point of view. During one of the *Weblabs* sessions the students were expected to create the factorial sequence by editing a chain of two robots generating the sequence of the partial sums of the natural numbers. The first robot, *Add 1*, would generate the natural numbers; the second, *Add up*, would add them up as they come along and would send out the sequence of partial sums to a bird.

This activity required an understanding of what the given chain of robots does, and an analysis of the factorial sequence (as a process). It was intended that students would realise that they only needed to change the addition operation (represented by the Add Up robot) by a multiplication operation.

At the end of the session students were given questions designed to assess:

- students' understandings of the given sequence and how it is generated both within the ToonTalk environment and mathematically.
- how students relate (if at all) the two sequences and the way in which they are generated (similar recursive processes).
- students' understanding of the factorial.

To illustrate this *Weblabs* session through the eyes of a single student let's present an individual web-report [7]

*For warming up the teacher asked me to construct the ADD_1 and the ADD_UP robots. It was easy. We had done such robots last year but this time she wanted me to connect them. So, to run the ADD_UP robot I put the nest of ADD_1 in the first cell, 1 – in the second and another nest – in the third.*

**BUG #1**

*I expected to see the terms of the sequence*:

1, 3, 6, 10,...

*in the second cell since they were the consecutive sums of the natural numbers*:

1, 1+2, 1+2+3, etc.

*But to my surprise the numbers were instead;*

1, 2, 4, ....

*So I realized that the first number in the second cell should be* 0*, not* 1.

*Then I retrained the ADD_UP robot so as to generate the numbers*:

1, 1, 2, 6, 24, ...

*Instead of adding this time the robot should multiply the natural numbers coming from the nest in the first cell. I called it MULT_UP.*

**BUG #2**

*First I gave the number* 1 *to the second cell. Then I erased the number from the first cell (with the nest) and replaced it by* "*1". *Thus I replaced* "1" *by* "*1" *and I thought that the robot would replace the number in the nest with the "*" in front. But, of course, it replaced any further number with* "*1″.

**BUG #3**

*Then I figured out that I should add "*" in front of the number (whatever it would be). So I used the arrow key to put the cursor in front of the number and pressed the "*" key. To accelerate the process I left the house so as not to "embarrass" the robot and came back in a while. The numbers in the first two cells were:*

1142 *(in the nest) and* 427404 *– in the second one.*

*I showed this result to the teacher and she asked me if the number in the second cell (*427404 *in my case) was divisible by* 3.

*Of course it was – its figures when added up gave* 21 *which itself is divisible by* 3.

*Then she asked me: What about* 7*? First of all the rule for divisibility by* 7 *is disgusting and second of all - I don't know it... Then she said: What about* 1113*? I have to check, I said, but Mitty was sure that it is not divisible. Then the teacher (without checking) said: If it is not divisible by* 1113 *then your robot is not working properly.*

*How could she be so sure? She saw the start of my robot and she looked happy with what she saw.*

*Then she asked again: "How did you get this number? Did it fall from the sky? If you were playing outside and I had written this number on the board for you to check if it is divisible by* 3, 7 *or* 1113 *you could apply the rules for divisibility or a direct division. But now you should think about the process behind it."*

*Then it dawned on me! Of course the number* 427404 *should be divisible by* 1113 *because the ADD_1 robot had already given* 1113 *to MULT_UP. Even more, any number in the second cell should be divisible by all the natural numbers from* 1 *to the current number in the nest minus* 1. *For example if we see the number* 10 *in the nest (the first cell) the number in the second cell should be equal to* 1*2*3*4*5*6*7*8*9.

*And since* 1113 *is smaller than* 1141 *it should divide* 427404 *which (according to the way I taught the robot) should be equal to* 1*2*3...*1141.

*But there was something strange – my teacher thought this product should be much bigger than* 427404. *For a moment Mitty and I thought that we might have made a mistake with the operation and we checked if the number* 427404 *satisfies the formula for the numbers in the first sequence– N(N+1)/*2, *i.e. if* 427404 *is equal to* 1140*1141/2. *But this was not true either, so I decided to check the work of the robot more carefully... What I found was* ***that it worked correctly only till 9****. After the number in the nest would have more than* 1 *digit, e.g.* 10, *the second cell would get "*1*0*" instead of* "*10*" and this was treated as* 1 *(or as "something multiplied by* 0 *plus* 1*", as Mitty commented) and added to the current number. We experimented a little bit with the result of putting the "*" sign in front of the last digit but didn't reach a consensus. At this point it was more important for me to debug my robot. So I pressed the "*" sign after the number and* *realized that TT puts it immediately in front of it and thus multiplies it by the current number in the second cell. This time the robot seemed to work O.K. and the numbers in the second cell were growing very fast.*

*The teacher asked me what was the reason I hadn't realised immediately that a number in the second cell is divisible by a concrete number less than the number in the nest. I wasn't sure but maybe because the number was* 3 *and I knew another way to check it. Sometimes I wonder if I am genius or dumb...*

This reports illustrates well (we believe) the interplay between ideas from mathematics and informatics. The process of constructing the "factorial" robot helped the students become aware of some important properties of the factorial function. This mathematical awareness helped them in turn to debug their robots.

In a panel discussion on the educational value of computer programming diSessa [8] proposes the idea that *the intellectual power the programming representations can have for learning science is at least comparable to, if not greater than, algebra*. One can easily adopt it in the context of learning mathematics – gaining the flexibility of moving from a programming to algebraic representation of a sequence contributes to a deeper understanding of the mathematical ideas. And such understanding helps back in verifying one's programs.

**4. Convergence and divergence – exploring advanced mathematical ideas in a friendly environment.** A key characteristics of *ToonTalk* is that the system employs exact arithmetic instead of floating point numbers (used by most other computer environments). This means that investigations of infinite number sequences and series can be as precise as desired. The *Weblabs* research team has designed a sequence of activities which challenge students' intuition of infinity and engage them in discussions and reasoning about the ideas of convergence, divergence, limits. In the course of these activities the students constructed robots generating infinite numbers sequences and series and explored issues such as:

- *Can a decreasing sequence contain only positive numbers? Give examples.*
- *When you add the terms of a convergent sequence, is the sequence of the partial sums also convergent*?

In order to convey the flavor of these activities, let us present them through the eyes of the teacher [9].

*After the students gave several examples of decreasing number sequences we asked them to build robots generating the sequences* $\{1/n\}$ *and* $\{1/n^2\}$*. Before they started we asked them which sequence was easier.*

**Student 1:** $\{1/n\}$. *Since there is one operation fewer in his sequence (raising to a power). No, wait.*

*In fact, the sequence* $\{1/2^n\}$ *is easier since every member can be obtained from the previous by dividing by* $\ldots {}^1\!/_2$*, I mean — by dividing by* 2.

**Student 2:** *Yes, the second sequence is easier.*

*They started building their robots* (Fig. 4).

Fig. 4. Building robots which generate the sequences $\{1/n\}$ *and* $\{1/n^2\}$.

*After a while*:

**Student 1**: *You know, it depends on which key is closer to you – to divide by* 2*, or...*

**Student 2:** *Or first to add* 1 *and then – to divide* 1 *by this number.*

**Student 1:** *(looking at the numbers his robot generates). Ha, look how long my numbers are getting.*

**Teacher**: *Does this mean they are "big"?*

**Student 1**: *Not big at all – these are a million times, a billion times smaller than* 1 *since they are obtained after halving* 1 *many times.*

**Teacher:** *One more time – which sequence was easier to build?*

**Student 2**: *The sequence* $\{1/2^n\}$ *is easier.* Pauses in reflection. *Wait, maybe both are equally easy ... if we have different versions of ADD1 (generating the denominators) – one for counting, and the other for doubling the numbers.*

***Can the partial sums of the harmonic series exceed 2?***

In the next session we started working on the questions related to the behaviour of the harmonic series [10]

**Student 1**: *Every member of the sequence* $1+1/2+\ldots+1/n$ *... is bigger than the previous one but with a number which becomes smaller and smaller (as n grows).*

(He reflected for a while and made a couple of calculations with his pocket calculator.

Surprisingly he didn't immediately figure out that the difference between the third and the second partial sum of the harmonic series was just $1/3$ and started calculating $1+1/2+1/3$ and then subtracted $3/2$ from it.)

*My first feeling is that no matter how small the added number is the sum can exceed 2 because we add as many members as we wish.*

**Teacher***: I don't know. This is what we want to check.*

**Student 1:** *But you know, this is different from Zeno's paradox.*

**Teacher:** *In what way*?

**Student 1:** *The turtle keeps doing the same thing – it always moves by a very small stretch, whereas Achilles...* (thinks hard again)

**Teacher**: *But there is a certain similarity, right? You add endlessly some numbers (the distances Achilles covers each time) which get smaller and smaller. The question is* *whether the covered distance will become larger than a concrete number?*

**Student 1:** *O.K. Let me build first the robot that generates $1/n$ and than the ADD_UP robot which will accumulate these numbers.*

**Teacher**: *Is it strange that the partial sums of the harmonic series became bigger than* 2?

**Student 2**: ***Yes and no***. *Because the further we go the smaller the added number is.* ***On one hand we can go as far as a billion members of $1/n$ and the feeling is that we can go over 2, but on the other we add hellishly small numbers and it is not clear if it will reach 2***.

**Student 2**: *I think that both sequences would not exceed* 2.

*(He started watching the robot generating the harmonic series. His ADD_UP robot was with a 2-hole box. In the first he had put the nest of the robot, generating the sequence $1/n$, and the second hole was for accumulating the consecutive members of this sequence. This version turned out to be the tidiest one for watching the relation between n and* $1+1/2+\ldots 1/n$.)

*Ha, my robot doesn't seem to work properly!*

**Teacher:** *Why?*

**Student 2:** *Because it already generates numbers bigger than 2!*

**Teacher:** *Maybe your conjecture was not right...*

**Student 2:** *Maybe...* (After watching the robot generating the geometric progression) *And these numbers get closer to 2 never seeming to exceed it ...*

**Teacher:** *I can show you on the blackboard why this is so in a formal way* [11]

After comparing our observations with those of our project partners we realized that we could explore together interesting research questions concerning the *complexity of the sequences*, such as*: How to measure their mathematical, computational, cognitive and pedagogical complexity? How are they correlated? How much of perceived complexity is inherent to the object under study, and how much – to the representational infrastructure being chosen?*

**5. Conclusions.** The effect of working with young students in the framework of *WebLabs* is not reduced to learning specific aspects of certain subject areas but has a much larger scope. On one hand, *the scientist in the learner is enhanced* – the students get used to pose questions, to look for answer no matter how sophisticated they might be. They develop an understanding of mathematics as a science in which formulating hypotheses, carrying out experiments, and attacking open problems plays a crucial part. The students are partners in a research process and can influence both the development of the computer environment and the design of the educational activities. They can communicate among themselves, with teachers and researchers locally and globally alike. During this communication they acquire specific social experience and are stimulated to build valuable personal skills and abilities, such as:

- to generate and verbalize ideas;
- to present their results according to a concrete standard;
- to share their experience by means of electronic communication
- to work in a team and discuss their work
- To be (self-)critical to the work published in the virtual environment

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Evgenia Yovkova Sendova
Liliana Ivanova Moneva
Institute of Mathematics and Informatics
Bulgarian Academi of Sciences
Acad. G. Bonchev, Bl. 8
1113 Sofia, Bulgaria
e-mail: jsendova@mit.edu, lylymo@yahoo.com

Iliana Hristova Nikolova
Georgi Stamov Gachev
University of Sofia
Faculty of Mathematics and Informatics
5, J. Bourchier Blvd.
1164 Sofia, Bulgaria
e-mail: iliana@fmi.uni-sofia.bg, gachev@fmi.uni-sofia.bg


## WEBLABS – СРЕДА ЗА ИЗСЛЕДВАНИЯ В ОБЛАСТТА НА МАТЕМАТИКАТА, ИНФОРМАТИКАТА И ПРИРОДНИТЕ НАУКИ ПОСРЕДСТВОМ ВИРТУАЛНО СЪТРУДНИЧЕСТВО


**Евгения Й. Сендова, Илиана Х. Николова, Георги С. Гачев, Лиляна И. Монева**



Представена е работата на българския екип в рамките на проекта *WebLabs: "New representational infrastructure for elearning"* (Нова инфраструктура за електронно обучение), европейски научно-изследователски проект, в който изследователи в областта на обучението по математика, информатика и природни науки, учители и ученици работят като партньори в среда, обогатена със съвременни компютърни и комуникационни технологии.