МАТЕМАТИКА И МАТЕМАТИЧЕСКО ОБРАЗОВАНИЕ, 2006
MATHEMATICS AND EDUCATION IN MATHEMATICS, 2006
Proceedings of the Thirty Fifth Spring Conference of
the Union of Bulgarian Mathematicians
Borovets, April 5–8, 2006

MODELING OF THE HOUSEHOLDS' CONSUMPTION UNDER CREDIT OPPORTUNITIES

Plamena B. Petrova, Stefan G. Tsvetkov

The main aim of this work is to model the process of optimization of the households' consumption under credit opportunities.
After acquainting with the available works on this topic was found that most of them were econometric models or very isolated cases. This article presents an original model of the crediting process and its relation to the households' consumption. The existing of the solution of the model is shown as a result of the presented connection with the theory. Some of the numerical results of the made simulations are shown.
The results can be used in the description of the current situation in Bulgaria. The parameters of the model could be interpret in the terms of the real conditions according which the commercial banks give credits and aiming to describe best the households' behavior. The output of the model could support the process of macroeconomic forecasting and policy making.

1. Problem formulation. We consider a closed economy, in which households take decision between consuming or saving their income in order to maximize their total utility. They receive a wage and in each moment have the opportunity to borrow or not a loan. The loan is consequently repaid according to a given amortization schedule. By amortization schedule we mean the kind of the amortization payments: increasing, equal, decreasing or random, paid in the beginning or at the end of the period.

3. General model specification. Each one representative household maximizes the sum of the discounted future utilities – U from consumption c_t. The discounting factor $\beta \in (0,1)$ reflects the time preferences. Formally, it solves the following problem:

$$\max_{\{c_t,k_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} U(c_t)\beta^t \tag{1}$$

under the condition:

$$a_{t+1} = (1+r)a_t + k_t + w_t - P_t c_t - \sum_{i=1}^{m} \alpha_t(R_t, m_t)k_{t-i}. \tag{2}$$

The control variables are the consumption c_t and k_t – the borrowed new loan in the moment t. Equation (2) gives the cash holdings a at the following moment. In each one considered period the household has a given cash position a_t (phase variable), which is deposited under a constant interest rate r (known), receives wage income w_t (known) and has the opportunity to borrow a loan k_t, with an amortization schedule – equal payments, paid at the end of the period, maturity m_t and an interest rate R_t. In case the household does not wish to borrow, it follows that $k_t = 0$. The consumption c_t is measured in units of consumed goods as utility is determined by the volume of consumption rather than its price. The expression $P_t c_t$, where P_t is the price level is already in monetary units and thus all variables in (2) share one and the same measuring unit. For the purpose of facility, we assume $P_t \equiv 1$. The sum in the right-hand side of (2) gives the due payments in time t on the borrowed funds in previous periods. We assume that the parameters of the loan are known in advance and are constant for the model or $R_t = R$ and $m_t = m$. According to the schedule described above, a borrowed loan k_t in time t is repaid during the $t+1$ till $t+m$ period, by a number of m equal installments, amounting to αk_t each, where $\alpha = \dfrac{R(1+R)^m}{(1+R)^m - 1}$. The expression for α is obtained through the relation between the installment υ and the loan principal k, which, given the specified amortization schedule, is equal to:

$$k = \frac{\upsilon}{1+R} + \cdots + \frac{\upsilon}{(1+R)^m}$$

We make the following transformations:

$$k = \frac{\upsilon}{1+R}\left(1 + \frac{1}{1+R} + \cdots + \frac{1}{(1+R)^{m-1}}\right) = \frac{\upsilon}{1+R}\frac{1-\left(\dfrac{1}{1+R}\right)^m}{1-\dfrac{1}{1+R}} = \upsilon\frac{(1+R)^m - 1}{(1+R)^m R}$$

$$\upsilon = \frac{(1+R)^m R}{(1+R)^m - 1}k.$$

This shows that α assumes the above given expression. Taking into account all these points, equation (2) can be written in the following way:

$$a_{t+1} = (1+r)a_t + k_t + w_t - c_t - \alpha\sum_{i=1}^{m} k_{t-i}. \tag{3}$$

The cash position in time $t+1$ is calculated as the expenditures in time t (consumption and payments on loans from previous periods) are deducted from all income flows in time t (wage, cash holdings plus accumulated interest on them and borrowed loans). The cash position in each moment might be positive or zero but it cannot be a negative number. We show that this condition follows as a corollary from one of the two additional requirements to the model. Firstly:

$$c_t + \alpha\sum_{i=1}^{m} k_{t-i} \le w_t + k_t + a_t, \tag{4}$$

that is the expenditures must not be higher than the available cash. If we consider the right-hand side of (4), then it is obvious that we can easily derive the desired inequality for the cash position. Since $a_t \leq (1+r)a_t$, where the inequality is not strict, because a_t might be zero (the interest rate r is strictly positive), it follows that

$$c_t + \alpha \sum_{i=1}^{m} k_{t-i} \leq w_t + k_t + (1+r)a_t.$$

By transferring all addends to the right, we arrive at the right-hand side of (3), which just gives us the needed inequality $a_{t+1} \geq 0$. The second additional requirement to the model is:

$$\alpha \sum_{i=1}^{m} k_{t-i} \leq w_t - \delta, \tag{5}$$

that is the payments on all loans in time t must not exceed the wage minus δ. The latter represents the mandatory minimum balance, necessary for basic consumption. The parameter δ is constant for the model and is defined by the agent, who also sets the remaining parameters of the credit process (a commercial bank for example).

The model, constructed in this way, is discrete and is solved using the methods of the dynamic programming (see [2]).

The model is described by (1), (3), (4) and (5). We additionally want $c_t \geq 0$ and $k_t \geq 0$ for $\forall t$, since the consumption and the credit should not be negative (these conditions are natural). We have already showed how $a_t \geq 0$ follows from condition (4). The parameters of the model are known in advance and strictly positive constants – $\beta \in (0,1)$, m, R, r, δ and α, which is unequivocally determined by m and R. It is realistic to assume that the income from a labor w_t increases at a fixed rate, i.e. $w_t = w_0(1+p)^t$. For facility we assume that $w_t = w$ for $\forall t$, where $w > 0$.

Proposition. *There exists a solution to the above specified model.*

Proof. Reducing the model to the terms of the theory, described in [1] Chapter 4, shows that the necessary conditions for the existence of a solution are satisfied. (See [4]). $\square$

3. Numerical results. We used the following form of the utility function $U(c)$ in simulations to get numerical results from the described model:

$$U(c) = \ln(1+c).$$

We present two of all simulations that were tried. Fig. 1 and 2 graph the output variables – consumption (dotted line), credit (dotted and dashed line) and cash holdings (straight line). The x-axis and y-axis measure respectively the time (in quarters) and the value (in BGN). The specific values of the parameters, used in the particular simulation, are listed in the following order: number of time periods in quarters (N), maturity (m), interest rates on deposits (r), interest rates on loans (R), mandatory balance (δ), discounting factor (β), starting value of the cash position (a_0) and quarterly wage (w). The MATLAB software, Optimization toolbox *fmincon* function was used for running the simulations and generating the results with an original code. *Fmincon* finds minimum of a constrained nonlinear multivariable function.

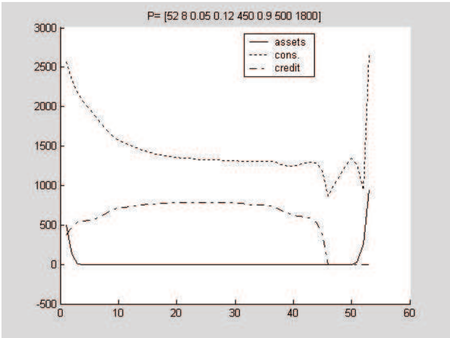

Fig. 1

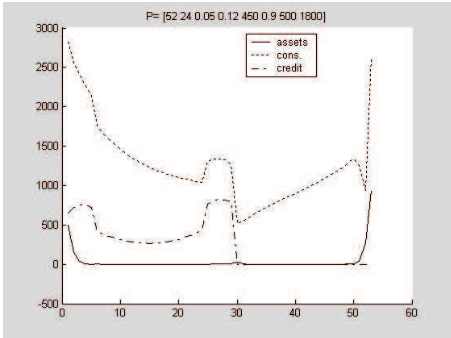

Fig. 2

The behavior of the households under the particular values of the parameters as on Fig. 1 can be described in the following manner. Consumption is high in the beginning of the period, financed by a starting loan significantly below the upper limit. In subsequent periods these variables reach and stay at equilibrium values, characterized by rolling over borrowed loans and accumulation of assets at the end of the period to ensure high consumption. Additional calculations allow us to conclude that the credit availability provides opportunities for a pattern of consumption, in which utility is higher than in the case where the wage is the only source of income.

One of the primary objectives of constructing the model was to simulate the impact of the loan maturity on the household's behavior. The significance of the maturity parameter can be deduced by comparing Fig. 1 and 2, for values of m respectively 8 and 24. The increase of the maturity leads to a marked change in the household's behavior. The starting loan is higher with higher maturity, which induces higher consumption in the beginning of the period. The total average level of the credit is lower than in the case of $m = 8$. The initial high consumption levels drop to relatively lower values in the middle of the monitored period, compared to the simulation with $m = 8$. The reason for this result is that the due amortization installment on a given loan is lower under a higher maturity. Observing the behavior with $m = 24$ shows that the credit growth rate accelerates in time 24 or in the period when the maturity of the initial loan expires. At this moment the amount of the total borrowed funds decreases or, conversely, the free resources are increasing and the households have the opportunity to loan more in order to boost their consumption and respectively their utility. The initial growth of the credit is followed by its sharp and subsequently a more gradual fall as the second sharp hike of the credit takes place in period 24. Interestingly, in the case of $m = 24$ the cash position assumes a low positive value in the time when the credit goes to 0 and in the following period drops to 0 again. This fluctuation is due to a change in the active restrictions when solving the optimization problem. In conclusion, we would like to point out that the results of all the numerical simulations show a behavior, which is in line with the economic reasoning and is intuitively expected.

The direction for potential future work is complicating the model by way of a more general specification of parameters like the interest rate on loans and assets, wages and maturity. This would complicate the description of the loan repayment process but would present a more realistic picture.

REFERENCES

[1] Nancy L. Stokey, Robert E. Lucas, Jr. with Edward C. Prescott. Recursive Methods in Economic Dynamics. Harvard University Press, 1989.
[2] Lars Ljungqvist, Thomas J. Sargent. Recursive Macroeconomic Theory. The MIT Press Cambridge, Massachusetts London, England, 2000.
[3] R. J. Barro, X. Sala-i-Martin. Economic Growth. MIT Press, Second Edition, 1999.
[4] P. Petrova. Modeling of the households' consumption under credit opportunities, Master thesis, 2005.

Plamena Petrova
Agency for Economic
Analysis and Forecasting
31, Aksakov Str.
1000 Sofia, Bulgaria
e-mail: p.petrova@aeaf.minfin.bg

Stefan Tsvetkov
Agency for Economic
Analysis and Forecasting
31, Aksakov Str.
Sofia, Bulgaria
e-mail: s.tsvetkov@aeaf.minfin.bg

МОДЕЛИРАНЕ НА ПОТРЕБЛЕНИЕТО НА ДОМАКИНСТВАТА ПРИ НАЛИЧИЕ НА КРЕДИТНИ ВЪЗМОЖНОСТИ

Пламена Б. Петрова, Стефан Г. Цветков

Целта на настоящата работа е моделирането на процеса, при който домакинствата оптимизират своето потребление, в условията на наличие на кредитни възможности.
В статията е предложен оригинален модел, представящ във възможно най-голяма точност процесът на кредитиране и връзката му с потреблението на домакинствата. Показана е връзката на конкретния модел с общата теория. Като следствие е показано съществуване на решение. Представени са някои от получените числени резултати от направените симулации.