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INVESTIGATION ON THE INFLUENCE OF THE BOUNDARY CONDITIONS BY SIMULATION WITH FINITE ELEMENT METHOD OF MICROINDENTATION PROCESS*

Sabina Ch. Cherneva, Rumen Zl. Iankov

Numerical simulations are implemented through Finite element method of micro indentation process of one deformed isotropic axis symmetric specimen with rotary-symmetric hard indenter. We propose appropriate correlation between size of the indenter's imprint and size of the area, where we resolve boundary task, which describe micro indentation process. It is established through numerical experiments, that fields of deformation in the examined task are not influenced from boundary conditions in displacements on the boundary $G4$. The influence of the boundary condition on the boundary $G4$ (Fig. 1) on field of stresses and displacements is investigated, as upon load-displacement curves of the indenter. It is shown, that load-displacement curves, which are got by numerical simulations with and without boundary condition, coincide on $G4$. Therefore, the correlation dependence, which we propose is suitable for this boundary task.

1. Introduction. Microindentation is very widespread technique which was used in the beginning for determination hardness of materials and it was created like instrument for quality control of metallurgic processes. Nowadays, microindentation is used successfully for determination material constants and properties like: elastic modulus, yield strength and tensile strength of different materials, alloys and thin films, crack-stability of ceramic products, and, moreover, adhesion of thin films. The microindentation is pressing over investigated specimen with hard indenter with different shape and after measuring with microscope imprint's geometry. Depending on indenter's shape, microindentation tests are divided into Vickers test, Brinell test, Knoop test, Rockwell test, Shore test, etc. In the present work we examine Vickers microindentation test. Numerical simulation of the microindentation process allow the determination of mechanical properties of films, when they cannot be separated from substratum. A number of authors resolve the boundary task, that describe indentation process, numericaly. For resolving such tasks most frequently the finite-element method is used. A number of authors examine tasks for microindentation of different kind of materials, alloys and films. Toonder and Govaert [4] made this for glassy polymers, Franco [2] – for aluminium, gold, steel, glass, Burgarolas [3] – for copper and for two kinds of stainless steel. Similar simulations were made for polymer thin films by Wang [6], for porous materials – by Chen, Xiang and Vlassak [8] and for ion-implanted layers – by Knapp, Folstaedt, Barbour and Myers [5].

***Key words:** finite-element method, microindentation, Vickers hardness

2. Theoretical part. We investigate microindentation process of brass alloy (metallurgic brass alloy type $CuZn36$). For that reason, we use model of elastic-plastic solid with linear strengthening. The process of indenter's penetrating and separation is simulated like process of contact between indenter and material for definitely time, through that the material become deformed. Penetrating and separation of the indenter are realized in 20 seconds. Therefore, this task can be in hand like quasistatic task. Usually, the indenter's material is very hard (usually this is diamond). Therefore, it is accepted, that the indenter is perfectly hard. Geometry of the indenter is a regular quadrangular pyramid. The indenter's penetrating cause deformation of the material in a local area around it. For that reason, we examine one cylindrical area around the indenter's tip with sufficiently big radius, upon whose contour the influence of deformation from the indenter is neglectfully small. Besides, relative displacements between the indenter and the deformed material in the process of quasistatic load are small. This allow to ignore friction forces. For modeling indentation process, we resolve boundary under the following assumptions:

- Quasistatic interaction between one deformed isotropic axissymmetric specimen and one rotary-symmetric hard indenter;
- We apply load over the indenter;
- We neglect friction force in the contact area.

Because of geometrcal symmetry and isotropy of the material, we reduce the boundary task to examining $\frac{1}{4}$ from the deformed material. Some authors [1] propose to resolve boundary tasks an for indenters-tetrahedral pyramid, by working with substitute cone with angle 70.3°. The aim is the correlation cross section area-depth of indentation to be selfsame, like the Vickers indenter. This leads to simplification and the task is reduced to two-dimensional axissymmetric in rectangular area. We choose correlation $\frac{l_3}{l_4} = \frac{1}{20}$ (Fig. 1), thereby deformations do not influence on boundary $G4$. $l_2 < 10\%.l_1$, thereby fields of stress and deformation during indentation loads don't influence on the boundary $G5$. This is important, because the numerical load-displacement curve will be compared with the experimental curve and applied the boundary conditions must not influence on the resolve of the task. Stress-strain state in the experimental specimen is described by the equations for equilibrium:

$$\sigma_{ij,j} = 0. \tag{1}$$

The full deformation ε_{ij} is the sum of the elastic ε_{ij}^{e} and plastic ε_{ij}^{p} deformations:

$$\varepsilon_{ij} = \varepsilon_{ij}^{e} + \varepsilon_{ij}^{p}. \tag{2}$$

The tensor of elastic strains is connected with the tensor of the stresses through Huk's law :

$$\varepsilon_{ij}^{e} = \frac{1}{2G} s_{ij}. \tag{3}$$

Mises' condition for plasticity has to be in force for metals:

$$\frac{1}{2} s_{ij} s_{ij} = \tau_p^2, \tag{4}$$

where s_{ij} is deviator of the stresses.

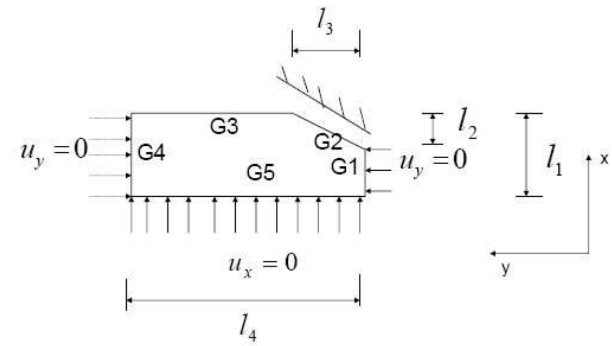

Fig. 1. Scheme of applied on the substratum boundary conditions

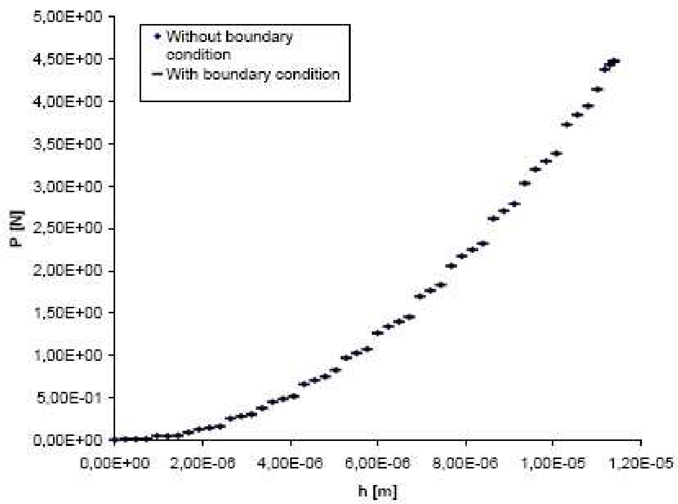

Fig. 2. Load-displacement curve with and without applied boundary condition on $G4$

A second numerical simulation was realized, in which additionally third boundary condition $u_y = 0$ was applied on the boundary $G4$. We again determinated the load-displacement curve (marked by horizontal lines in Fig. 2), fields of displacements (Fig. 5) and stresses (Fig. 6).

As a result of this numerical simulation, we got load-displacement curve (marked by rhombs of Fig. 2), as well as fields of stresses (Fig. 3) and fields of displacements (Fig. 4).

3. Numerical simulation. We realized 2 two-dimensional numerical simulations through a finite-element method of process of Vickers' microindentation by means of the finite-element code MSC.MARC [7] with purpose to investigate the influence of the boundary conditions. We examined a rectangular isotropic axissymmetric specimen from metallurgic brass alloy type CuZn36 with width 0.5 mm and length 2 mm, that was pressed with a hard cone with angle 70.3° for 10 seconds. After that it is unloaded for

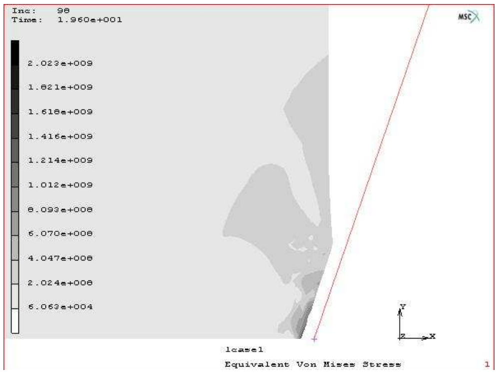

Fig. 3. Field of stresses without applied boundary condition on $G4$

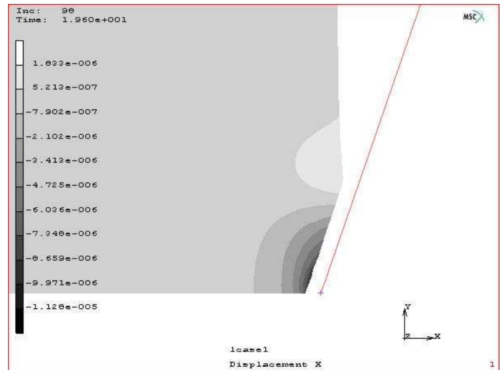

Fig. 4. Field of displacements without applied boundary condition on $G4$

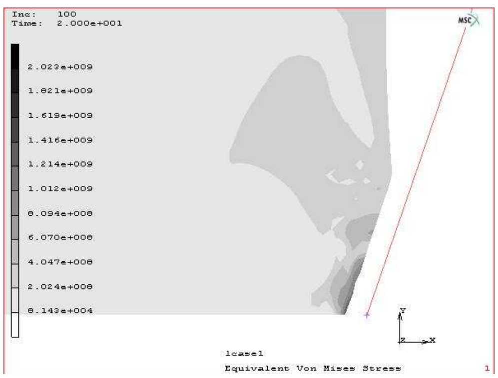

Fig. 5. Field of stresses with applied boundary condition on $G4$

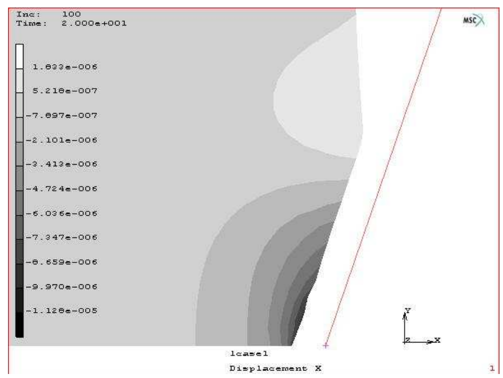

Fig. 6. Field of displacements with applied boundary condition on $G4$

10 seconds, too. Indenter's velocity after the contact was $1,2.10^{-6} m/s$. The substratum have following mechanical characteristic: yield strength $\sigma_p = 2,44.10^8 [N/m^2]$, tensile strength $\sigma_m = 5,19.10^8 [N/m^2]$, Young's modulus $E = 3,85.10^{11} [N/m^2]$ and Poason's coefficient $\nu = 0.36$, which was determined at previous investigation from the authors of this report. We divided the deformed solid as built grid from 3600 axissymmetric 4-node isoparametric finite elements with full integrating. In the first simulation we applied boundary condition $u_y = 0$ on the boundary $G1$ and $u_x = 0$ on the boundary G5 (Fig. 1). Figure 2 dislpays, that load-displacement curves, which were determinated through the already mentioned two simulations, coincide, as $P_{\max} = 4.4821$ without boundary condition, and $P_{\max} = 4.4824$ with it, therefore, we have a difference smaller than $O(h^{-3})$. Therefore, the correlation $\dfrac{l_3}{l_4} = \dfrac{1}{20}$ is well-chosen.

4. Conclusion. Two-dimensional numerical simulations were realized through Finite-element method of process of Vickers' microindentation with and without applied boundary condition $u_y = 0$ on the boundary $G4$ with purpose to investigate the influence of this boundary condition. We proposed correlation $\frac{l_3}{l_4} = \frac{1}{20}$ between the size of the indenter's imprint and the radius of axissymmetric area. In this case, the applied boundary condition in displacement do not influence on the solution in displacement. We determinated and compared load-displacement curves as well as fields of stresses and displacements in the two cases. Results showed that proposed correlation between l_3 and l_4 is appropriate.

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Sabina Ch. Cherneva
Institute of Mechanics
Bulgarian Academy of Sciences
Acad. G. Bonchev Str., Bl. 4
1113 Sofia, Bulgaria
e-mail: sabina_cherneva@yahoo.com

Rumen Zl. Iankov
Institute of Mechanics
Bulgarian Academy of Sciences
Acad. G. Bonchev Str., Bl. 4
1113 Sofia, Bulgaria
e-mail: iankovr@yahoo.com

ИЗСЛЕДВАНЕ НА ВЛИЯНИЕТО НА ГРАНИЧНИТЕ УСЛОВИЯ ПРИ МОДЕЛИРАНЕ С МЕТОДА НА КРАЙНИТЕ ЕЛЕМЕНТИ НА ПРОЦЕС НА МИКРОИНДЕНТАЦИЯ

Събина Ч. Чернева, Румен Зл. Янков

Проведени са числени симулации с помощта на метода на крайните елементи на процеса на микроиндентация на един деформируем изотропен ососиметричен образец с ротационно-симетричен твърд индентор. Предложено е подходящо отношение между размера на следата на индентора и размера на областа, в която се решава граничната задача, описваща процеса на микроиндентация. С числени експерименти е установено, че полето на деформации в разглежданата задача не се влияе от граничните условия в премествания по границата $G4$. Изследвано е влиянието на граничното условие по границата $G4$ (Фиг. 1) върху полетата на напреженията и преместванията, както и върху кривите на натоварване-преместване на индентора. Съвпадането на получените вследствие на числени симулации със и без гранично условие по $G4$ криви на натоварване-преместване, дава основание да се направи извода, че предложеното съотношение е подходящо при тази гранична задача.