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APPLICATIONS OF THE SPACE SHAPE OF THE TRIANGLE*

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We define affine shape coordinates of a point with respect to a fixed triangle. In conformity with the Kimberling's definition for a center and a center-function we determine a shape-center-function as a function of a space shape of a triangle. We apply this definition to solve the problem **axy** and to describe some loci in terms of shapes.

1.Affine shape coordinates of a point in the plane. J. Lester introduced a complex analytic formalism for the study of Euclidean plane in [6]. A main tool of this formalism is the shape of a triangle. We recall briefly her definition. Identify the Euclidean plane $\mathbb{E}^2$ with the field of the complex numbers. If a, b and c are three distinct points in $\mathbb{E}^2$, then the shape of the triangle $\triangle\mathbf{abc}$ is the ratio

$$(1)\qquad \triangle_{\mathbf{abc}} = \frac{\mathbf{a}-\mathbf{c}}{\mathbf{a}-\mathbf{b}} = \frac{|\mathbf{a}-\mathbf{c}|}{|\mathbf{a}-\mathbf{b}|}(\cos\sphericalangle\mathbf{bac} + i.\sin\sphericalangle\mathbf{bac}).$$

This means that, up to similarity, any triangle is determined completely by a single complex number.

In the paper [1] the notion of a shape of a triangle is carried over the three-dimensional Euclidean space $\mathbb{E}^3$. A definition and some properties are given below. Let $\mathbb{H}$ be the quaternion algebra. Identify $\mathbb{E}^3$ with the imaginary space $\operatorname{Im}\mathbb{H}$ of pure quaternions. The properties of quaternions and their applications are known from [4] and [5]. Three distinct points $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ in $\mathbb{E}^3$ determine a non degenerate or degenerate triangle $\triangle\mathbf{abc}$.

Definition 1. *Let* $\mathbf{a}$, $\mathbf{b}$ *and* $\mathbf{c}$ *be three points in* $\mathbb{E}^3$ *such that* $\mathbf{a}\neq\mathbf{b}$. *A space shape of the ordered triple of points* $\mathbf{a}$, $\mathbf{b}$ *and* $\mathbf{c}$ *is called the quaternion*

$$p = S(\mathbf{a},\mathbf{b},\mathbf{c}) = (\mathbf{a}-\mathbf{c})(\mathbf{a}-\mathbf{b})^{-1}.$$

Triangles with the same vertices have generally different shapes, which can be determined from the properties: $S(\mathbf{b},\mathbf{c},\mathbf{a}) = (1-p)^{-1}$, $S(\mathbf{c},\mathbf{a},\mathbf{b}) = 1-p^{-1}$, $S(\mathbf{a},\mathbf{c},\mathbf{b}) = p^{-1}$ and $S(\mathbf{a},\mathbf{b},\mathbf{c})S(\mathbf{b},\mathbf{c},\mathbf{a})S(\mathbf{c},\mathbf{a},\mathbf{b}) = -1$. If $A = \sphericalangle\mathbf{bac}$, $B = \sphericalangle\mathbf{cba}$, $C = \sphericalangle\mathbf{acb}$, then

$$p = \frac{|\mathbf{a}-\mathbf{c}|}{|\mathbf{a}-\mathbf{b}|}(\cos A + l.\sin A),$$

$$\text{where } l = \frac{(\mathbf{a}-\mathbf{b})\times(\mathbf{a}-\mathbf{c})}{|(\mathbf{a}-\mathbf{b})\times(\mathbf{a}-\mathbf{c})|} \in \operatorname{Im}\mathbb{H} \text{ and } |l| = 1.$$

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The space shape of a triangle contains all the usual information about its angles and ratios of side lengths: for example, we have $\cos A = \dfrac{\operatorname{Re} p}{|p|}$, $\sin A = \dfrac{|\operatorname{Im} p|}{|p|}$, $\dfrac{|\mathbf{a}-\mathbf{c}|}{|\mathbf{a}-\mathbf{b}|} = |p|$. The normal vector to the triangle plane $\operatorname{Im} p$ determines an orientation in the same plane. If $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ are distinct and collinear points in $\mathbb{E}^3$, then $S(\mathbf{a}, \mathbf{b}, \mathbf{c}) = p \in \mathbb{R}$ is the so called signed (or affine) ratio of the points $\mathbf{c}$, $\mathbf{b}$, $\mathbf{a}$, which is a fundamental affine invariant.
The next lemma affords an opportunity to define affine shape coordinates of any point with respect to a non degenerate base triangle $\triangle\mathbf{abc}$. It is proved in [1].

Lemma 1. *Let $\triangle\mathbf{abc}$ be a non degenerate triangle with a space shape p, and let $\mathbf{d}$ be an arbitrary point in $\mathbb{E}^3$. Then, the points $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{d}$ are coplanar if and only if there exists a unique ordered pair (α, β) of real numbers such that $S(\mathbf{a}, \mathbf{c}, \mathbf{d}) = \alpha + \beta p^{-1}$.*

Definition 2. *Let $\mathbf{d}$ be an arbitrary point lying in the plane of the triangle $\triangle\mathbf{abc}$. Then, the ordered pair (α, β) of real numbers is called affine shape coordinates of $\mathbf{d}$ with respect to the triangle $\triangle\mathbf{abc}$ if $S(\mathbf{a}, \mathbf{c}, \mathbf{d}) = \alpha + \beta p^{-1}$.*

It is clear that
(i) $\mathbf{d} \in \mathbf{ab} \iff \alpha = 0$, $\mathbf{d} \in \mathbf{bc} \iff \alpha + \beta = 1$, $\mathbf{d} \in \mathbf{ca} \iff \beta = 0$;
(ii) $\mathbf{d}$ is an inner point of the triangle $\triangle\mathbf{abc}$ if and only if $\left| \begin{array}{l} 0 < \alpha < 1 \\ 0 < \alpha + \beta < 1 \end{array} \right.$;
(iii) $\mathbf{a} - \mathbf{d} = \alpha(\mathbf{a} - \mathbf{c}) + \beta(\mathbf{a} - \mathbf{b})$.
The following properties can be easily checked : If the affine shape coordinates of a point $\mathbf{d}$ with respect to the triangle $\triangle\mathbf{abc}$ are (α, β), then (β, α) are the affine shape coordinates of $\mathbf{d}$ with respect to the triangle $\triangle\mathbf{acb}$, and $(\alpha, 1 - \alpha - \beta)$ are the affine shape coordinates of $\mathbf{d}$ with respect to the triangle $\triangle\mathbf{bac}$.
Simple calculations imply the next Proposition about some triangle centers.

Proposition 1. *Let $\triangle\mathbf{abc}$ be a non degenerate triangle with a space shape p and let $\mathbf{d}$ be an arbitrary point in the triangle plane. If (α, β) are the affine shape coordinates of the point $\mathbf{d}$ with respect to the triangle $\triangle\mathbf{abc}$, then*
(i) $\mathbf{d}$ *is the centroid of the triangle $\triangle\mathbf{abc}$ if and only if $\alpha = \frac{1}{3}$, $\beta = \frac{1}{3}$;*
(ii) $\mathbf{d}$ *is the orthocenter of the triangle $\triangle\mathbf{abc}$ if and only if*

$$\alpha = \frac{\operatorname{Re} p(1 - \operatorname{Re} p)}{|\operatorname{Im} p|^2}, \quad \beta = \frac{\operatorname{Re} p(|p|^2 - \operatorname{Re} p)}{|\operatorname{Im} p|^2};$$

(iii) $\mathbf{d}$ *is the circumcenter of the triangle $\triangle\mathbf{abc}$ if and only if*

$$\alpha = \frac{|p|^2 - \operatorname{Re} p}{2|\operatorname{Im} p|^2}, \quad \beta = \frac{|p|^2(1 - \operatorname{Re} p)}{2|\operatorname{Im} p|^2};$$

(iv) $\mathbf{d}$ *is the incenter of the triangle $\triangle\mathbf{abc}$ if and only if*

$$\alpha = \frac{1}{|1 - p| + |p| + 1}, \quad \beta = \frac{|p|}{|1 - p| + |p| + 1}.$$

The above statements can be directly assigned to the Gaussian plane and then $p \in \mathbb{C} \setminus \mathbb{R}$.

2. Shape-center-function. The affine shape coordinates also afford an opportunity to determine a center of a triangle. We shall consider more general case of a triangle in Euclidean space. Let us recall Kimberling' s definition for a center and a center-function in [3]:

Definition 3. *Let $\mathbb{T}$ be the set of all triples (a_1, a_2, a_3) of real numbers that are sidelengths of a triangle, i.e.*

$$\mathbb{T} = \{(a_1, a_2, a_3) : 0 < a_1 < a_2 + a_3,\ 0 < a_2 < a_1 + a_3,\ 0 < a_3 < a_1 + a_2\}.$$

On any subset $\mathbb{U}$ of $\mathbb{T}$, define a center-function as a nonzero function $f(a_1, a_2, a_3)$ homogeneous with respect to a_1, a_2, a_3 and symmetric with respect to a_2 and a_3 (i. e. $f(a_1, a_2, a_3) = f(a_1, a_3, a_2)$ for all $(a_1, a_2, a_3) \in \mathbb{U}$.) A center on $\mathbb{U}$ is an equivalence class $x_1 : x_2 : x_3$ of ordered triples (x_1, x_2, x_3) given by

$$x_1 = f(a_1, a_2, a_3), \quad x_2 = f(a_2, a_3\, a_1), \quad x_3 = f(a_3, a_1, a_2)$$

for some center-function f defined on $\mathbb{U}$.

Using complex triangle coordinates, J. Lester gives another definition of a center of a triangle. In the remaining part of the paper we use only the above mentioned Kimberling' s definition.

Definition 4. *Define a shape-center-function $\Phi : \mathbb{H} \setminus \mathbb{R} \longrightarrow \mathbb{R}^*$ as a nonzero function which satisfies the conditions:*
(i) $\Phi(p) = \Phi(\overline{p}) = \overline{\Phi(p)}$ for any $p \in \mathbb{H} \setminus \mathbb{R}$;
(ii) $\Phi(p) = \Phi(1 - p)$ for any $p \in \mathbb{H} \setminus \mathbb{R}$.
A center of the triangle $\triangle\mathbf{abc}$ with a space shape $p = S(\mathbf{a}, \mathbf{b}, \mathbf{c})$ is called the ordered pair (α, β) of real numbers such that $\alpha = \Phi(p)$, $\beta = \Phi(p^{-1})$ for some shape - center - function Φ defined on $\mathbb{H} \setminus \mathbb{R}$.

Cycling the vertices of the triangle $\triangle\mathbf{abc}$ with a space shape p, we obtain for the shape-center-function Φ that

$$\Phi(p) + \Phi((1 - p)^{-1}) + \Phi(1 - p^{-1}) = 1 \quad \text{for any } p \in \mathbb{H} \setminus \mathbb{R}.$$

The next theorem solves the problem **axy** from [2] in terms of shapes.

Theorem 2. [*Problem* **axy**] *Let $X = (\Phi(p), \Phi(p^{-1}))$ be a center. Let $\mathbf{x}$ be its value in the triangle $\triangle\mathbf{abc}$ with a space shape p and let $\mathbf{y}$ be its value in the triangle $\triangle\mathbf{xbc}$ with a space shape p_1. Then, the points $\mathbf{a}$, $\mathbf{x}$, $\mathbf{y}$ are collinear if and only if Φ satisfies the functional equation*

$$\Phi(p^{-1})\Phi(p_1) = \Phi(p)\Phi(p_1^{-1}), \tag{2}$$

where

$$p_1 = \{[\Phi(p) - 1]p + \Phi(p^{-1})\}\{\Phi(p)p + \Phi(p^{-1}) - 1\}^{-1}. \tag{3}$$

Proof. Let (α, β) be the affine shape coordinates of the point $\mathbf{x}$ with respect to the triangle $\triangle\mathbf{abc}$, (α_1, β_1) be the affine shape coordinates of $\mathbf{y}$ with respect to the same triangle and $(\widehat{\alpha}, \widehat{\beta})$ be the affine shape coordinates of $\mathbf{y}$ with respect to the triangle $\triangle\mathbf{xbc}$. The collinearity of the points $\mathbf{a}$, $\mathbf{x}$, $\mathbf{y}$ is equivalent to $\beta\alpha_1 = \alpha\beta_1$. From $S(\mathbf{x}, \mathbf{c}, \mathbf{y}) = \widehat{\alpha} + \widehat{\beta}p_1^{-1}$ we have that $\mathbf{a} - \mathbf{y} = \widehat{\alpha}(\mathbf{x} - \mathbf{c}) + (\widehat{\beta} - 1)(\mathbf{x} - \mathbf{a}) + (\mathbf{a} - \mathbf{b})\widehat{\beta}$. Hence, $S(\mathbf{a}, \mathbf{c}, \mathbf{y}) = (\mathbf{a} - \mathbf{y})(\mathbf{a} - \mathbf{c})^{-1} = \widehat{\alpha}S(\mathbf{c}, \mathbf{a}, \mathbf{x}) + (1 - \widehat{\beta})S(\mathbf{a}, \mathbf{c}, \mathbf{x}) + \widehat{\beta}S(\mathbf{a}, \mathbf{c}, \mathbf{b}) = \widehat{\alpha} + (1 - \widehat{\alpha} - \widehat{\beta})S(\mathbf{a}, \mathbf{c}, \mathbf{x}) + \widehat{\beta}p^{-1}$. Since $S(\mathbf{a}, \mathbf{c}, \mathbf{x}) = \alpha + \beta p^{-1}$, then we find that

$$\alpha_1 = \widehat{\alpha} + \alpha(1 - \widehat{\alpha} - \widehat{\beta}), \quad \beta_1 = \widehat{\beta} + \beta(1 - \widehat{\alpha} - \widehat{\beta}). \tag{4}$$

Now, (2) follows from (4) by substituting. The remaining equality (3) holds from $p_1 = S(\mathbf{x}, \mathbf{b}, \mathbf{c}) = (\mathbf{x} - \mathbf{c})(\mathbf{x} - \mathbf{b})^{-1}$.

3. Locus of orthocenters of moving triangles in Euclidean space. Consider a variable triangle $\triangle\mathbf{abc}$ in $\mathbb{E}^3 \cong \mathrm{Im}\,\mathbb{H}$ with a space shape $p = S(\mathbf{a},\,\mathbf{b},\,\mathbf{c})$, where $\mathbf{a}$ and $\mathbf{b}$ are fixed and the third vertex is allowed to vary. Let $i,\,j,\,k$ be the canonical quaternions in $\mathbb{H}$ so that $i^2 = j^2 = k^2 = -1 = ijk$. We use $\mathbf{i}, \mathbf{j}$ and $\mathbf{k}$ to denote the standard orthonormal basis for $\mathbb{E}^3 \cong \mathrm{Im}\,\mathbb{H}$. Without loss of generality we may assume that $\mathbf{a} = \mathbf{i}$, $\mathbf{b} = -\mathbf{i}$ and $\mathbf{c} = x^1\mathbf{i} + x^2\mathbf{j} + x^3\mathbf{k}$, where $x^i \in \mathbb{R}\ \ i = 1,\,2,\,3$ are the Cartesian coordinates of $\mathbf{c}$. Therefore, $p = S(\mathbf{a},\,\mathbf{b},\,\mathbf{c}) = (\mathbf{a}-\mathbf{c})(\mathbf{a}-\mathbf{b})^{-1} = \dfrac{1-x^1}{2} - \dfrac{x^2}{2}k + \dfrac{x^3}{2}j$. If we denote by $\mathbf{d}$ any center of the triangle $\triangle\mathbf{abc}$, then $\mathbf{d} = y^1\mathbf{i} + y^2\mathbf{j} + y^3\mathbf{k}$, where $y^i \in \mathbb{R}\ \ i = 1,\,2,\,3$ are Cartesian coordinates of $\mathbf{d}$ and $S(\mathbf{a},\,\mathbf{b},\,\mathbf{d}) = \dfrac{1-y^1}{2} - \dfrac{y^2}{2}k + \dfrac{y^3}{2}j$. Let $(\alpha,\,\beta)$ be the affine shape coordinates of $\mathbf{d}$ with respect to the triangle $\triangle\mathbf{abc}$. Since $(\beta,\,\alpha)$ are the affine shape coordinates of $\mathbf{d}$ with respect to the triangle $\triangle\mathbf{acb}$ with a space shape p^{-1} we obtain $S(\mathbf{a},\,\mathbf{b},\,\mathbf{d}) = \beta + \alpha p$. Replacing in the last equality $S(\mathbf{a},\,\mathbf{b},\,\mathbf{d})$ and p by the above expressions, we get

$$
\begin{aligned}
y^1 &= 1 - \alpha - 2\beta + \alpha\, x^1 \\
y^2 &= \alpha\, x^2 \\
y^3 &= \alpha\, x^3.
\end{aligned} \tag{5}
$$

Thus, we have a relation between the moving vertex $\mathbf{c}$ of the triangle $\triangle\mathbf{abc}$ and any center $\mathbf{d}$ of the same triangle. This relationship allows us to describe the locus of centers of moving triangles with two fixed vertices in the Euclidean space. The case, when $\mathbf{d}$ is the centroid of the triangle $\triangle\mathbf{abc}$ is trivial. Here we deal with the orthocenter of the triangle $\triangle\mathbf{abc}$.

Proposition 2. *Let $\triangle\mathbf{abc}$ be a non degenerate triangle in $\mathbb{E}^3 \cong \mathrm{Im}\,\mathbb{H}$ with fixed vertices $\mathbf{a} = \mathbf{i}$ and $\mathbf{b} = -\mathbf{i}$. The map $\mathcal{F} : \mathbb{E}^3\backslash(\mathbf{ab}) \longrightarrow \mathbb{E}^3\backslash(\mathbf{ab})$ defined by*

$$
\begin{aligned}
y^1 &= x^1 \\
y^2 &= \frac{1-(x^1)^2}{(x^2)^2 + (x^3)^2}\, x^2 \\
y^3 &= \frac{1-(x^1)^2}{(x^2)^2 + (x^3)^2}\, x^3
\end{aligned} \tag{6}
$$

maps any point $x^1\mathbf{i} + x^2\mathbf{j} + x^3\mathbf{k} = \mathbf{c} \in \mathbb{R}^3\backslash(\mathbf{ab})$ in the orthocenter of the triangle $\triangle\mathbf{abc}$ and vice versa.

Proof. Applying the condition (ii) in Proposition 1 and replacing α and β in (5), we obtain (6). Conversely, since the map (6) is an involution, $\mathcal{F}$ maps any point $\mathbf{d} \in \mathbb{E}^3\backslash(\mathbf{ab})$ into a point $\mathbf{c} \in \mathbb{E}^3\backslash(\mathbf{ab})$ such that $\mathbf{d}$ is the orthocenter of the triangle $\triangle\mathbf{abc}$.

Now, let $\mathcal{H} \subset \mathbb{E}^3\backslash(\mathbf{ab})$ be a plane perpendicular to the line $(\mathbf{ab})$. From (6) it follows immediately that $\mathcal{H}$ is an invariant under the map $\mathcal{F}$ and the restriction $\mathcal{F}_{|\mathcal{H}} : \mathcal{H} \to \mathcal{H}$ is an inversion in $\mathcal{H}$. The next examples are direct applications of this inversion.

Example 1. Let $\triangle\mathbf{abc}$ be a non degenerate triangle in $\mathbb{E}^3$ with two fixed vertices $\mathbf{a}$ and $\mathbf{b}$. Then, the vertex $\mathbf{c}$ moves along:
(i) a line in the plane $\mathcal{H}$ non intersecting the line $(\mathbf{ab})$ if and only if the orthocenter of the triangle $\triangle\mathbf{abc}$ describes a circle in the same plane through the point $\mathcal{H}\bigcap(\mathbf{ab})$;
(ii) a circle in the plane $\mathcal{H}$ passing through the point $\mathcal{H}\bigcap(\mathbf{ab})$ if and only if the

orthocenter of the triangle $\triangle\mathbf{abc}$ describes a line in the same plane non intersecting the line $(\mathbf{ab})$;
(iii) a circle in the plane $\mathcal{H}$ not passing through the point $\mathcal{H}\bigcap(\mathbf{ab})$ if and only if the orthocenter of the triangle $\triangle\mathbf{abc}$ describes a circle in the same plane not passing through the point $\mathcal{H}\bigcap(\mathbf{ab})$.

Example 2. Let $\triangle\mathbf{abc}$ be a non degenerate triangle in $\mathbb{E}^3$ with two fixed vertices $\mathbf{a}$ and $\mathbf{b}$. Then, the vertex $\mathbf{c}$ moves along a logarithmic spiral in the plane $\mathcal{H}$ with a pole at the point $\mathcal{H}\bigcap(\mathbf{ab})$ if and only if the orthocenter of the triangle $\triangle\mathbf{abc}$ describes a logarithmic spiral in $\mathcal{H}$ with the pole at the same point and with an opposite orientation.

Another application of the map $\mathcal{F}$ can be obtained if the third vertex $\mathbf{c}$ of the triangle $\triangle\mathbf{abc}$, where the vertices $\mathbf{a}$ and $\mathbf{b}$ are fixed, moves along a surface of revolution in $\mathbb{E}^3$ with an axis of rotation $(\mathbf{ab})$. So, let $\mathbf{a}=\mathbf{i}$, $\mathbf{b}=-\mathbf{i}$ and S be a surface of revolution with axis of rotation $(\mathbf{ab})$. Then, we may represent S in Cartesian coordinates by

$$S : x = x(f_1(u),\ f_2(u)\cos v,\ f_2(u)\sin v) \quad f_i : \mathrm{I} \to \mathbb{R},\ i = 1,\ 2,\ v \in (0, 2\pi]$$

Using the expressions (6), we get the image of S under the map $\mathcal{F}$, i.e.

$$\mathcal{F}(S) : y = y(f_1(u),\ \frac{1-f_1^2(u)}{f_2(u)}\cos v,\ \frac{1-f_1^2(u)}{f_2(u)}\sin v). \tag{7}$$

Obviously, $\mathcal{F}(S)$ is a surface of revolution with the same axis of rotation. Applying the representation (7), we obtain generalizations in $\mathbb{E}^3$ of well-known assertions in the Euclidean plane.

Example 3. Let $\triangle\mathbf{abc}$ be a non degenerate triangle in $\mathbb{E}^3$ with two fixed vertices $\mathbf{a}$ and $\mathbf{b}$. Then, the third vertex $\mathbf{c}$ moves along:
(i) a cylinder of revolution with axis of rotation $(\mathbf{ab})$ if and only if the orthocenter of the triangle $\triangle\mathbf{abc}$ describes a surface of revolution with the same axis of rotation and meridians - parabolas through the points $\mathbf{a}$ and $\mathbf{b}$ (see Fig. 1.);
(ii) a cone of revolution with axis of rotation $(\mathbf{ab})$ if and only if the orthocenter of the triangle $\triangle\mathbf{abc}$ describes a surface of revolution with the same axis of rotation and meridians - hyperbolas through the points $\mathbf{a}$ and $\mathbf{b}$ (see Fig. 2.).

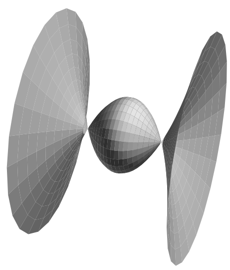

Fig. 1.

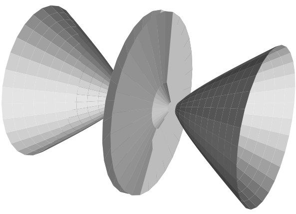

Fig. 2.

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ПРИЛОЖЕНИЯ НА ПРОСТРАНСТВЕНИЯ ШЕЙП НА ТРИЪГЪЛНИКА

Радостина Петрова Енчева

Дефинираме афинни шейп координати на точка относно фиксиран триъгълник. В съответствие с дефинициятана Кимберлинг за център и център-функция дефинираме шейп-център-функция като функция от пространствения шейп на триъгълника. Прилагаме тази дефиниция за да решим задачата **axy** и за да опишем някои геометрични места в термините на шейпа.