*МАТЕМАТИКА И МАТЕМАТИЧЕСКО ОБРАЗОВАНИЕ, 2008*
*MATHEMATICS AND EDUCATION IN MATHEMATICS, 2008*
*Proceedings of the Thirty Seventh Spring Conference of the Union of Bulgarian Mathematicians*
*Borovetz, April 2–6, 2008*

# MINTY VARIATIONAL PRINCIPLE IN SET-VALUED OPTIMIZATION*

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In scalar optimization it is well known that a solution of a Minty variational inequality of differential type is a solution of the related optimization problem. This relation is known as "Minty variational principle". In the vector case, the links between Minty variational inequalities and vector optimization problems have been investigated in [8] and subsequently in [7]. In this paper we extend such results to the case of a set-valued optimization problem distinguishing two different kinds of solutions, namely ideal (or absolute) efficient points and weakly efficient points.

**1. Introduction.** Let $X$ be a linear space, and let $K$ be a convex subset of $X$. In [2] and [3] we studied the Minty-type variational inequality (for short VI) of differential type

$$f'_-(x,\, x^* - x) \le 0, \quad x \in K\,. \tag{1}$$

Here $f'_-(x,u)$ denotes the lower Dini directional derivative of $f : X \to \mathbb{R}$ in direction $u \in X$ defined for $x \in X$ as an element of $\overline{\mathbb{R}} = \mathbb{R} \cup \{-\infty\} \cup \{+\infty\}$ by

$$f'_-(x,u) = \liminf_{t \to 0^+} \frac{1}{t}(f(x+tu) - f(x))\,.$$

Clearly, inequality (1) is a generalization of the classical Minty VI of differential type [9]. It has been shown in [2], [3] that, under certain regularity assumptions on $f$, if $x^*$ is a solution of VI (1), then it is also a solution of the minimization problem

$$\min f(x)\ ,\ x \in K\,. \tag{2}$$

This, is usually called the "Minty variational principle". Further, under convexity assumptions on the function $f$, every solution of problem (2) is a solution of the VI (1).

In [8] the Minty vector variational inequality (of differential type) has been introduced and its solutions have been linked to the solutions of a vector optimization problem. Subsequently, in [7] such results have been extended to a not necessarily differentiable function $f$. In this paper, we extend the investigation started in [7], to the case of set-valued optimization problems.

**2. Preliminaries.** In the sequel $X$ denotes a real linear space and $K$ is a convex subset of $X$. Further, $Y$ is a finite-dimensional normed space with dual space $Y^*$, $C \subset Y$ is a pointed closed convex cone with nonempty interior, $C' \subset Y^*$ is its positive polar

*__2000 Mathematics Subject Classification:__ 49J40, 49J53, 90C29, 47J20
**Key words:** Set-valued optimization, variational inequalities, Minty variational principle.

cone, and $S$ is the unit sphere in $Y^*$. Recall that a vector $\xi \in C'$ is said to be an extreme direction of $C'$ when $\xi \in C' \backslash \{0\}$ and $\xi = \xi^1 + \xi^2$, $\xi^1$, $\xi^2 \in C'$, implies $\xi^1 = \lambda_1 \xi$, $\xi^2 = \lambda_2 \xi$ for some positive reals $\lambda_1$, $\lambda_2$. We denote by $\operatorname{extd} C'$ the set of extreme directions of $C'$.

In this paper we focus on the set-valued optimization problem

$$\min\nolimits_C F(x), \quad x \in K\,, \tag{3}$$

where $F : K \rightsquigarrow Y$ is a set-valued function (for short, *svf*) with nonempty values. In the following we assume that $F$ has also compact and convex values. Like in [6], the solutions to (3) (minimizers) are defined as pairs $(x^0, y^0)$, $y^0 \in F(x^0)$. In this paper we deal with global minimizers and next we recall some definitions.

The pair $(x^0, y^0)$, $y^0 \in F(x^0)$, is said to be a $w$-minimizer (weakly efficient point) if $F(K) \cap \big(y^0 - \operatorname{int} C\big) = \emptyset$. The pair $(x^0, y^0)$, $y^0 \in F(x^0)$, is said to be $a$-minimizer (absolute or ideal efficient point) if $F(K) \subset y^0 + C$. For a given set $M \subset Y$ we define the $a$-frontier (absolute or ideal frontier) by $a\text{-}\mathrm{Min}_C M = \{y \in M \mid M \subset y + C\}$. Let us point out that the $a$-frontier with respect to a pointed cone $C$, if not empty, is a singleton.

Besides the previous solution concepts which are classical in set-valued optimization, we consider the following notions, introduced in [6]. The point $x^0 \in K$ is said to be a set $a$-minimizer of problem (3), if $F(x) \subset F(x^0) + C$ for all $x \in K$. We say that $x^0 \in K$ is a set $w$-minimizer of the set-valued problem (3) with a *svf* $F : K \rightsquigarrow Y$, if for each $x \in K$ there exists $y^0 \in F(x^0)$ such that $F(x) \cap (y^0 - \operatorname{int} C) = \emptyset$.
Following the approach of [5], the space $Y$ can be extended with infinite elements. Indeed, we introduce the set of infinite elements $Y_\infty = \{v_\infty \mid v \in Y \setminus \{0\}\}$. The element $v_\infty$ is interpreted as the infinite element in direction $v$. We accept that $v^1_\infty = v^2_\infty$ if and only if $v^2 = \lambda v^1$ for some $\lambda > 0$. We put $\tilde{Y} = Y \cup Y_\infty$.

A topology on $\tilde{Y}$ can be introduced in terms of local bases of neighbourhoods. If $y \in Y$ and $\mathcal{B}(y)$ is a local base of neighbourhoods of $y$ in $Y$, then we accept that $\mathcal{B}(y)$ is also a local base of neighbourhoods of $y$ in $\tilde{Y}$. The family $\mathcal{B}(v_\infty) = \{(y + W) \cup W_\infty \mid y \in Y,\ v \in W,\ W \text{ open cone in } Y\}$ constitutes a local base of neighbourhoods of $v_\infty$. Here $W_\infty = \{w_\infty \mid w \in W \setminus \{0\}\}$. Saying that $W$ is an open cone in $Y$, we mean that $W$ is an open set in $Y$ such that $\lambda W \subset W$ for all $\lambda > 0$. The extended topological space $\tilde{Y}$ is compact [5]. Since $\tilde{Y}$ is a topological space, we can apply topological operations on $\tilde{Y}$. Obviously $\operatorname{cl} C = \tilde{C} := C \cup C_\infty$, where $C_\infty = \{v_\infty \mid v \in C \setminus \{0\}\}$. We have also $\operatorname{int} \tilde{C} = \operatorname{int} C \cup C^\circ_\infty$, where $C^\circ_\infty = \{v_\infty \mid v \in \operatorname{int} C\}$. Take into consideration that the topological operations in $Y$ in general lead to different results than those in $\tilde{Y}$, e. g. in $Y$ it holds $\operatorname{cl} C = C$. To avoid ambiguity, we will notice usually the cases when topological operations in $\tilde{Y}$ are applied.

Dini derivatives for set-valued functions have been studied for instance in [4]. The Dini derivative of svf $F : K \rightsquigarrow Y$ at $(x, y)$, $y \in F(x)$, in the feasible direction $u \in X$ is

$$F'(x, y; u) = \operatorname*{Limsup}_{t \to 0^+} \frac{1}{t}\big(F(x + tu) - y\big)\,, \tag{4}$$

where the Limsup is calculated in $\tilde{Y}$. Following the scheme developed in [5] and [7], we propose the following variational inequalities (VI)

$$F'(x, y; x^* - x) \cap (-\tilde{C}) \neq \emptyset\,, \quad x \in K,\ \ y \in F(x)\,; \tag{5}$$

$$F'(x, y; x^* - x) \not\subseteq \operatorname{int} \tilde{C}\,, \quad x \in K,\ \ y \in F(x)\,. \tag{6}$$

We call a solution of (5) (resp. of (6)) a point $x^* \in K$, such that for all $x \in K$ and all

$y \in F(x)$ the property in (5) (resp. (6)) holds.

We recall that the *svf* $F : K \rightsquigarrow Y$ is said to be $C$-convex when for every $x^1, x^2 \in K$ and $t \in [0,1]$ it holds $tF(x^1) + (1-t)F(x^2) \subseteq F(tx^1 + (1-t)F(x^2)) + C$.

**Theorem 2.1** [1] *The svf $F : K \rightsquigarrow Y$ is $C$-convex if and only if the function $\phi_\xi(x) = \inf_{y\in F(x)}\langle \xi, y\rangle$ is convex for every $\xi \in C'$.*

We close this section recalling that a single valued function $f : K \to Y$ is said to be radially lower semicontinuous along the rays starting at $x^* \in K$ and we write $f \in RLSC(K, x^*)$, if for every $x \in K$ the function $g : [0,1] \to Y$, $g(t) = f((x^* + t(x - x^*))$ is lower semicontinuous.

## 3. Minty vector VI and vector optimization.

We investigate the relationships between solutions of VI (5) and (6) and of problem (3).

### 3.1. Case 1: VI (5) and $a$-minimizers.

**Lemma 3.1.** *Let $x^* \in K$ and assume that for every $x \in K$ and $\xi \in C'$, it holds $(\phi_\xi)'_-(x, x^* - x) \le 0$. Then, $x^*$ is a solution of VI (5). Conversely, assume that $x^* \in K$ is a solution of VI (5), which satisfies $F'(x, y; x^* - x) \cap (-\operatorname{int}\tilde{C}) \neq \emptyset$ (the interior is taken in $\tilde{Y}$) when $F'(x, y; x^* - x) \cap -C = \emptyset$. Then, for every $x \in K$ and $\xi \in C'$, it holds $(\phi_\xi)'_-(x, x^* - x) \le 0$.*

**Proof.** Consider a point $x \in K$ and suppose that for every $\xi \in C'$ it holds $(\phi_\xi)'_-(x, x^* - x) \le 0$. For a fixed $\xi \in C'$, let $t_k \to 0^+$ be a sequence such that

$$(\phi_\xi)'_-(x, x^* - x) = \lim_{k\to+\infty} \frac{1}{t_k}(\phi_\xi(x + t_k(x^* - x)) - \phi_\xi(x))$$

$$= \lim_{k\to+\infty} \frac{1}{t_k}\left(\min_{y\in F(x+t_k(x^*-x))}\langle \xi, y\rangle - \min_{y'\in F(x)}\langle \xi, y'\rangle\right) \le 0 \;.$$

If we take a vector $\bar{y} \in F(x)$, such that $\langle \xi, \bar{y}\rangle = \min_{y\in F(x)}\langle \xi, y\rangle$ and a sequence of vectors $y^k \in F(x+t_k(x^*-x))$ such that $\langle \xi, y^k\rangle = \min_{y\in F(x+t_k(x^*-x))}\langle \xi, y\rangle$, then we have $(\phi_\xi)'_-(x, x^*-x) = \lim_{k\to+\infty} \frac{1}{t_k}\langle \xi, y^k - \bar{y}\rangle$. Assume first $\frac{1}{t_k}\langle \xi, y^k - \bar{y}\rangle$ is a bounded sequence. Hence, without loss of generality, we have $\lim_{k\to+\infty} \frac{1}{t_k}\langle \xi, y^k - \bar{y}\rangle = v \in Y$ and we obtain $\langle \xi, v\rangle \le 0$, for every $\xi \in C'$ that is $v \in -C$. If $\frac{1}{t_k}\langle \xi, y^k - \bar{y}\rangle$ is unbounded, then, due to the compactness of $\tilde{Y}$, we can assume, without loss of generality that

$$\lim_{k\to+\infty} \frac{1}{t_k}\langle \xi, y^k - \bar{y}\rangle = v_\infty \in \tilde{Y}\,.$$

If ab absurdo $v_\infty \notin -\tilde{C}$, i.e. $v \notin -C$, then choose an open convex cone $W \subseteq (-C)^c$ with $v \in W$ and $v + W \subseteq (-C)^c$. From the definition of convergence in $\tilde{Y}$ we have

$$\frac{1}{t_k}\langle \xi, y^k - \bar{y}\rangle \in v + W\,.$$

Since $v \in (-C)^c$, there exists $\bar{\xi} \in C'$, such that $\langle \bar{\xi}, v\rangle > 0$. Hence, it is possible to choose the cone $W$ such that $\langle \bar{\xi}, w\rangle > 0$ for every $w \in W$ and so it holds $\frac{1}{t_k}\langle \bar{\xi}, y^k - \bar{y}\rangle > \langle \bar{\xi}, v\rangle > 0$, eventually, which contradicts $(\phi_\xi)'_-(x, x^* - x) \le 0$ for every $\xi \in C'$.

Conversely, assume $x^*$ is a solution of VI (5). Then, for every $x \in K$ and $y \in F(x)$, there exists $\hat{y} \in F'(x, y; x^* - x)$, such that $\hat{y} \in -\tilde{C}$. Let $\xi \in C'$ be fixed and choose $y = \bar{y} \in F(x)$ such that $\langle \xi, \bar{y} \rangle = \min_{y \in F(x)} \langle \xi, y \rangle$. Then, according to the assumptions, the following alternative has place:

i) $\hat{y} \in Y$. In this case, we have for some sequence $t_k \to 0^+$

$$\hat{y} = \lim_{k \to +\infty} \frac{1}{t_k}(y^k - \bar{y}),$$

with $y^k \in F(x + t_k(x^* - x))$. Since $\hat{y} \in -C$, for every $\xi \in C'$ it holds $\langle \xi, \hat{y} \rangle \leq 0$ and we obtain

$$(\phi_\xi)'_-(x, x^* - x) \leq \lim_{k \to +\infty} \frac{1}{t_k} \langle \xi, y^k - \bar{y} \rangle = \langle \xi, \hat{y} \rangle \leq 0.$$

ii) $\hat{y} = \hat{v}_\infty \in (-\tilde{C}) \backslash (-C)$. Let now $t_k \to 0^+$ be such that $\hat{y} = \lim_{k \to +\infty} \frac{1}{t_k}(y^k - \bar{y})$ (we consider the convergence in $\tilde{Y}$). From the definition of convergence in $\tilde{Y}$, it follows easily the existence of a sequence $\alpha_k \in \mathbb{R}$, $\alpha_k \to 0^+$, such that

$$\lim_{k \to +\infty} \alpha_k \frac{1}{t_k}(y^k - \bar{y}) = \hat{v} \in Y.$$

From $\hat{y} \in (-\tilde{C}) \backslash (-C)$ it follows $\hat{v} \in -C$ and, hence, $\langle \xi, \hat{v} \rangle \leq 0$. This easily entails $(\phi_\xi)'_-(x, x^* - x) \leq 0$. □

**Theorem 3.1.** *Let $F : K \rightsquigarrow Y$, $x^* \in K$ and $\phi_\xi \in RLSC(K, x^*)$ for all $\xi \in C'$. If for every $x \in K$ and $\xi \in C'$ it holds $(\phi_\xi)'_-(x, x^* - x) \leq 0$, then $x^*$ is a set a-minimizer of problem* (3).

**Proof.** Under the assumptions we made the point $x^*$ is a minimizer of function $\phi_\xi$ over the set $K$, when the vector $\xi \in C'$ [2]. It follows [6] that $x^*$ is a set $a$-minimizer for problem (3). □

The proof of the next result follows immediately from Lemma 3.1 and Theorem 3.1.

**Theorem 3.2.** *Let $F : K \rightsquigarrow Y$, $x^* \in K$ and $\phi_\xi \in RLSC(K, x^*)$ for all $\xi \in C'$. Let $x^*$ be a solution of the VI* (5) *which satisfies $F'(x, y; x^* - x) \cap (-\operatorname{int} \tilde{C}) \neq \emptyset$ (the interior is taken in $\tilde{Y}$) when $F'(x, y; x^* - x) \cap -C = \emptyset$. Then, $x^*$ is a set a-minimizer of the associated optimization problem* (3).

We can say that Theorem 3.2 states a Minty vector variational principle for set $a$-minimizers.

**3.2. Case 2: VI (6) and $w$-minimizers.** In [7] in the case of vector optimization problems it has been pointed out how this case marks the difference with respect to the scalar case. Indeed, results analogous to Theorem 3.1 and Theorem 3.2 do not hold in general, but only under convexity of the considered function. In this subsection we give some extension of such results to the case of set-valued functions. We need first the following lemma whose proof is similar to that of Lemma 3.1 and, hence, is omitted.

**Lemma 3.2.** *Assume for every $x \in K$ there exists $\xi \in \operatorname{extd} C'$, such that $(\phi_\xi)'_-(x, x^* - x) \leq 0$. Then, $x^*$ is a solution of VI (6)). Conversely, assume $x^* \in K$ is a solution of VI (6)) which satisfies $F'(x, y; x^* - x) \cap \tilde{C}^c \neq \emptyset$ when $F'(x, y; x^* - x) \cap (\operatorname{int} C)^c = \emptyset$ (the interior is taken in $\tilde{Y}$). Then, for every $x \in K$ there exists $\xi \in \operatorname{extd} C'$, such that $(\phi_\xi)'_-(x, x^* - x) \leq 0$.*

We observe that in Lemma 3.2, the vectors $\xi$ can be chosen in $\operatorname{extd} C' \cap S$.

**Theorem 3.3.** *In problem* (3) *assume $C$ is a polyhedral cone and $F : K \rightsquigarrow Y$ is a $C$-convex svf. Assume further that $\phi_\xi \in RLSC(K, x^*)$, for $\xi \in \operatorname{extd} C'$. If $x^* \in K$ is such that for every $x \in K$ there exists $\xi \in C'$, such that $(\phi_\xi)'_-(x, x^* - x) \le 0$ and $a$-$\mathrm{Min}_C F(x^*) \neq \emptyset$, then $x^*$ is a set w-minimizer for problem* (3).

**Proof.** Assume the contrary. Then, one can find a point $\bar{x} \in K$, such that $F(\bar{x}) - y \cap -\operatorname{int} C \neq \emptyset$, for every $y \in F(x^*)$. If we choose in particular, $y = y^* \in a$-$\mathrm{Min}_C F(x^*)$, then we can find $\bar{y} \in F(\bar{x})$ such that $\bar{y} - y^* \in -\operatorname{int} C$. Consider the function $\tilde{\phi}_\xi(t) = \inf_{y \in F(x^* + t(\bar{x} - x^*))} \langle \xi\,, y - y^* \rangle$, with $\xi \in \operatorname{extd} C' \cap S$ and $t \in [0,1]$. From $y^* \in a$-$\mathrm{Min}_C F(x^*)$, we obtain $\tilde{\phi}_\xi(0) = 0$. Since $\phi_\xi \in RLSC(K, x^*)$, we obtain that for every $\xi \in \operatorname{extd} C' \cap S$, the function $\tilde{\phi}_\xi(t)$, attains a global minimum at a point $\bar{t} = \bar{t}(\xi) \in (0,1]$ and we have

$$\tilde{\phi}_\xi(\bar{t}(\xi)) \le \min_{t \in [0,1]} \max_{\gamma \in \operatorname{extd} C' \cap S} \tilde{\phi}_\gamma(t) \le \max_{\gamma \in \operatorname{extd} C' \cap S} \phi_\gamma(1)$$

$$= \max_{\gamma \in \operatorname{extd} C' \cap S} \inf_{y \in F(\bar{x})} \langle \gamma\,, y - y^* \rangle \le \max_{\gamma \in \operatorname{extd} C' \cap S} \langle \gamma\,, \bar{y} - y^* \rangle = \mu < 0$$

(observe that $\operatorname{extd} C' \cap S$ is a finite set, since $C$ is polyhedral and, hence, the use of $\max_{\xi \in \operatorname{extd} C' \cap S}$ is allowed). Since $F$ is $C$-convex, the functions $\tilde{\phi}_\xi(t)$, $\xi \in \operatorname{extd} C' \cap S$ are convex on the interval $[0,1]$. It follows that for every $\xi \in \operatorname{extd} C' \cap S$ one can find a number $\delta \in (0, \bar{t}(\xi))$, such that $\tilde{\phi}_\xi(t)$ is strictly decreasing on $(0, \delta)$. Put

$$\bar{\delta}(\xi) = \inf\{\delta \in (0,1) \text{ such that } \tilde{\phi}_\xi(t) \text{ is strictly decreasing on } (0,\delta)\}\,.$$

Clearly, $\bar{\delta}(\xi) > 0$ and the point $\bar{\delta}(\xi)$ is a minimum point of $\tilde{\phi}_\xi(t)$ on $[0,1]$ and, hence, $\phi_\xi(\bar{\delta}(\xi)) \le \mu < 0$. Further, we have $\bar{\delta} = \max_{\xi \in \operatorname{extd} C' \cap S} \bar{\delta}(\xi) > 0$.

For $t_1, t_2 \in (0, \bar{\delta})$ with $t_2 > t_1$, we have $\tilde{\phi}_\xi(t_2) > \tilde{\phi}_\xi(t_1)$ for every $\xi \in \operatorname{extd} C' \cap S$, i.e. $\phi_\xi(x^* + t_2(\bar{x} - x^*)) > \phi_\xi(x^* + t_1(\bar{x} - x^*))$. Since $\phi_\xi$ is convex, this entails $(\phi_\xi)'_-(x^* + t_1(\bar{x} - x^*)) < 0$. Observe now that the convex functions $\phi_\xi$, $\xi \in \operatorname{extd} C' \cap S$ are differentiable a.e. and since $\operatorname{extd} C' \cap S$ is a finite set, without loss of generality we can assume they are differentiable at $x^* + t_1(\bar{x} - x^*)$. This entails $(\phi_\xi)'_-(x^* + t_1(\bar{x} - x^*)) = -(\phi_\xi)'_-(x^* + t_1(x^* - \bar{x})) < 0$, for every $\xi \in \operatorname{extd} C' \cap S$, which is a contradiction. $\square$

**Theorem 3.4.** *In problem* (3) *assume $C$ is polyhedral and $F : K \rightsquigarrow Y$ is $C$-convex. Assume further that $\phi_\xi \in RLSC(K, x^*)$, for $\xi \in \operatorname{extd} C'$. Let $x^* \in K$ be a solution of the vector VI (6)), such that $F'(x, y; x^* - x) \cap \tilde{C}^c \neq \emptyset$ when $F'(x, y; x^* - x) \cap (\operatorname{int} C)^c = \emptyset$. Then, $x^*$ is a set w-minimizer for problem* (3).

**Proof.** It follows immediately from Lemma 3.2 and Theorem 3.3. $\square$

It remains an open question whether Theorems 3.3 and 3.4 can be extended to the case of an arbitrary closed convex pointed cone $C$ with nonempty interior.

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## ВАРИАЦИОНЕН ПРИНЦИП НА МИНТИ В ОПТИМИЗАЦИОННИ ЗАДАЧИ ЗА МНОГОЗНАЧНИ ФУНКЦИИ

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За скаларна оптимизация е известно, че всяко решение на вариационно неравенство на Минти от диференциален тип е решение на съответната оптимизационна задача. Тази връзка е позната като “вариационен принцип на Минти”. За векторния случай отношението между вариационни неравенства на Минти и векторни оптимизационни задачи е изследвана от Джанеси [F. Giannessi, T. Rapcsác, S. Komlósi (eds.), New trends in mathematical programming, 93–99, Kluwer Acad. Publ., Dordecht 1997], както и в няколко работи на Креспи, Гинчев и Рока. В настоящата статия тези резултати се пренасят за оптимизационни задачи с многозначни функции, като се разглеждат два типа решения, именно идеални ефективни решения и слаби ефективни решения.