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IMPROVING THE VISUALIZATION OF SCALOGRAMS BY MEANS OF TRANSFORMATION IN THE COMPLEX PLANE*

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In the last years the Fourier and wavelet transformations are extensively used for investigation and identification of objects, processes etc. In the present paper we propose, to change the usual rectangular shape of the scalograms using transformations, realized by complex valued functions. This is suitable for visualization and classification of real signals. Some interesting special effect for acoustic signals imagery is obtained. The notion of "sound print" of an object is introduced and discussed.

1. Introduction. Acoustics is the interdisciplinary science that deals with the study of all mechanical waves in gases, liquids, and solids including vibration, sound, ultrasound and infrasound. There is a great number of sources of sound (including ultrasound and infrasound). The human hearing is not sensible for ultrasound and infrasound (too less or very high frequencies). On the other hand the dynamic range of some sound sources exceeds the human hearing harmless limit of 120 dB. These characteristics require the use of special measuring and analyzing equipment as well as appropriate microphones.

Combination of the acoustic equipment with special software enables obtaining of visual representation of important characteristics of the sound.

2. Visualized characteristics of the sound. Here we show three of the most important characteristics:

– damping of the sound – the change of the dynamic range is shown (Fig. 1);

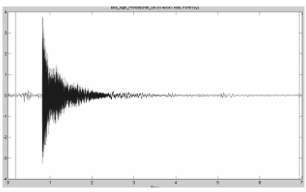

Fig. 1

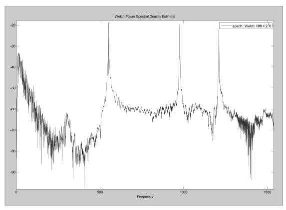

Fig. 2

– spectrum – shows the distribution of the frequencies (Fig. 2);

– scalogram – continuous wavelet transform (CWT) (Fig. 3).

We turn our attention to the scalograms. Especially, to change their rectagular shape to another one, which will be more convenient for further investigations.

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Fig. 3

For that purpose we think the scalogram (the rectangular) „placed" in the complex plane.

3. Transformations in the complex plane. Let $f(z)$, $z = x + \mathrm{i}\, y$, be a complex valued function, defined in a simply-connected subset of the complex plane. If $f(z)$ is defined in the point z_0 and $f(z_0) \neq 0$, then in some neighborhood of the point z_0 the function $f(z)$ is univalent, i.e.

$$f(z_1) = f(z_2) \Rightarrow z_1 = z_2.$$

The exponential function $w = e^z$ is a very important as well as very interesting function from analytic and geometric point of view:

- (i) e^z is defined and different from zero for each $z \neq \infty$;
- (ii) $(e^z)' = e^z \neq 0$ for each $z \neq \infty$; (cf. above remark for univalence);
- (iii) $e^z = e^{x+iy} = e^x \cdot e^{iy}$ shows that:
    - (a) lines parallel to the abscise, mapped by e^z become rays starting from the origin;
    - (b) lines parallel to ordinate axis, mapped by e^z become circles centered at the origin;
    - (c) the exponential function is periodic with basic period $2\pi\, i$.

The last property says that the exponential function is univalent, in the above described sense, in each horizontal strip with (vertical) width 2π.

Let $-\infty < a < b < +\infty$. Now consider a rectangle with vertices a, b, $b + 2\pi i$ and $a + 2\pi i$. The image of this rectangle, mapped by e^z, is a concentric ring centered at the origin with radii e^a and e^b respectively [1].

4. Transformation of the scalogram. We are in the complex plane.

a) Place the scalogram (rectangular) in the fourth quadrant, the left upper vertex (Fig. 3) to coincide with the origin;

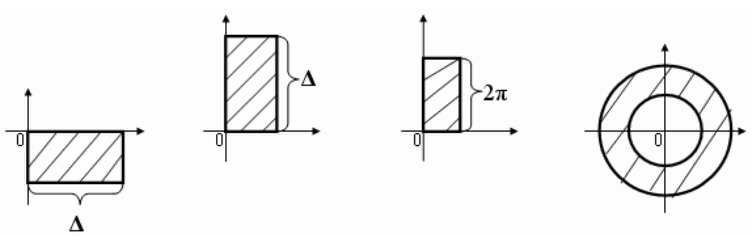

Fig. 4

b) Denote by Δ the length of the horizontal side. Rotate the scalogram on the angle 90° with centre the origin (multiplication by $e^{\frac{\pi i}{2}}$);

c) Change the size of the side Δ – multiply by $2\pi/\Delta$;

d) Map the new rectangle (scalogram) by the exponential function (Fig. 4).

The scalogram, transformed into circular ring we call *sound print* of the source of sound. This name is chosen as an analogue of the well-known *finger print*.

5. Examples. 5.1. Scalogram and sound print of a church bell.

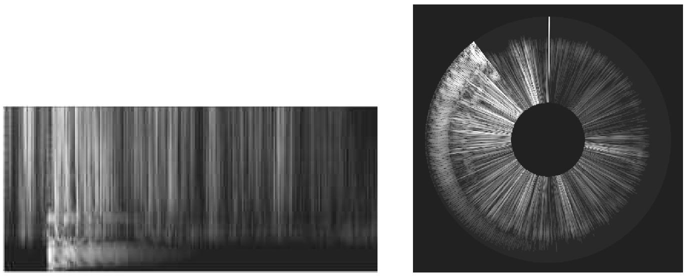

Fig. 5

5.2. Scalogram and sound print of a pistol shot.

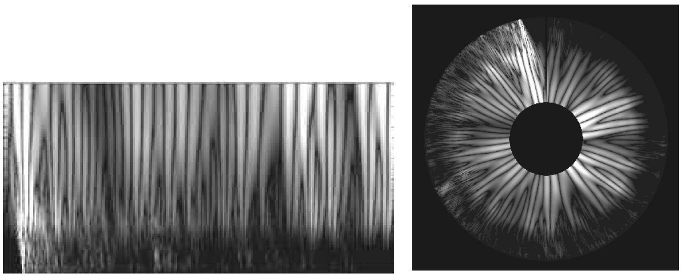

Fig. 6

5.3. Scalogram and sound print of a thunder.

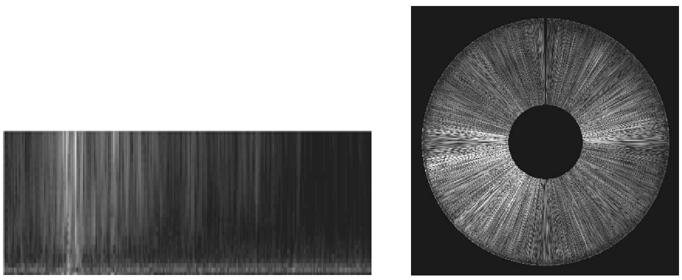

Fig. 7

6. Conclusions. The foregoing examples show that the sound prints give better possibilities for investigation of the special characteristics of the sound sources. They can be organized in libraries similar to the finger print libraries. Such libraries could serve as a basis for identification of the sound source.

7. Final remarks. The presented results are obtained by use of the system PULSE (hardware and software) together with the appropriate accessories [2] and Matlab&Simulink [3].

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ПОДОБРЯВАНЕ НА ВИЗУАЛИЗАЦИЯТА НА СКАЛОГРАМИ ЧРЕЗ ПРЕОБРАЗУВАНЕ В КОМПЛЕКСНА ОБЛАСТ

Георги Димков, Тихомир Трифонов, Иван Симеонов

През последните години Фурие и уейвлет трансформациите все по-често се използват за изследване и идентификация на обекти, процеси и др. В настоящата статия се предлага промяна на вида на скалограмите: от правоъгълник в концентричен кръгов венец. За целта си използва преобразуване в комплексна област.