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ON A METHOD OF BL. SENDOV*

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In this paper implementation of algorithm due to Bl. Sendov for approximate calculation of all positive roots of polynomial equations in MATHEMATICA package and interesting numerical examples are presented.

A method, originally due to Bl. Sendov [1], for simultaneous approximate calculation of all positive roots of the equation

$$f(x) = a_0 + a_1 x + \cdots + a_m x^m = 0 \tag{1}$$

is based on the following theorem given by Poincare:

Let f be a polynomial with real coefficients. If k is a large enough natural number, then the number of positive roots of (1) is equal to the number of changes in sign in the sequence of the coefficients of the polynomial

$$g(x) = (1+x)^k f(x).$$

Let $0 < x_1 \le x_2 \le \cdots \le x_p$, $p \le m$ be positive roots of (1) and

$$(1+x)^k f(x) = \sum_{\nu=0}^{m+k} b_k(\nu) x^\nu. \tag{2}$$

Let $\nu_k(1)$ denote the smallest integer for which $b_k(\nu_k(1)) \ge 0$ and $b_k(\nu_k(1)+1) < 0$, $b_k(0) = a_0 > 0$. In general $\nu_k(s)$ is the smallest integer for which

$$(-1)^{s-1} b_k(\nu_k(s)) \ge 0,$$

$$(-1)^{s-1} b_k(\nu_k(s)+1) < 0.$$

Then we obtain the numbers

$$\nu_k(1), \nu_k(2), \ldots, \nu_k(s). \tag{3}$$

According to Poincare, there exists a number $k_0 = k_0(f)$, such that for every $k \ge k_0$, we have $s_k = p$, where p is the number of positive roots of (1).

The numbers (3) satisfy (see Sendov [1]):

$$\frac{\nu_k(s)}{k - \nu_k(s) + 1} \le \xi(k, \nu, s) \le \frac{\nu_k(s)+1}{k - \nu_k(s)}, \tag{4}$$

$$\lim_{k\to\infty} \frac{\nu_k(s)}{k - \nu_k(s) + 1} = \lim_{k\to\infty} \xi(k, \nu, s) = x_s, \tag{5}$$

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$$f(x_s) = 0,\ s = 1, 2, \ldots, p.$$

For other results, see [2]:

$$\lim_{k\to\infty}\left(\frac{\nu}{k-\nu+1} - \frac{\frac{\nu}{k-\nu+1} - 1}{\left(\frac{\nu}{k-\nu+1}\right)^k}\left(1+\frac{1}{k}\right)^k\right) = \lim_{k\to\infty}\xi(k,\nu,s) = x_s, \tag{6}$$

$$f(x_s) = 0,\ s = 1, 2, \ldots, p.$$

Implementation of Sendov's method in MATHEMATICA package ([3]).

```
Print["The Sendov algorithm:"]
k = Input["Input value of k:"];
Print[k];
v = CoefficientList[(1 + x)^k (x^5 - 15. x^4 + 85. x^3 - 225. x^2 + 274. x - 120), x];
step = k;
n = 1;
i = 1;
key = 1;
While[key == 1 && i < k,
 If[v[[i]] < 0 && v[[i + 1]] > 0,
  Print["The change in sign in the sequence (3):"]
   Print[" v", i - 1, "=", v[[i]], " v", i, "=", v[[i + 1]]];
  Print["Root: \!\(\*FractionBox[\(v\), \(k - v + 1\)]\) (see (5))"]
   Print[(i - 1)/(k - i)];
  Print[N[(i - 1)/(k - i)]];
 ];
 If[v[[i]] > 0 && v[[i + 1]] < 0,
  Print["The change in sign in the sequence (3):"]
   Print[" v", i - 1, "=", v[[i]], " v", i, "=", v[[i + 1]]];
  Print["Root: \!\(\*FractionBox[\(v\), \(k - v + 1\)]\) (see (5))"]
   Print[(i - 1)/(k - i)];
  Print[N[(i - 1)/(k - i)]];
 ];
 i++;
]
Plot[x^5 - 15. x^4 + 85. x^3 - 225. x^2 + 274. x - 120, {x, 0., 6.}]
```

The Sendov algorithm:

100 000

The change in sign in the sequence (3):

$v49\,998 = -1.008858415 \times 10^{30\,097}$ $v49\,999 = 1.411385374 \times 10^{30\,097}$

Root: $\frac{v}{k - v + 1}$ (see (5))

$\frac{16\,666}{16\,667}$

0.99994

The change in sign in the sequence (3):

$v66\,665 = 8.82008309 \times 10^{27\,637}$ $v66\,666 = -2.19696288 \times 10^{27\,637}$

Root: $\frac{v}{k - v + 1}$ (see (5))

$\frac{66\,665}{33\,334}$

1.99991

The change in sign in the sequence (3):

$v75\,000 = -1.12889828 \times 10^{24\,416}$ $v75\,001 = 1.24036907 \times 10^{24\,415}$

Root: $\frac{v}{k - v + 1}$ (see (5))

$\frac{25\,000}{8333}$

3.00012

The change in sign in the sequence (3):

$v80\,004 = 4.0287481 \times 10^{21\,723}$ $v80\,005 = -6.35755930 \times 10^{21\,723}$

Root: $\frac{v}{k - v + 1}$ (see (5))

$\frac{26\,668}{6665}$

4.0012

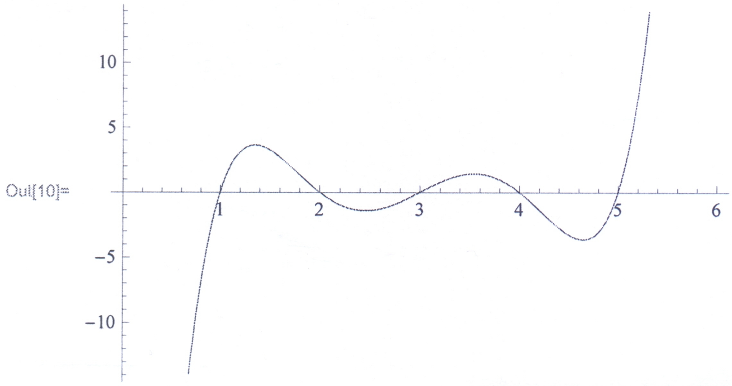
The change in sign in the sequence (3):

$v83\,344 = -2.49624133 \times 10^{19\,555}$ $v83\,345 = 3.66279672 \times 10^{19\,554}$

Root: $\frac{v}{k - v + 1}$ (see (5))

$\frac{83\,344}{16\,655}$

5.00414



Sendov's method can be applied to elegant study of financial and balance flows and determination of relevant solutions when studying investment projects – see [4] and [5].

When this methodology is implemented in MATHEMATICA programming enviroment the user did not receive the required information for the maximal polynomial degree when he apply the operator CoefficientList[...].

This is evident from the comment's message of our chosen parameter $k = 120000$:

```
The Sendov algorithm:

120 000


No more memory available.
Mathematica kernel has shut down.
Try quitting other applications and then retry.
```

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ВЪРХУ ЕДИН МЕТОД НА БЛ. СЕНДОВ

Николай Кюркчиев

В тази статия се разглежда имплементирането в програмната среда MATHEMATICA на един метод на Сендов за намиране на всички положителни корени на алгебричен полином. Методът намира елегантно приложение при изследване на финансово-балансови потоци и определяне на релевантни решения при изследване на инвестиционни проекти.