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MATHEMATICAL MODELING AND DESIGN IN THE DEVELOPMENT OF A PROFESSIONAL LEARNING COMMUNITY FOR STEM TEACHERS

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The paper describes how mathematical modeling and design were employed in developing a professional learning community for Science, Technology, Engineering, and Mathematics (STEM) teachers. A modeling perspective and design were then used as a vehicle to better understand, analyze and support the content knowledge, pedagogical content knowledge, attitudes and beliefs that mathematics and science teachers bring to their in-service training programs. The study explored the impact of a modeling-based training program on 14 mathematics and science teachers who worked together, and also with their students' parents in designing and implementing interdisciplinary tasks in their classrooms. Results revealed that teachers improved their knowledge and pedagogical approaches to interdisciplinary teaching, through modeling. Changes in their attitudes and motivation in collaborating with other teachers, were also evident. We conclude that such approaches could serve in supporting an interdisciplinary teacher learning community.

Introduction and theoretical framework. The purpose of the paper is to address how mathematical modeling and design [6] were employed in developing an Interdisciplinary Teacher Professional Learning Community for Science, Technology, Engineering, and Mathematics (STEM) teachers. In the relevant research study, on which the paper is based, researchers have adopted a multitiered research design that involved a modeling and design approach to learning, in order to understand the nature and development of science and mathematics teacher knowledge, and what it means for a teacher to develop science and mathematics content, pedagogy, and an understanding of how students develop their mathematical ideas when working with inquiry-based interdisciplinary modeling problems.

Reform movements in school science and mathematics include the notion that the two subjects should be integrated as a means of strengthening students' understanding of and appreciation for the many connections and applications that link science and mathematics [1]. This new setting places a great demand on teacher knowledge and skills, necessary to appropriately teach through an interdisciplinary approach. Further, the proposed major shift from learning science and mathematics as an accumulation of rote facts and procedures to learning science and mathematics in authentic contexts – as socially negotiated constructions and explanations used to make sense of the world [2] clearly identifies new guidelines and structure for the provided teacher training programs.

Increasingly, researchers are realising that it is crucial to work collaboratively with teachers and their students, in the reality of their own classrooms, in an attempt to better inform practice [13]. Such collaborative paradigms include design studies, which are process oriented, and theory driven, such as the multitiered teaching experiments [7], and professional development approaches that involve content-based collaborative inquiry [4]. In the latter, professional learning communities are developed that support teachers' shift and professional development to reform-oriented approaches, through a focus on their students' developments and understandings through inquiry based approaches [5].

Recent research underlined the importance of working collaboratively with teachers (and their students), in the reality of their own classrooms. Support for teacher professional development has been undergoing change from typical short-course approaches to more longitudinal, interdisciplinary, and collaborative approaches related to workplace learning. Such collaborative paradigms include, among others, professional development approaches that involve content-based collaborative inquiry. In the latter, professional learning communities are developed that support teachers' shift and professional development to reform-oriented approaches, through a focus on their students' developments and understandings through inquiry based approaches [5].

The goal in a teacher learning community is to advance the collective knowledge, and in that way to support the growth of individual teacher knowledge and skills. A successful and productive learning community must have four distinct characteristics, namely: (a) diversity of expertise and experiences among its members, who are valued for their contributions and given support to develop, (b) a shared vision of continually advancing the collective (and individual) knowledge and skills, (c) an emphasis on the development of participants' metacognitive abilities, reflective thinking, and the notion of learning how to learn, and (d) mechanisms for sharing what is learned [5].

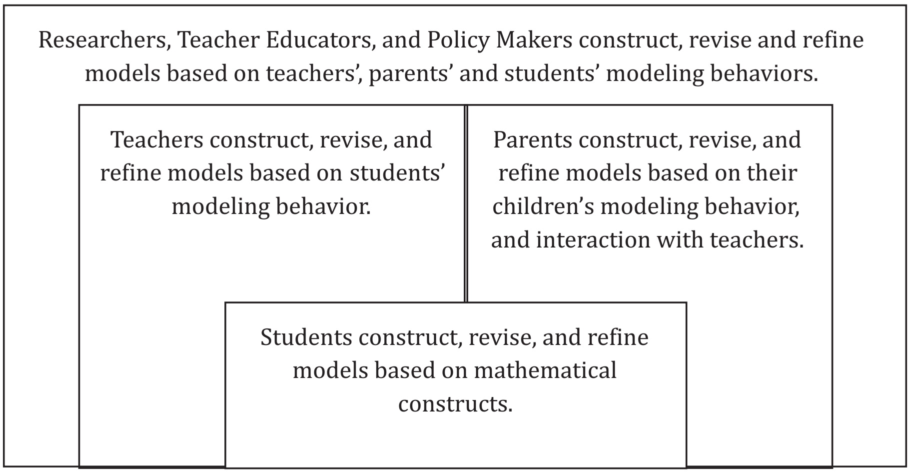


Fig. 1. Tiers of the Learning Community Development

The learning community developed in our research project was organized around a three-tiered research paradigm (see Fig. 1) that mainly addresses the development of teachers, but also the development of researchers, teachers, parents, and students, in an attempt to provide a learning and development environment for classroom practitioners and researchers to work together in generating meaningful change within modeling learning contexts [7].

In the learning community that was developed, collaboration aimed at designing and implementing experiences (involving the construction and application of models) that maximize learning at each level, while the collaboration also focused on the documentation and analysis of learning, together with reflection on learning. As students are engaged in complex modeling situations (for instance (i) provide mathematically based arguments to validate a statement (young people involved in car accidents during weekends are usually drunk), (b) why a person is gaining weight, although she is doing sports on a daily basis) that repeatedly challenge them to reveal, test, refine, and revise important aspects of mathematical constructs (e.g., various concepts from Statistics and Algebra), teachers are focused on their own thought revealing problems that focus their attention on their students' modeling behavior. Further as the teacher revises and refines her/his model, this in turn affects the students' models and vice versa. At the same time researchers (and teacher educators) are focused on the nature of teachers' and students' developing knowledge and abilities which in turn are constantly affecting each other [8].

The present study.

Purpose. The purpose of the study was to examine the impact of a multi-tiered professional learning community on mathematics and science teachers' knowledge and skills in designing and implementing interdisciplinary modeling problems in their classrooms. We hypothesized that a collaborative modeling based professional learning community would have a positive impact on teachers' mathematical knowledge, attitudes and motivation, and their pedagogical approaches in teaching interdisciplinary problems.

Participants and procedures. Fourteen teachers (eleven females and three male) teaching 6th graders (11–12 year old) from two urban situated schools agreed on participating in the present study (more schools were engaged in the project). Six participants held master's degrees in mathematics or science education. Other participants of the learning community developed included the students, the parents (around 40% of parents actively participated in the community), three inspectors who served as teacher trainers, and researchers.

The overview of the design of the learning community is presented in Fig. 2. In a period of two months, teachers (and parents) participated in five 3-hour workshops on mathematical modeling, inquiry based learning, connections between school subjects and the World of Work (WoW), and pedagogical approaches on teaching through modeling and problem solving.

Teachers were then invited to participate in a collaborative design approach to develop and implement a number of model-eliciting activities in their classrooms. Two interdisciplinary thematic areas have emerged, namely 'Car Accidents and Road Safety' and 'Health and Exercise'.

Over the course of the next two months, teachers worked collaboratively with researchers, and teacher educators to develop their own lesson plans and activities, and to implement the activities in their classrooms. Participants met weekly, communicated via email and via a blog designed for the study, and developed five 80-minute lessons for each modeling activity. With the collaboration of the research team (and a sub-contracted

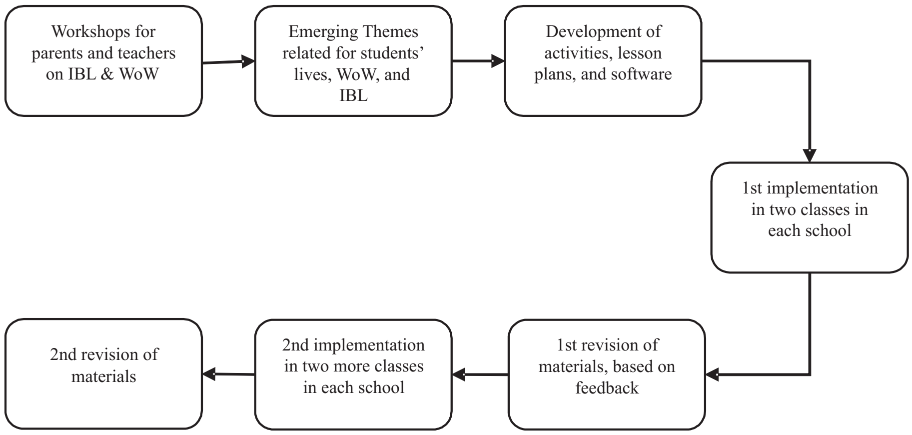


Fig. 2. Stages of the Learning Community Design

company) two interactive applets were also designed to support and facilitate students' work in the two modeling activities. Screenshots of the 'Car Accidents and Road Safety' applet and of the 'Health and Exercise' applets are presented in Fig. 3 and 4. The Health and Exercise' activity is presented in more detail in the next session.

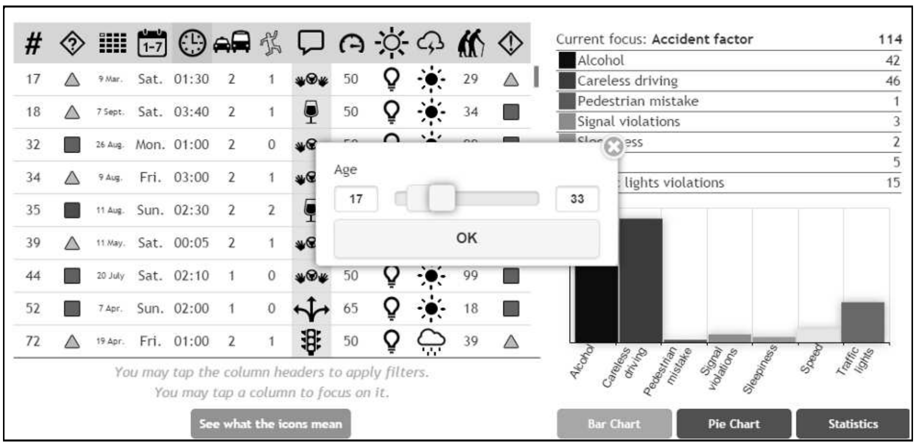


Fig. 3. Screenshot of the 'Car Accidents and Road Safety' applet

The implementation of the first modeling activity took place in two classes. Teachers, teacher trainers, parents, and the research team observed each lesson, debriefed and analyzed teacher approaches and methods, student work immediately following each observed lesson, and reflected on their understandings throughout the process. Following each lesson implementation individual interviews with the teachers implemented the lesson, and a group interview with all teachers took place. Revised activities were then tried out in other classrooms, followed by interviews (see above), and final modifications in the activities. Following a similar track, the second activity was also delivered in all eight

Fig. 4. Screenshot of the 'Health and Exercise' applet

classes. The study presented here is based on data from the 2014–2015 school calendar year.

The health and exercise' activity. The first component of the activity presented the case of Mary, a teenage girl who cannot fit in her favorite clothes. The students then considered the general question, "Is stop eating the right method to lose weight?" Students quickly realized that the question needed to be refined in order to answer it meaningfully, and statistically correct. On refining the question in their own way, students acknowledged that data on nutrition, and also on physical activity are needed. Following a class discussion on how food consumption is calculated, students agreed to work with their parents at home to anonymously complete a questionnaire regarding their nutrition and physical activity, over a period of one week (Fig. 5).

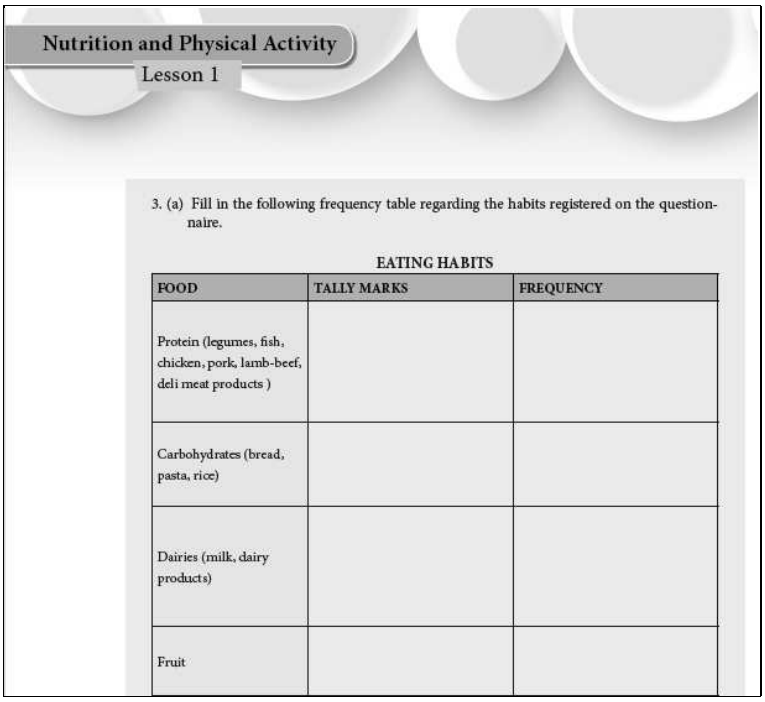
Working with their own data, each group of students summarized their results, by categorizing their data into the different food categories (e.g., protein, carbohydrates, dairy products, fruits, vegetables, sweets, etc.), and by discussing the advantages and disadvantages of each food category.

The students then used a spreadsheet software to enter their data and to use spreadsheet functions for calculating the sum and average. Students had also opportunities to explore the representation possibilities of the software to generate more detailed representations (see Fig. 6).

On completion of their representations, the students were to respond to the questions, "What does your representation tell you? How does it help to answer your question? Are more data needed?"

The second component of the activity required students to identify which factors determine a person's daily calorie intake (age, gender, height and body mass). Students worked on analyzing some tables and graphs, and by working on an applet software, designed for the purposes of the projects. Using the applet, students could work on various subjects (different age, height, mass, activity, and sports), trying to identify which factors and how they determine a person's daily calorie intake (see Fig. 7).

The third part of the activity commenced by a student statement, indicating that



Nutrition and Physical Activity

Lesson 1

3. (a) Fill in the following frequency table regarding the habits registered on the questionnaire.

EATING HABITS

FOOD	TALLY MARKS	FREQUENCY
Protein (legumes, fish, chicken, pork, lamb-beef, deli meat products)		
Carbohydrates (bread, pasta, rice)		
Dairies (milk, dairy products)		
Fruit		

Fig. 5. Students' data collection and reporting

since he likes chocolates, he could just eat nine chocolates to meet his daily calorie needs. Students had to comment on that statement and then worked with a graph presenting the recommended daily intake of servings from each food group for both girls and boys. Students recorded their responses to the following: "From which food category should you consume most in your daily servings? Boys or girls should receive more carbohydrates?" Students then moved to the software, to suggest a balanced diet plan for a day, taking also into consideration the daily amount of energy a person needs. Students could use the provided 'food database' for creating the person's diet for a day (see Fig. 4) and then explore the appropriateness of the diet with regards to the calories taken and the food categories (see Fig. 8). After completing the tasks and share their results in whole class discussion, students then moved to the last part of the task, in which they designed their own balanced nutrition and physical activity case.

Students finally returned to the first question of the activity (Is stop eating the right method to lose weight?), and documented their answers. Groups of students shared their conclusions with the class, indicating the data they used, their strategies for analyzing their data, and how certain they felt about their conclusion.

Data sources and analysis. Extensive field notes from lesson observations were collected, and during the group discussions and observations. All discussions through the planning stages, observations, debriefing meetings, lesson implementations, and lesson revisions were videotaped and transcribed. Data were triangulated with individual and group interviews, and written reflections from each participating teacher. A grounded

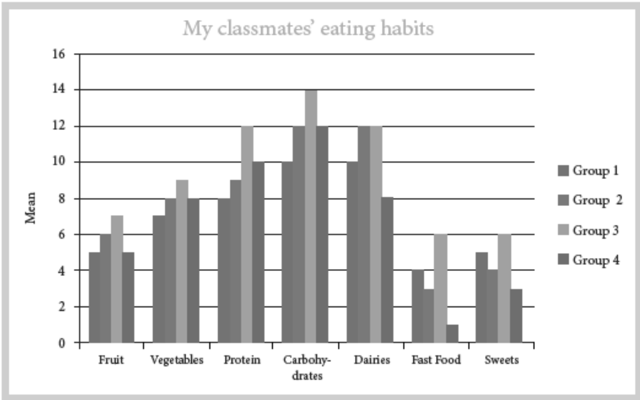


Fig. 6. A bar chart presenting the results for four groups of students

theory approach to qualitative data collection and analysis was adopted [3]. Data analysis involved applying the constant comparative method, by coding and categorizing data from all sources in order to "fracture the data and force interpretation" [9, p. 55].

Three themes were revealed through coding and categorizing patterns in teachers' discourse and teaching practice, namely teacher knowledge and pedagogical approaches, teacher attitudes and motivation, and teacher collaboration and communication.

Results. The results are presented with regards to the three themes emerged, namely: (a) teacher knowledge and pedagogical approaches to interdisciplinary teaching, emerged as teachers gradually adopted and use more appropriate teaching methods, and sought to assist students in improving their solutions, (b) teacher attitudes, self-confidence and motivation, emerged as an outcome of participants' responses in interviews, and in their reflections, and (c) communication and collaboration with teachers from the other school subject, emerged both in group and individual interviews, as well as in the collaborative lesson plan design.

Teacher knowledge and pedagogical approaches to interdisciplinary teaching. Throughout the design, implementation, and modification of the activities, teachers (and researchers) identified a number of changes in teacher knowledge and pedagogical approaches. Specifically, the following sub-themes emerged:

(i) Their ideas and conceptions on interdisciplinary teaching. Through their engagement in the course, all teachers conveyed strong convictions about the importance of interdisciplinary teaching, and how useful connecting mathematics and science instruction was for students. One teacher mentioned that: "although difficult to find appropriate tasks, it is important to integrate them [maths and science] because they can understand the real problems they face." A second teacher shared the same feeling, by expressing her acceptance of the designed tasks: "The activity [Health and Exercise] shows exactly how connecting mathematics and concepts from science can help you understand how diet and exercise are connected [...] you cannot have this understanding in any other way."

(ii) Types of questions and time spent on tasks. Gradually teachers moved from

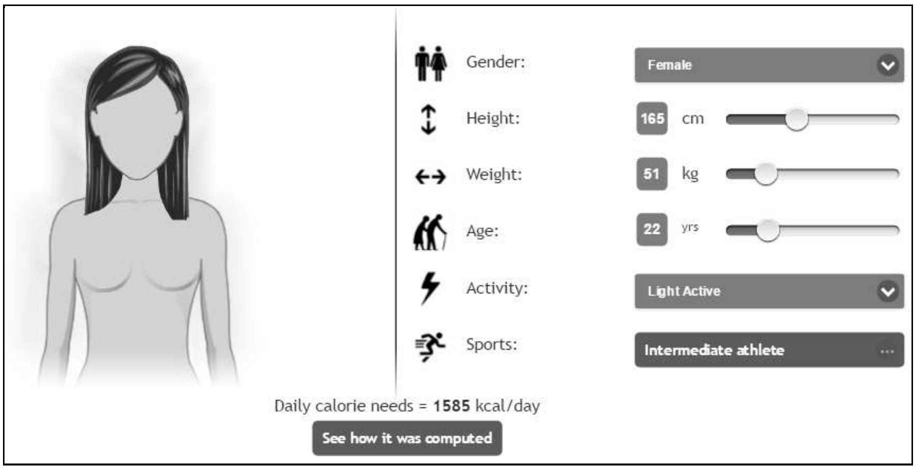


Fig. 7. Applet's screen for selecting the person characteristics

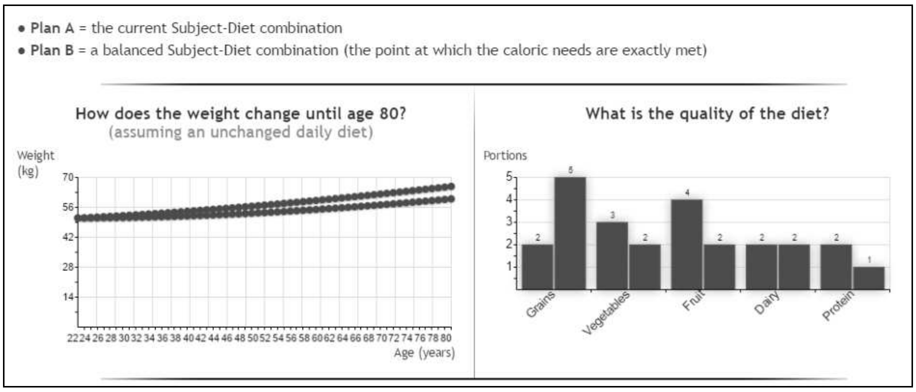


Fig. 8. Examining the appropriateness of a diet in terms of calories and food quality

addressing to students more closed questions to more open and inquiry-oriented ones, and towards questions that address topics from an interdisciplinary point of view. This shift in the nature of the questions asked was in line with a shift in the provided time for student work. Results revealed that gradually teachers provided more time for students to work on an interdisciplinary task. Evident in their reactions, teachers gradually started devoting more time in orchestrated discussions between student teams, encouraging them to focus on the interdisciplinary nature of the problems rather than on isolated mathematics or science concepts. All but one teachers reported that the training they had [PD meetings on modeling and IBL] impacted the way they asked questions. A teacher mentioned that: "We all pose more open questions, more appropriate, like those we discussed during the training [...] gradually I also address more interdisciplinary questions, rather than mathematical ones."

(iii) Better feedback to students. Provided feedback significantly improved, as teachers focused on noticing important elements (from both science and mathematics) and aspects of students' work, and addressed crucial interdisciplinary suggestions for the improvement of student models. Teachers, following recommendations from the research team and the inspectors, shifted the provided feedback to more abstract concepts rather than on wrong calculations and syntax. For example, in a task that required students to provide an argument on whether young adults involved in accidents drove under the influence of alcohol, three teachers developed assessment rubrics that only focused on the 'evidence' and 'argument clarity' reported by students. During a group interview, one of those teachers supported her choice as follows: "I tried to identify in each task the important elements and then target my feedback to these elements." A second teacher agreed and reported that it was not easy for him to shift his attention from checking and providing feedback on the average calculations, but it was necessary and much more important for students. He continued by pointing out that "you miss the big picture and the value of the modeling activity if you only check students' work for wrong calculations."

Teacher attitudes, confidence and motivation towards interdisciplinary teaching. The results of the study revealed the following two sub-themes:

(i) Teachers' improved confidence to work with interdisciplinary modeling tasks. During interviews all six teachers gradually appeared more confident to teach, using interdisciplinary modeling tasks. They, however, mentioned that it was too difficult and demanding to prepare such an activity and lesson plans, and to identify good interdisciplinary modeling tasks. However, it was a shared understanding among participants that their engagement in the learning community was beneficial, and gradually they could see themselves becoming more independent in working with such activities. At the beginning, during the first PD sessions, Mary (pseudonyms are used throughout the paper), one of the teachers, expressed her concerns by noting: "I certainly believe I do not lack in the science nor the mathematics background, but I still feel not very comfortable about having to connect the two subjects." Similarly, another teacher referred to her studies. She mentioned that she was taught the two subjects (content and didactics courses) in a totally isolated approach, and that resulted in her "limited capabilities to teach through an integrated approach." Gradually teachers appeared more confident to teach an interdisciplinary task, like those developed for the study. For instance in a follow-up interview, Mary expressed not only her willingness but also her comfortableness to teach an interdisciplinary task: "It seems easier now to teach like this [...] you are not stressed on how to teach the mathematics and the science. You focus on the big ideas and the real problem and you have the mathematics and the science concepts help you to solve the problem."

(ii) Teachers' increased motivation and their positive attitudes and task value to integrate interdisciplinary activities in their day-to-day science and mathematics teaching. During the individual and group interviews, and in the reports provided in their reflective-diaries, teachers appeared motivated to more frequently use an interdisciplinary approach in their subject lessons. Quite often, all teachers referred to the great benefits (for students) from using interdisciplinary modeling tasks in their lessons, albeit mentioning the various systemic and other constraints (e.g. time consuming, separated curricula for the various school subjects). "Although at the beginning I was hesitant about doing this, since I started working like this, it was very helpful", a teacher noted. Another teacher reflected that: "It is a great way to teach both subjects, demanding but great!

[...] It helped me realize how many concepts and procedures I could teach through a single activity, and to help students understand how concepts are connected." A third teacher referred to the task value of the interdisciplinary activity: "I never experienced teaching maths or science like this [...] such an approach helped me realize how rich the connections between them [maths and science] could be." He continued, by documenting the added value of his collaboration with others: "My interactions with others helped me to reflect on better ways to teach like this [interdisciplinary methods] and to devote more time to the design of our lesson plans and activities."

Communication and collaboration. Collaboration between teachers became more constant and productive. The collaboration between teachers shifted from discussing issues related to time devoted in each task to sharing challenging questions, and from 'how to' discussions on using applets and other tools to developing shared assessment rubrics for assessing student models. One of the teachers noted that since all teachers shared similar sentiments, they all well received the collective wealth of knowledge in the group, and how important it was to collaborate on a daily basis. Another teacher agreed by explaining that: "collaboration was useful from the beginning. It was nice to see how useful our knowledge was, as it was uncovered through our conversations. [...] We learnt from the process. We stopped talking about simple things, like how much time to spend on each question and started discussing about the student models and solutions."

Teachers also mentioned their shared goals, and how this progressively improved: "At the beginning each of us [teacher] focuses on her classroom and her students ... how to use the activity in her classroom. We did not pay a lot of attention on what was going on in other classrooms, although we attended the lessons as observers. [...] The collaboration helped us to share more. How to convince our students that maths and science are strongly connected, how to pose challenging questions, how to assess our students work." In line with the above extract, another teacher wrote in her reflective diary: "The sharing of real challenges and problems furthers that goal [use a real world interdisciplinary approach to teach maths and science]. I could not be in this position if I was alone. This cannot be an isolated effort, at least at the beginning. My collaboration with Gina (teacher) and Mike (researcher) helped me improve my approach and my understanding of the activities."

Teachers also appreciated the collaboration developed as a method to improve their content knowledge and their pedagogical content knowledge. "I soon realized that I needed to use some effective mathematics teaching strategies, and that I was not the only one" a teacher noted. She continued: "I think that it is absolutely essential to accept that you have to improve your knowledge on the subject and that someone else could help you doing so [...] I can collaborate with Mary because we both aim to teach in an interdisciplinary and constructivist style and can help each other with what we might lack in mathematics [...] you have to be honest to yourself and to the others. You cannot know everything, especially when it comes to the two connected subjects and modeling. At the beginning it seemed like we were only able to scratch the surface!"

Of course collaboration was not straightforward or easy. As documented in both interviews and in teacher reflective diaries, "collaboration takes time, both in conversation and observation." The lack of time was reported by all teachers and it is probably one of the biggest roadblocks for collaborative efforts. The fact that teachers did not have any time release for their participation in the program, made it hard for them to find time during the day to talk. Therefore, it was quite common that teachers talk to each other after school hours and very often during weekends. Further, it was also hard for teachers to get out of their own classrooms to watch other teachers due to the lack of time release. However, it should be noted that in some cases the school head provided on her own some free time for teachers to watch other teachers.

Discussion. In this study, the participating teachers were engaged in the design and modification of their interdisciplinary modeling learning context through their collaborative activities selection and design, co-planned lessons, observations, and collaborative analysis of classroom implementation. This teacher-driven and interdisciplinary based professional learning community offered unique opportunities for participants to collaborate, synthesize and integrate more interdisciplinary pedagogies and teaching methods in their teaching [1]. As participants investigated how various approaches to teaching in a more interdisciplinary way engaged their students, and assisted them in building better and more coherent solutions, they raised their expectations and confidence in teaching in a more interdisciplinary way, and lessened their focus on the difficulties in implementing interdisciplinary activities in their day-to-day teaching. Participants improved in a number of ways; they shifted towards more 'social constructivist' and interdisciplinary approaches, collaborated with their students in developing interdisciplinary multifaceted solutions, improved their self-confidence in teaching more complex and modeling based tasks, became more motivated in designing interdisciplinary tasks, and explored how beneficial collaboration between teachers was for improving their teaching practices.

The idea of learning communities in classrooms will grow as we try to address the needs of being able to reason through complex issues and problems, direct one's own learning, communicate and work with people from diverse backgrounds and views, and share what one learns with others. More research on teachers' developments as they construct knowledge and skills for teaching interdisciplinary modeling is needed in order to illustrate ways that professional learning communities can meet the learning needs of teachers so that teachers can meet the learning needs of their students.

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МАТЕМАТИЧЕСКО МОДЕЛИРАНЕ В КУРСОВЕ ЗА РАЗВИТИЕТО НА ПРОФЕСИОНАЛНА ОБЩНАСТ ОТ УЧИТЕЛИ ПО ТОЧНИ НАУКИ В ИЗСЛЕДОВАТЕЛСКИ СТИЛ

Николас Г. Мусолидис

Предмет на доклада е как математическото моделиране и проектиране могат да се използват като подходи за развиването на общност от учители, които да преподават *природни науки, технологии, инженерство и математика* (на англ. STEM) в изследователски стил. Моделирането и проектирането се използват като инструменти за по-добро разбиране и анализ както на знанията (свързани със съдържанието и с методиката), така и с отношението на учителите по природо-математически науки към курсовете за професионално развитие. Изследването е свързано с ефекта от един, основан на моделиране, обучителен курс, проведен с 14 учители по математика и природни науки, в рамките на който те проектираха, разработваха (заедно с родители на учениците си) и прилагаха в клас сценарии на интердисциплинарни проекти. Резултатите показват, че учителите са обогатили знанията си и педагогическите подходи чрез интердисциплинарно преподаване, основано на моделиране. Очевидни бяха и промени в тяхното отношение и мотивация към сътрудничество с други учители. В заключение смятаме, че такъв подход е в подкрепа на общност от учители, които прилагат работа по интердисциплинарни проекти в изследователски стил.