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RAREFIED GAS HEAT CONDUCTION BETWEEN TWO COAXIAL CYLINDERS UNDER PULSATING TEMPERATURE BOUNDARY CONDITION*

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The heat transfer in a monatomic rarefied gas between two stationary concentric cylinders in the case of the inner cylinder temperature harmonically pulsating is studied. The obtained numerical results are applicable in the thermal conductivity gauges modeling.

1. Introduction. The thermal conductivity gauges are widely used for the measurement of the pressures in different vacuum systems. The Pirani gauge consists of a metal wire (inner cylinder) suspended in a cylindrical tube (outer cylinder). The wire is connected to an electrical Wheatstone bridge circuit.

The heat flux between two coaxial cylinders was measured in the range from the free molecular to the early transitional flow regimes for extraction of the thermal accommodation coefficient using an approximate relation on the pressure dependence of the heat flux [10]. In [11] the transient heat flux between two coaxial cylinders is studied on the basis of the numerical solution of the nonlinear unsteady S-model kinetic equation.

If, a thermocouple gauge is used to monitor a pumpdown cycle, the wire will become hotter and hotter as the pressure drops and fewer and fewer molecules are available to transfer heat away from the wire [7].

Various authors have studied mathematical models that are related to heat transfer and thermo acoustic waves occurring in rarefied gas between solid walls. Heat flux through a rarefied gas confined between two coaxial cylinders is calculated on the basis of the kinetic equation [8]. Acoustic waves propagating through a rarefied gas between two plates induced by unsteady heating of one of them are considered [9] on the basis of a model of the linearized Boltzmann equation. The gas flow is considered as fully established so that the dependence of all quantities on time is harmonically pulsating.

A special form of the Pirani gauge – the pulsed Pirani vacuum gauge is a new trend imposed in the last 15 years, although it has been used more than 100 years in different vacuum installations [6, 7].

Heat transer under pulsating tempeature condititon studied is the first step to the pulsated Pirani gauge modeling. This paper is the sequel to [12, 13] in order to be closer to the real conditions. Here we study heat conduction in the rarefied gas between two

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stationary cylinders. The waves arising are generated by the inner cylinder temperature harmonic oscillation (the amplitude is ± 10%) compare to initial one. We consider a continuous Navier-Stokes-Fourier model (NSF), and a statistical one based on Direct Simulation Monte Carlo (DSMC) Method.

2. Formulation of the problem and methods of solution. We study a rarefied monatomic gas in the gap between two coaxial unconfined cylinders.

2.1. Continuous model (NSF) and numerical simulation. The continuous model is based on the Navier-Stokes-Fourier equations for compressible fluid, completed with the equations of continuity and energy transport. For details see [5].

A rather standard notation is used: u and v are the velocity components in r and φ directions, ρ is density and T is the temperature, P is the pressure. The stress tensor components are $\tau_{i,j}$ and Φ is the dissipation function [5]. For a perfect monatomic gas, the viscosity and the heat transfer coefficient read as [3]:

$$\mu = \mu(T) = C_\mu \rho_0 l_0 V_0 \sqrt{T}, \quad C_\mu = \frac{5}{16}\sqrt{\pi}, \tag{2.1}$$

$$\lambda = \lambda(T) = C_\lambda \rho_0 l_0 V_0 \sqrt{T}, \quad C_\lambda = \frac{15}{32}\sqrt{\pi}. \tag{2.2}$$

The governing equations are normalized by using the following scales: for density, $\rho_0 = m n_0$ (m is the molecular mass, n_0 – the average number density), for velocity $V_0 = \sqrt{2RT_0} - R$ is the gas constant, for length – the distance between the cylinders $L = R_2 - R_1$, for time $t_0 = L/V_0$, for temperature T_0 – the wall temperature of both cylinders. The Knudsen number is $\mathrm{Kn} = l_0/L$, where the mean free path is l_0 and $\gamma = c_P/c_V = 5/3$ (c_P and c_V are the heat capacities at constant pressure and constant volume respectively). In this way in the dimensionless model the characteristic number Kn and the constants C_μ and C_λ take part.

For the problem formulated first-order slip boundary conditions are imposed at both walls, which can be written directly in dimensionless form as follows [4]:

$$v \mp A_\sigma \mathrm{Kn}_{local} \left(\frac{\partial v}{\partial r} - \frac{v}{r} \right) = V_i, \tag{2.3}$$

$$u = 0, \tag{2.4}$$

$$T_i \pm \zeta_T \, \mathrm{Kn}_{local} \frac{\partial T}{\partial r} = 1, \tag{2.5}$$

at $r = R_i$, $i = 1, 2$. In Eqs (2.3)–(2.5) $V_i = v_i/V_0$ is the dimensionless wall velocity, (v_i, $i = 1, 2$ is the dimensional wall velocity). The dimensionless wall pulsating temperature on the inner cylinder wall T_1 depends on the dimensionless time t harmonically:

$$T_1 = T_{1,0} \left[1 + A \sin(\omega t) \right], \tag{2.6}$$

where $T_{1,0} = 1$ is the mean temperature value and $A = 0.1$ is the oscillations amplitude.

$T_2 = 1$ is the outer cylinder wall constant temperature. For diffuse scattering we have used the viscous slip and temperature jump coefficients $A_\sigma = 1.1466$ and $\zeta_T = 2.1904$ calculated, respectively in [1], from the kinetic BGK equation. The boundary conditions are modeled by using the local Knudsen number Kn_{local}.

$$\mathrm{Kn}_{local} = \frac{l}{L} = \left(L\sqrt{2}\pi\sigma^2 \frac{\bar{\rho}}{\rho_0} . n_0 \right)^{-1} = \frac{\mathrm{Kn}}{\rho} \tag{2.7}$$

In (2.7) l denotes the local mean free path, σ – molecular diameter and $\bar{\rho}$ is the dimensional density.

2.2. Direct simulation monte carlo (DSMC) method. The gas considered is simulated as a stochastic system of N particles [4, 5]. All quantities used are non-dimensional, so that the mean free path at equilibrium is equal to 1. The basic steps of simulation are as follows:

• The time interval $\left[0;\hat{t}\right]$ over which the solution is found, is subdivided into subintervals with step Δt.

• The space domain is subdivided into cells with sides Δz, Δr.

• Gas molecules are simulated in gap G using a stochastic system of N points (particles) having position $z_i\,(t)$, $r_i\,(t)$ and velocities $\left(\xi_z^i\,(t)\,,\xi_r^i\,(t)\,,\xi_\varphi^i\,(t)\right)$,

• N_i particles are located in the i-th cell at any given time. This number varies during the computer simulation by the following two stages:

Stage 1. Binary collisions in each cell are calculated, whereas particles do not move. Collision modeling is realized using Bird's scheme "no time counter".

Stage 2. Particles move with new initial velocities acquired after collisions, and no external forces act on particles. No collisions are accounted for at this stage.

• Stage 1 and Stage 2 are repeated until $t = \hat{t}$.

• Flow macro-characteristics (density, velocity, temperature) are calculated as time-averaged.

• Boundary conditions are diffusive at the cylinder walls and periodic along axis Oz.

The number of particles (simulators) used in DSMC calculations is 1.28×10^7.

3. Numerical results. In present paper we consider the macro-characteristics (pressure, temperature and density) gas flow between two stationary coaxial unconfined cylinders. The inner cylinder temperature harmonic pulsating on time by the formula (2.6), where $T_{1,0} = 1$, $A = 0.1$ and $\omega = 0.5$, the outer cylinder temperature $T_2 = 1$. In the numerical calculation we have used the Knudsen number $\mathrm{Kn} = 0.02$, the inner and the outer cylinder radiuses $R_1 = 1$, respectively $R_2 = 2$.

The Figs. 1 and 2 show that the pressure fluctuates around a constant value with small constant amplitudes when a sustained pulsating flow is reached. The pressure profile along the radius is almost constant. The pressure values that correspond to the local temperature maximum and which is measured from a discrete transmitter also are constant over the time. This allows using the device proposed by authors [6] which switches only at certain times instead of using expensive Wheatstone bridge. The density and temperature graphics on Figs. 3, 4, 5 and 6 show sustained pulsing values. The temperature and density phase shift in the gas as compared to the temperature of the wall is approximately 4 dimensionless time units, which is approximately one third of the period (Figs. 3 and 5).

4. Conclusions. The proposed solutions allow to estimate the application limits of the various models NSF, DSMC and to study the heat transfer between stationary concentric cylinders under pulsating temperature conditions. The results can be applied to adjustment and calibration of some dynamically thermo conductivity gauges for pressure measuring. The heat conduction modeling in the rarefied gas between two stationary cylinders under pulsating heat condition can be seen as the first step towards an overall pulsating Pirani sensor modeling.

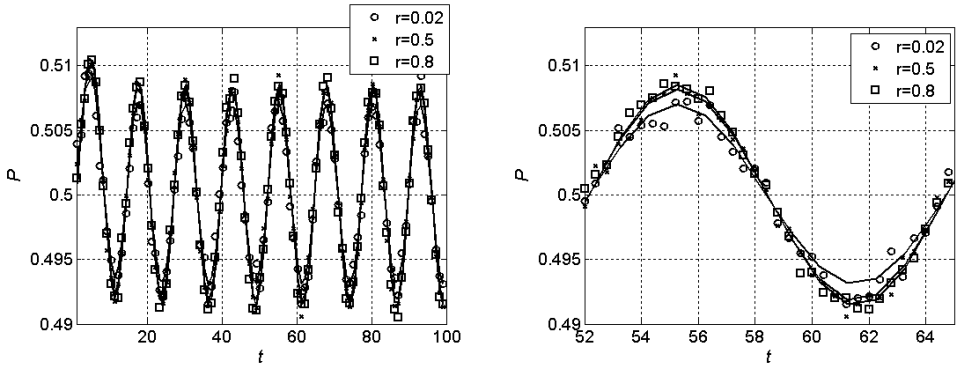

Fig. 1. The pressure time variation at the fixed radius. The solid line – NSF, $(\circ, \times, \square)$ – DSMC

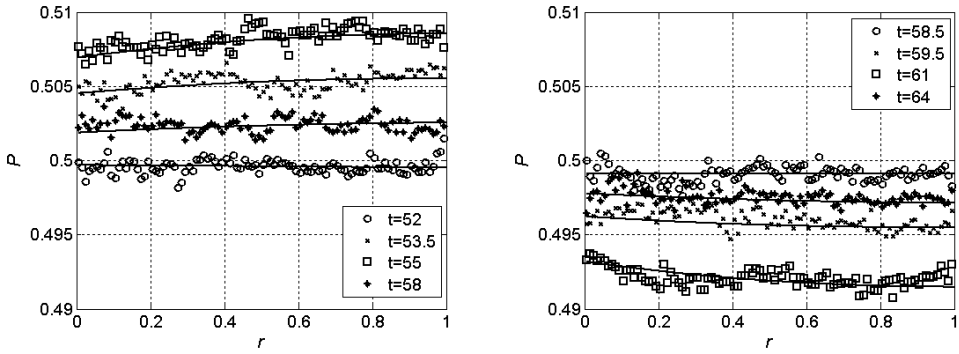

Fig. 2. The pressure profile at the fixed time. The solid line – NSF, $(\circ, \times, \square, +)$ – DSMC

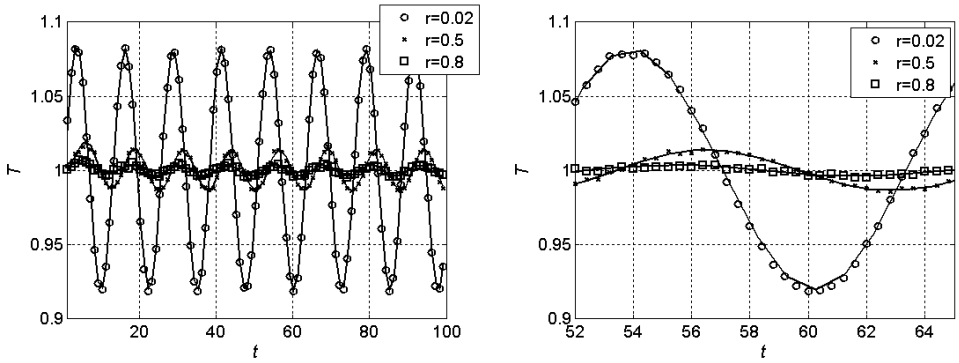

Fig. 3. The temperature time variation at the fixed radius. The solid line – NSF, $(\circ, \times, \square)$ – DSMC

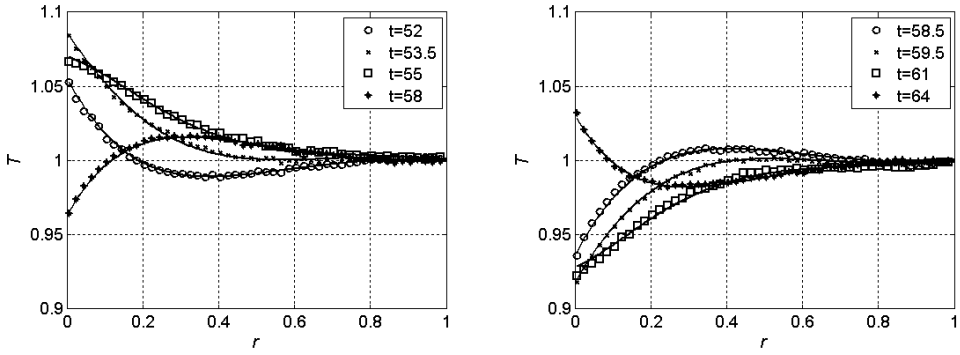

Fig. 4. The temperature profile at the fixed time. The solid line – NSF, $(\circ, \times, \square, +)$ – DSMC

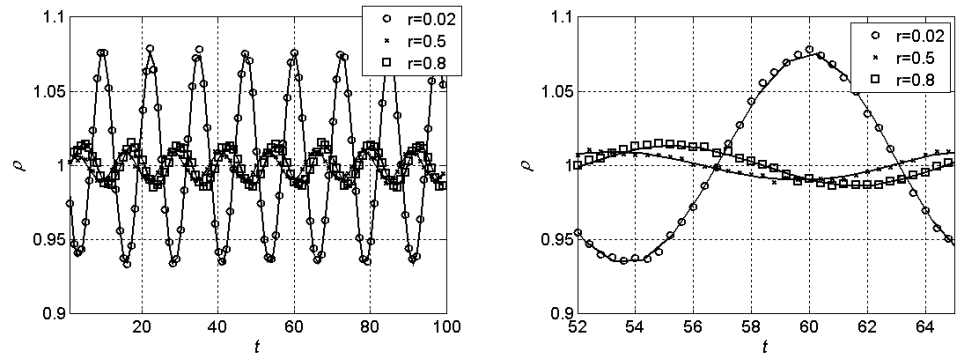

Fig. 5. The density time variation at the fixed radius. The solid line – NSF, $(\circ, \times, \square)$ – DSMC

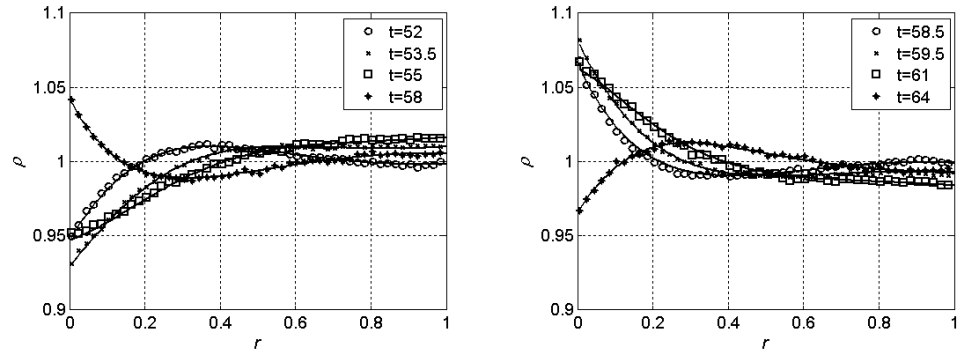

Fig. 6. The density profile at the fixed time. The solid line – NSF, $(\circ, \times, \square, +)$ – DSMC

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ТОПЛОПРОВОДИМОСТ НА РАЗРЕДЕН ГАЗ МЕЖДУ ДВА КОАКСИАЛНИ ЦИЛИНДЪРА С ПУЛСИРАЩО ТЕМПЕРАТУРНО ГРАНИЧНО УСЛОВИЕ

Петър Господинов, Владимир Русинов, Добри Данков

Изследван е топлообмен в едноатомен разреден газ между два стационарни концентрични цилиндъра в случай на хармонично пулсираща температура на вътрешния цилиндър. Получените числени резултати са приложими в моделирането на топлопроводими устройства.