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STRATEGIC TRADE WITH PARTIAL LOCAL CONSUMER PROTECTION*

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The paper presents a strategic model of the trade relations between two countries. Local consumers have preferential access to a part of the available quantity of the traded good on the home market. Depending on the outcome of the trade, prices are adjusted over time to correct for excess supply or demand. Using numerical simulations implemented in IPython/Jupyter Notebook, we establish that the model is capable of generating diverse types of price dynamics, including degenerate (zero price) outcomes and cyclicality.

1. Introduction. This paper presents a model of the trade relations between two countries or regions, where the participating agents compete strategically for the fixed supply of a good. Depending on the outcome of the trade, prices are adjusted over time to correct for excess supply or demand on the respective markets. The setup is similar to our previous works [2] and [4], with one important generalization: local consumers enjoy partial protection in terms of preferential access to the good on their home market, unlike the cited works, where protection is full. Thus, the present model subsumes the previous formulations. In this work, we focus on discrete-time dynamics.

Using numerical simulations implemented in IPython/Jupyter Notebook (see [3]), we establish that, under the chosen price adjustment rules, the model is capable of generating diverse types of price dynamics, including degenerate (zero price) outcomes and cyclicality.

2. Model description. We study the interaction of two consumers from different countries (regions), labelled 1 and 2, who compete for a good supplied on both markets. Consumers are endowed with constant monetary income Y_i and the good is supplied in fixed quantities q_i, $i = 1, 2$. Income cannot be saved and is therefore available only in the current period.

The price of the good on the market of region 1 for the local consumer at time t is denoted $p_{1,t}$ and the local price of the good in region 2 is $p_{2,t}$. If consumer 1 wants to import from the other region, he pays an additional cost ρ_2 per unit of good. This cost may have various interpretations, including transportation costs, customs duties or other transaction costs associated with foreign trade. Analogously, consumer 2 pays a cost ρ_1 per unit of goods imported from region 1. Thus, the total price of goods imported from

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region 2 for consumer 1 is $p'_{2,t} = p_{2,t} + \rho_2$ and the total price of imports from region 1 for consumer 2 is $p'_{1,t} = p_{1,t} + \rho_1$.

Consumer 1 can place orders for quantities α and β on the markets of region 1 and 2, respectively. The orders of consumer 2, placed in regions 1 and 2, are denoted γ and δ. The orders of the two consumers form their strategy spaces (see [1], Ch. 3, for details of the game-theoretic terminology and results employed below), denoted $S_{1,t}$ and $S_{2,t}$, and assumed to be nonempty. They are defined as follows:

$$S_{1,t} = \left\{(\alpha, \beta) \in \mathbb{R}^2 \mid p_{1,t}\alpha + p'_{2,t}\beta \le Y_1,\ \alpha, \beta \ge 0\right\},$$

$$S_{2,t} = \left\{(\gamma, \delta) \in \mathbb{R}^2 \mid p'_{1,t}\gamma + p_{2,t}\delta \le Y_2,\ \gamma, \delta \ge 0\right\}.$$

When effecting a trade, each consumer enjoys partial protection on the local market. This means that a fixed share $\epsilon \in (0, 1]$ of the quantity q_1 is reserved for consumer 1 and, similarly, a share $\xi \in (0, 1]$ of the quantity q_2 is preferentially available to consumer 2. Consumers have the right to buy the respective quantities ϵq_1 and ξq_2 but are not obliged to do so. After the local consumer buys a part or all of the preferentially available quantity, the remaining quantity of the good is offered to the foreign consumer. In turn, the foreign consumer can purchase part or all of this remainder and, if there is anything left, it is again offered to the local consumer.

We assume that under the above conditions, each consumer wants to maximize the quantity ot the good purchased. This implies the following payoff function for consumer 1:

$$\begin{aligned} P_1(\alpha, \beta, \gamma, \delta) =& \min(\alpha, \epsilon q_1) + \min(\beta, q_2 - \min(\delta, \xi q_2)) + \\ & \min(\alpha - \min(\alpha, \epsilon q_1), q_1 - \min(\alpha, \epsilon q_1) - \min(\gamma, q_1 - \min(\alpha, \epsilon q_1))) \\ =& \min(\beta, q_2 - \min(\delta, \xi q_2)) + \min(\alpha, q_1 - \min(\gamma, q_1 - \min(\alpha, \epsilon q_1))). \end{aligned}$$

Similarly, the payoff function for consumer 2 is

$$P_2(\alpha, \beta, \gamma, \delta) = \min(\gamma, q_1 - \min(\alpha, \epsilon q_1)) + \min(\delta, q_2 - \min(\beta, q_2 - \min(\delta, \xi q_2))).$$

The best reply for consumer 1, for given strategies $\bar{\gamma}, \bar{\delta}$ played by consumer 2, is defined as

$$BR_1(\bar{\gamma}, \bar{\delta}) = \underset{(\alpha,\beta)\in S_{1,t}}{\operatorname{argmax}} P_1(\alpha, \beta, \bar{\gamma}, \bar{\delta}).$$

Analogously, the best reply for consumer 2 is defined as

$$BR_2(\bar{\alpha}, \bar{\beta}) = \underset{(\gamma,\delta)\in S_{2,t}}{\operatorname{argmax}} P_2(\bar{\alpha}, \bar{\beta}, \gamma, \delta).$$

Since P_i are continuous and $S_{i,t}$ are compact, there exist solutions to the best-reply problems, i.e. the best-reply sets are nonempty.

The best-reply mapping BR for the game associates each strategy profile $(\alpha, \beta, \gamma, \delta)$ with $BR_1(\gamma, \delta) \times BR_2(\alpha, \beta)$, i.e. $BR : (\alpha, \beta, \gamma, \delta) \to BR_1(\gamma, \delta) \times BR_2(\alpha, \beta)$. A *Nash equilibrium* is a strategy profile $(\alpha^*, \beta^*, \gamma^*, \delta^*)$ for which $(\alpha^*, \beta^*, \gamma^*, \delta^*) \in BR(\alpha^*, \beta^*, \gamma^*, \delta^*)$.

At the end of each period, after trading has been concluded and a Nash equilibrium has been reached, prices are decreased if the quantity available in the respective region has not been entirely consumed. Prices are increased if there is unspent income in the respective region. These two possibilities are mutually exclusive.

The above principle for the change in prices can be formalized through different price adjustment rules.

In discrete time, an example of price adjustment rules might be

(1) $$p_{1,t+1} = p_{1,t} q_1^{\text{cons}}/q_1 + Y_1^{\text{res}}/q_1 - p_{1,t},$$

(2) $$p_{2,t+1} = p_{2,t} q_2^{\text{cons}}/q_2 + Y_2^{\text{res}}/q_2 - p_{2,t},$$

where $Y_1^{\text{res}} = Y_1 - p'_{2,t}\beta^*$, $Y_2^{\text{res}} = Y_2 - p'_{1,t}\gamma^*$, $q_1^{\text{cons}} = \alpha^* + \gamma^*$ and $q_2^{\text{cons}} = \beta^* + \delta^*$.

As another example, the price adjustment rules can take the form

(3) $$p_{1,t+1} = p_{1,t} q_1^{\text{cons}}/q_1 + (Y_1 - Y_1^{\text{cons}})/q_1,$$

(4) $$p_{2,t+1} = p_{2,t} q_2^{\text{cons}}/q_2 + (Y_2 - Y_2^{\text{cons}})/q_2,$$

where $Y_1^{\text{cons}} = p_{1,t}\alpha^* + p'_{2,t}\beta^*$ and $Y_2^{\text{cons}} = p'_{1,t}\gamma^* + p_{2,t}\delta^*$.

The above rules are versions of the rules used in [2] and [4].

3. Results. The model described above was implemented in Python, using the infrastructure provided by IPython/Jupyter Notebook. To find a solution to the best-reply problem of the respective consumers, functionality for optimization under inequality constraints from the library `scipy.optimize` was employed. The Nash equilibrium points for the game were computed using fixed-point routines, again from `scipy.optimize`. In essence, our simulations were directed at exploring the types of price dynamics that can be obtained under the price rules by varying incomes, quantities and transportation costs. Below we report several representative outcomes.

Table 1. Parametrization of the model simulations

Sim. №	Rules	Y_1	Y_2	q_1	q_2	ρ_1	ρ_2
1	(1),(2)	100	120	30	40	2	2
2	(1),(2)	100	320	500	40	2	2
3	(3),(4)	50	4	30	40	2	2
4	(3),(4)	200	320	30	40	2	2
5	(3),(4)	100	320	50	40	2	200

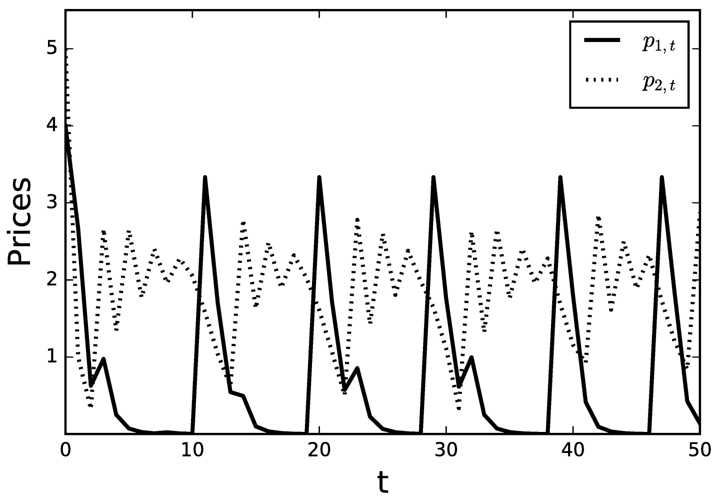

Fig. 1. Complex price dynamics with multi-period transition to the cyclical orbit (Simulation 1)

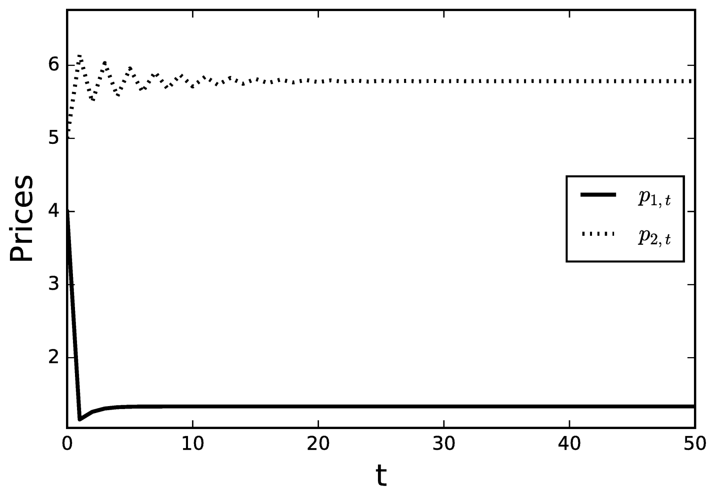

Fig. 2. Convergence to steady state prices with dampened oscillations in the transition dynamics (Simulation 2)

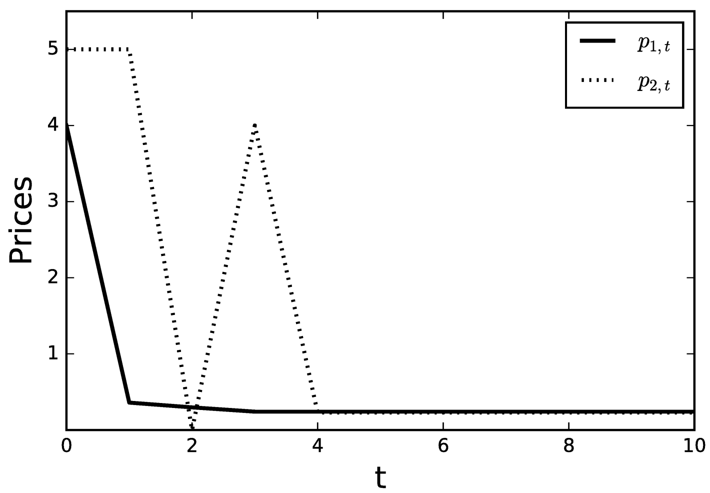

Fig. 3. Simple transition to steady state prices (Simulation 3)

The simulations were parametrized as follows. The common parameters across all simulations are $\epsilon = 0.2$, $\xi = 0.15$, $p_{1,0} = 4$ and $p_{2,0} = 5$. Table 1 presents the subset of model parameters that changes across simulations. The numbering of the simulations corresponds to the numbering of the figures presenting price dynamics.

The results indicate that the model is capable of producing a variety of dynamic patterns, ranging from simple one-period convergence to settling into cyclical mode after a transition period. In some of the simulations degenerate outcomes, involving a zero price in one of the regions, were obtained.

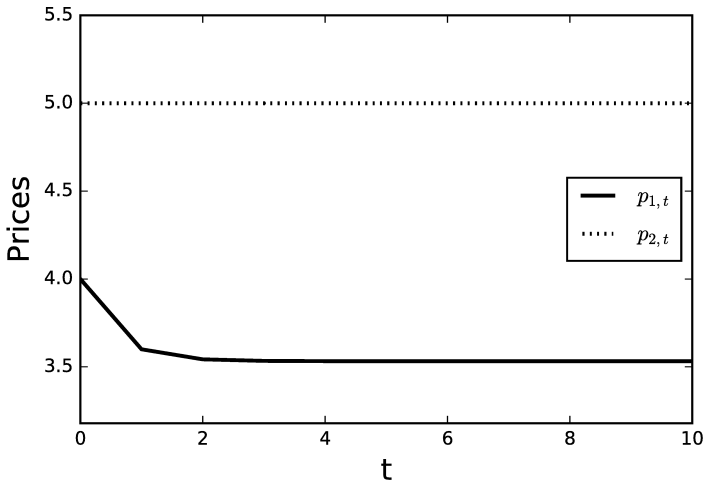

Fig. 4. Monotonic convergence to a steady state price (Simulation 4)

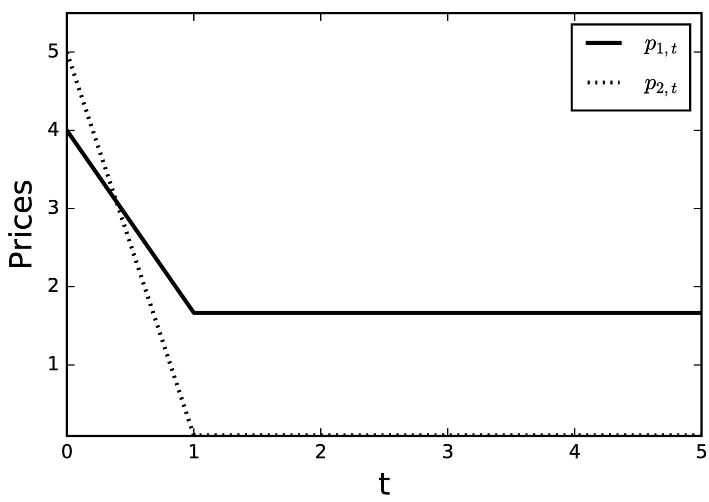

Fig. 5. Convergence to a steady state in one period (Simulation 5)

4. Possible extensions. The model suggests that rich dynamics can be generated for various parameter combinations. Apart from studying more rigorously the properties of the dynamic system through the current price rules to understand the sources of such dynamics, two other directions for extension present themselves. First, it would be natural to extend the model to the case of n regions. Second, developing and analyzing versions of the model with alternative specifications of the price equations, possibly arising from appropriate models for the behaviour of suppliers, would shed more light on the nature of the economic mechanisms involved.

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СТРАТЕГИЧЕСКА ТЪРГОВИЯ С ЧАСТИЧНА ЗАЩИТА НА МЕСТНИТЕ ПОТРЕБИТЕЛИ

Йордан Велинов Йорданов, Андрей Андреев Василев

В статията се разработва стратегически модел на търговските отношения между две страни. Местните потребители имат преференциален достъп до част от наличното количество от търгуваната стока на вътрешния пазар. В зависимост от резултата от търговията цените се променят, така че да коригират несъответствията между търсене и предлагане. С помощта на числени симулации, реализирани на IPython/Jupyter Notebook, се показва, че моделът е в състояние да генерира разнообразни форми на динамика на цените, включително изродени състояния (с нулеви цени) и различни циклични режими.