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MANIPULATION BY MERGING IN WEIGHTED VOTING GAMES*

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The problem of manipulation in voting is fundamental and has received attention in recent research in game theory. In this paper, we consider one case of manipulation in weighted voting games done by merging of coalitions into single players viewed from two perspectives: of the effect of swings of players and of the role of the Banzhaf index. We prove a theorem for manipulation by merging two players and show several attractive properties in the process.

1. Introduction. The modern notion of a simple game was introduced by John von Neumann and Oskar Morgenstern in their monumental book "Theory of Games and Economic Behavior" in 1944 [10]. Previous works on this problem were fragmentary and did not attract much attention. This book provided some new important developments, such as the introduction of information sets, the formal definitions, and the decision rules. According to Von Neumann and Morgenstern a simple game is a conflict in which the only objective is winning and the only rule is an algorithm to decide which coalitions of players are winning. For more information see [11], that is, the sixtieth anniversary edition which includes not only the original text but also an introduction by Harold Kuhn, an afterword by Ariel Rubinstein, and reviews and articles about the book.

The aim of this paper is to show a process of manipulation by merging two players in weighted voting games.

Let N be a nonempty finite set of players in game G and every subset $S \subset N$ is referred to as a *coalition*. The set N is called *the grand coalition* and $\emptyset$ is called *the empty coalition*. We denote the collection of all coalitions by 2^N and the number of players of $S \in 2^N$ by $|S|$. Let us label the players by $1,\ 2\,, dots\, n$, $n = |N| \geq 2$.

Definition 1. *A simple game in characteristic-function form is a pair $G = (N, v)$ where $N = \{1, 2, \ldots, n\}$ is the set of players and $v : 2^N \to \{0, 1\}$ is the characteristic function which satisfies the following three conditions:*

(1) $v(\emptyset) = 0$.

(2) $v(N) = 1$.

(3) *ν is monotonic, i.e. if $S \subset T \subset N$, then $v(S) \leq v(T)$.*

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Thus, we formalize the idea of coalition decision making. From the above definition it follows that the characteristic function v for a coalition $S \subset N$ indicates the value of S. This means that for each coalition $S \subset N$ we have either $v(S) = 0$ or $v(S) = 1$.

In this paper, we will consider a special class of simple games called *weighted voting games* (or weighted majority games) with dichotomous voting rule – acceptance ("yes") or rejection ("no"). These games have been found to be well-suited to model economic or political bodies that exercise some kind of control. A weighted voting game is a type of simple cooperative game and a formal model of coalition decision making in which decisions are made by vote.

A weighted voting game (N, v) is described by $G = [q; w_1, w_2, \dots, w_n]$ where q and $w_1, w_2, \dots, w_n$ are nonnegative integer numbers such that $q \leq \sum_{k=1}^{n} w_k = \tau$. For more information see [6], [8] and [9]. This game has the following properties:

(1) $1 \leq q \leq \tau$.
(2) $n = |N| \geq 2$ is the number of players.
(3) $w_i \geq 0$ is the number of votes of player $i \in N$.
(4) q is the needed quota so that a coalition can win.
(5) the symbol $[q; w_1, w_2, \dots, w_n]$ represents the weighted voting game G defined by

$$v(S) = \begin{cases} 1, & \sum_{k \in S} w_k \geq q \\ 0, & \sum_{k \in S} w_k < q \end{cases}, \text{ where } S \subset N.$$

For any weighted voting game G, the form $[q; w_1, w_2, \dots, w_n]$ is often called a *weighted presentation of game G*. Obviously, one weighted voting game has many representations. For example, the following three weighted voting games $G_1 = [51; 49, 49, 2]$, $G_2 = [9; 7, 6, 5]$, and $G_3 = [2; 1, 1, 1]$ represent the same voting rule - majority rule, that is, each coalition of 2 or 3 players is winning. It follows that they have the same characteristic functions.

2. Definitions and concepts.

We start our study with a consideration of two basic types of coalitions – winning and losing.

Definition 2. *For any coalition $S \subset N$ in game G, S is winning if $v(S) = 1$, and S is losing if $v(S) = 0$. The collections of all winning and all losing coalitions in game G are denoted by $W(G)$ and $L(G)$, respectively. If game G is fixed, we simply write W and L.*

By definition, any simple game has winning and losing coalitions, and this game is determined by the set of all its winning or losing coalitions. We have that $N \in W$ and $\emptyset \in L$; therefore, W and L are nonempty, $W \cap L = \emptyset$ and $W \cup L = 2^N$. Observe that a coalition having a winning sub-coalition is also winning and a sub-coalition of a losing coalition is also losing.

It is important to note that if the quota increases (decreases), then the set of all winning coalitions decreases (increases) and the set of all losing coalitions increases (decreases).

For any player $i \in N$, the collection of all winning coalitions including i is denoted by W_+^i, the collection of all winning coalitions excluding i is denoted by W_-^i, the collection of all losing coalitions including i is denoted by L_+^i, and the collection of all losing coalitions excluding i is denoted by L_-^i.

Definition 3. *For any coalition $S \in W$, S is called a minimal winning coalition if*

$S\backslash\{i\}$ is not winning for all $i \in S$. The collection of all minimal winning coalitions is denoted by MW. For any player $i \in N$, the collection of all minimal winning coalitions including i is denoted by MW_+^i and the collection of all minimal winning coalitions excluding i is denoted by MW_-^i.

It is easy to prove that MW and W are two finite sets, $MW \subset W$ and MW is nonempty.

Thus, a simple game (N, v) can alternatively be defined in winning-set form as (N, W) or minimal-winning-set form as (N, MW).

Definition 4. *A player who does not belong to any minimal winning coalition is called a* dummy*, i.e. player $i \in N$ is a dummy if and only if $i \notin S$ for all $S \in MW$. A player who belongs to all minimal winning coalitions is called a* veto player *or* vetoer*, i.e. player $i \in N$ has the capacity to veto if and only if $i \in S$ for all $S \in MW$. A player $i \in N$ is a dictator if $\{i\}$ is a winning coalition.*

For any player $i \in N$, it is easy to show that $MW_+^i = \emptyset$ is equivalent to player i being a dummy, and $MW_+^i = MW$ is equivalent to player i being a veto player.

Definition 5. *A weighted voting game G is called* proper *if $v(S) + v(N\backslash S) \leq 1$ for all $S \subset N$.*

Note that a weighted voting game to be proper is equivalent to the complement of a winning coalition to be not winning. This means that in a proper game both coalitions S and $N\backslash S$ cannot be winning. In this context, if S is winning, then $N\backslash S$ is losing, but the converse statement is not always true.

Clearly, the following statements are true:

(1) A proper game may have only one dictator and all other players are dummies. However, a proper game may have several veto players while a dictator is unique. A proper game with two or more veto players does not have a dictator.

(2) An improper game may have one pair of non-intersecting winning coalitions. In particular, an improper game may have more than one dictator.

In what follows, we will only study proper games with $n \geq 3$.

Definition 6. (*a*) *A proper game G is called* decisive *(or* strong*) if $v(S)+v(N\backslash S) = 1$ for all $S \subset N$.*

(*b*) *A proper game G is called* weak *if $v(S) + v(N\backslash S) < 1$ for all $S \subset N$.*

It is easy to show that weighted voting game $[5; 4, 3, 2, 1]$ is improper, game $[6; 4, 3, 2, 1]$ is proper but it is not decisive, and game $[3; 1, 1, 1, 1, 1]$ is decisive.

Definition 7. *For $i \in N$ and $S \in W_+^i$, player i is called a* negative swing *(also critical or pivotal) member of S if $S\backslash\{i\}$ is not winning. The collection of all winning coalitions including i as a negative swing member is denoted by W_s^i. For $i \in N$ and $S \in L_-^i$, player i is called a* positive swing *(also* critical *or* pivotal*)* member *of S if $S \cup \{i\}$ is not losing. The collection of all losing coalitions including i as a positive swing member is denoted by L_s^i.*

Note that each member of a minimal winning coalition is a negative swing player, a winning coalition may have a negative swing member and a losing coalition may have a positive swing member.

It is often said that $\left|W_s^i\right|$ and $\left|L_s^i\right|$ are the number of swings of player $i \in N$.

Remark 1. In classical theory each positive swing for player $i \in N$ corresponds to a pair of coalitions $(S, S \cup \{i\}) \in L^i_- \times W^i_+$ such that S is losing and $S \cup \{i\}$ is winning ($L \stackrel{i}{\longrightarrow} W$ process), and each negative swing for player $i \in N$ corresponds to a pair of coalitions $(S \backslash \{i\}, S) \in L^i_- \times W^i_+$ such that $S \backslash \{i\}$ is losing and S is winning ($L \stackrel{i}{\longleftarrow} W$ process). In the first case we say that player i is a *swing member of the pair* $(S, S \cup \{i\})$, but in the second case we say that that player i is a *swing member of the pair* $(S \backslash \{i\}, S)$.

It is easy to show that if a weighted voting game has a dictator, then he/she is the only swing player in this game.

Theorem 1 [2, Corollary 4.1]**.** *For any proper game* $|W^i_s| = |L^i_s|$ *for all* $i \in N$.

Remark 2. It is important to note that in the proof of Theorem 1 the authors construct a one-to-one mapping $m_i : W^i_s \to L^i_s$ such that coalition $S \in W^i_s$ only corresponds to coalition $S \backslash \{i\} \in L^i_s$ and conversely, coalition $S \backslash \{i\} \in L^i_s$ only corresponds to coalition $S \in W^i_s$, see also Remark 1.

3. Decision powers of the players. The concept of decision power of the players in weighted voting games is well-known. For example, let us consider a game $G = [51; 62, 27, 11]$. We may say that player 1 has 62% of the decision power, players 2 and 3 have 27% and 11%, respectively. But this is not true because player 1 has 100% of the power, and players 2 and 3 are powerless, i.e. player 1 is a dictator and players 2 and 3 are dummies. Weighted voting games use mathematical models to analyze the distribution of decision power of the players. These distributions of decision power are central in economics and political science.

These notes allow us to discuss Banzhaf power index. This index was introduced by the American jurist and law professor John Banzhaf III as a measure of the real power of players in a cooperative game [1]. It depends on the number of ways in which each player can affect a negative swing. The absolute Banzhaf index concerns the number of times each player $i \in N$ could change a coalition from winning to losing and it requires that we know the number of negative swings for each player i. For each player $i \in N$, the absolute Banzhaf index is denoted by η_i and it equals the number of negative swings for this player.

Theorem 2 [3], [4, Lemma 1]**.** *For any proper game* $\eta_i = |W^i_s| = |W^i_+| - |W^i_-|$ *for all* $i \in N$.

Remark 3. From Theorems 1 and 2 it follows that $\eta_i = |W^i_s| = |L^i_s|$ for all $i \in N$, i.e. η_i is either the number of negative swings or the number of positive swings of player i.

The normalized Banzhaf power index is the vector $\beta = (\beta_1, \beta_2, \ldots, \beta_n)$, given by $\beta_i = \frac{\eta_i}{\sum_{k=1}^n \eta_k}$ for $i = 1, 2, \ldots, n$.

The Banzhaf index is similar to the Penrose-Banzhaf (or Banzhaf-Coleman) index which is defined by $b_i = \frac{\eta_i}{2^{n-1}}$ for $i \in N$. The Banzhaf index was originally created in 1946 by Leonel Penrose, but was reintroduced by John Banzhaf in 1965.

Theorem 3 [7, Theorem 2]**.** *For any proper game, player* $i \in N$ *is a dummy is equivalent to* $\eta_i = 0$.

Remark 4. It is important to note that the Banzhaf power index is monotonic with respect to the weights when we are evaluating the power, i.e. for two different players $i, j \in N$, $\eta_i = \eta_j$ when $w_i = w_j$ and $\eta_i \geq \eta_j$ when $w_i > w_j$, see Theorem 3 and [3].

Theorem 4 [7, Theorem 2(b)]. *For any proper game if $i, j \in N$, $i \neq j$, player i is a dummy, and $w_i \geq w_j$, then player j is also a dummy.*

4. Concepts for manipulation. In the real world weighted voting games can be dynamic. Players might have an incentive to merge into voting blocks or split up into smaller players. Decision rules in voting games can be manipulated by coalitions merging into single players and players splitting into a number of smaller units [5]. Weighted voting games are cooperative games; therefore, analyses of manipulation are natural. The study of methods of manipulation also has practical applications.

In this paper, we will focus our attention on a specific case of manipulation – merging two or more players into a single player.

Consider a proper weighted voting game $G = [q; w_1, w_2, \ldots, w_n]$ when $n \geq 3$. We construct a new game $G' = [q; w'_1, w'_2, \ldots, w'_{n'}]$ such that $n > n'$, any player i' in game G' is a fixed coalition of players in game G, its weight $w'_{i'}$ is the sum of the weights of this fixed coalition and for any two different players i' and j' in game G' the condition $i' \cap j' = \emptyset$ holds. We denote the set of all players in games G and G' by N and N', respectively. Game G' is called a *derivative game of the original game G*.

Note that the converse process of manipulation by merging is the process of manipulation by splitting.

Example 1. Let $G = [17; 8, 7, 4, 4, 2, 1, 1]$ be an original game with $n = 7$, $q = 17$ and $\tau = 27$. For the derivative game, let the coalition of players 2, 3 and 5 in game G be a new player in game G' and the other players remain the same. So we get $G' = [17; 13, 8, 4, 1, 1]$ where $n' = 5$, $q' = 17$, $\tau' = 27$, $1' = \{2, 3, 5\}$, $2' = \{1\}$, $3' = \{4\}$, $4' = \{6\}$ and $5' = \{7\}$. The sum of Banzhaf power indices of players 2, 3 and 5 in game G is $\beta_2 + \beta_3 + \beta_5 = 0,4851$ but the index of player $1'$ in game G' is $\beta'_{1'} = 0,6000$. As a result we obtain $\beta_2 + \beta_3 + \beta_5 < \beta'_{1'}$.

Example 2. Let $G = [30; 9, 8, 5, 5, 4, 3, 1]$ be an original game with $n = 7$, $q = 30$ and $\tau = 35$. For the derivative game, let the coalition of players 1, 2 and 3 in game G be a new player in game G' and the other players remain the same. In this case we get $G' = [30; 22, 5, 4, 3, 1]$ where $n' = 5$, $q' = 30$, $\tau' = 35$, $1' = \{1, 2, 3\}$, $2' = \{4\}$, $3' = \{5\}$, $4' = \{6\}$ and $5' = \{7\}$. Here the sum of Banzhaf power indices of players 1, 2 and 3 in game G is $\beta_1 + \beta_2 + \beta_3 = 0,5789$ but the index of player $1'$ in game G' is $\beta'_{1'} = 0,3684$. Now we obtain $\beta_1 + \beta_2 + \beta_3 > \beta'_{1'}$.

Note that in Example 1 decision power increases but in Example 2 it decreases.

5. Main result. In this section, we present the basic theorem for manipulation by merging two players into a single player.

Theorem 5. *Transform game G to game G' by merging two different players i and j into player i', and the other players remain the same. The following statements are true.*

(*a*) $\eta_i + \eta_j = 2\eta'_{i'}$, $\left|W_s^i(G)\right| + \left|W_s^j(G)\right| = 2\left|W_s^{i'}(G')\right|$ *and* $\left|L_s^i(G)\right| + \left|L_s^j(G)\right| = 2\left|L_s^{i'}(G')\right|$.

(b) *If player j is a dummy in game G, then* $\eta_i = 2\eta'_{i'}$, $\left|W_s^i(G)\right| = 2\left|W_s^{i'}(G')\right|$ *and* $\left|L_s^i(G)\right| = 2\left|L_s^{i'}(G')\right|$.

(c) *Players i and j being dummies in game G is equivalent to player i' being a dummy in game G'.*

(d) *If* $w_i = w_j$, *then* $2w_i = w'_{i'}$, $\eta_i = \eta'_{i'}$, $\left|W_s^i(G)\right| = \left|W_s^{i'}(G')\right|$ *and* $\left|L_s^i(G)\right| = \left|L_s^{i'}(G')\right|$.

(e) *If* $k \neq i, j$ *and player k in game G transforms to player k' in game G', then* $\eta_k \geq \eta'_{k'}$, $\left|W_s^k(G)\right| \geq \left|W_s^{k'}(G')\right|$ *and* $\left|L_s^k(G)\right| \geq \left|L_s^{k'}(G')\right|$.

(f) *If player i' is a dummy in game G', then* $\beta_i + \beta_j = \beta'_{i'} = 0$.

(g) *If player i' is not a dummy in game G', then* $\beta_i + \beta_j < 2\beta'_{i'}$.

(h) *If* $w_j \leq w_i$ *and player i' is not a dummy in game G', then* $\beta_j < \beta'_{i'}$.

(i) *If player i is a dictator in game G, then player i' is a dictator in game G' and* $\beta_i + \beta_j = \beta'_{i'} = 1$.

Proof. (a) Let $S \in L_s^{i'}(G')$, $w_i + w_j = w'_{i'}$ and let us assume that $w_j \leq w_i$. This means that $0 < q - \sum_{h \in S} w_h \leq w'_{i'} = w_j + w_i$ and $S \cup \{i'\} \in W(G')$.

There are three cases for positive swings of each player i, j or i'.

Case 1. If $0 < q - \sum_{h \in S} w_h \leq w_j$, then $q \leq \sum_{h \in S} w_h + w_j$ and $q \leq \sum_{h \in S} w_h + w_i$. As a result we see that player i' is a swing member of pair $(S, S \cup \{i'\})$ in game G', and players j and i are swing members of pair $(S, S \cup \{j\})$ and pair $(S, S \cup \{i\})$ in game G, respectively. Hence, in this case we have $\eta_i + \eta_j = 2\eta'_{i'}$.

Case 2. If $w_j < q - \sum_{h \in S} w_h \leq w_i$, then $\sum_{h \in S} w_h + w_j < q \leq \sum_{h \in S} w_h + w_i$. Here we get that player i' is a swing member of pair $(S, S \cup \{i'\})$ in game G', and players j and i are swing members of pair $((S \cup \{i\}) \backslash \{j\}, S \cup \{i\})$ and pair $(S, S \cup \{i\})$ in game G, respectively. We also obtain $\eta_i + \eta_j = 2\eta'_{i'}$.

Case 3. If $w_i < q - \sum_{h \in S} w_h \leq w'_{i'}$, then $\sum_{h \in S} w_h + w_i < q \leq \sum_{h \in S} w_h + w_j + w_i$. So we have that player i' is a swing member of pair $(S, S \cup \{i'\})$ in game G', and players j and i are swing members of pair $((S \cup \{i\}) \backslash \{j\}, S \cup \{i\})$ and pair $((S \cup \{j\} \backslash \{i\}), S \cup \{j\})$ in game G, respectively. Here we get $\eta_i + \eta_j = 2\eta'_{i'}$ too.

In summary, we obtain $\eta_i + \eta_j = 2\eta'_{i'}$. From $\eta_i = \left|W_s^i(G)\right| = \left|L_s^i(G)\right|$, $\eta_j = \left|W_s^j(G)\right| = \left|L_s^j(G)\right|$ and $\eta'_i = \left|W_s^{i'}(G')\right| = \left|L_s^{i'}(G')\right|$ it follows that $\left|W_s^i(G)\right| + \left|W_s^j(G)\right| = 2\left|W_s^{i'}(G')\right|$ and $\left|L_s^i(G)\right| + \left|L_s^j(G)\right| = 2\left|L_s^{i'}(G')\right|$.

(b) If player j is a dummy, then $W_s^j(G)$ is empty, see Theorem 3. According to (a) we get that $\eta_i = 2\eta'_{i'}$, $\left|W_s^i(G)\right| = 2\left|W_s^{i'}(G')\right|$ and $\left|L_s^i(G)\right| = 2\left|L_s^{i'}(G')\right|$.

(c) This is immediate from (a) and Theorem 3.

(d) The proof follows from (a).

(e) Let us consider a player $k' \in N'$ in game G' such that $k' \neq i'$ and coalition $S \in L_s^{k'}(G')$. This means that $S \in L_s^k(G)$; therefore, we obtain $\left|L_s^k(G)\right| \geq \left|L_s^{k'}(G')\right|$. It also follows that $\eta_k \geq \eta'_{k'}$ and $\left|W_s^k(G)\right| \geq \left|W_s^{k'}(G')\right|$.

(f) If player i' is a dummy, then $W_s^{i'}(G')$ is empty, see Theorem 3. This means that $\left|W_s^{i'}(G')\right| = 0$, i.e. $\beta'_{i'} = 0$. Thus we find that $\beta_i = \beta_j = 0$; therefore, $\beta_i + \beta_j = \beta'_{i'} = 0$.

(g) If player i' is not a dummy, then $\eta'_{i'} > 0$. Clearly, we have that $\eta_i + \eta_j = 2\eta'_{i'} > 0$, $2\eta'_{i'} > \eta'_{i'}$ and $\eta_i > 0$ or $\eta_j > 0$, see (a) and (f).

Applying now (a) and (e) we calculate that

$$\beta_i + \beta_j = \frac{\eta_i + \eta_j}{\sum_{h \in N} \eta_h} = \frac{\eta_i + \eta_j}{\eta_i + \eta_j + \sum_{h \in N \setminus \{i.j\}} \eta_h} \leq$$

$$\frac{2\eta'_{i'}}{2\eta'_{i'} + \sum_{h \in N \setminus \{i'\}} \eta'_h} < \frac{2\eta'_{i'}}{\eta'_{i'} + \sum_{h \in N'} \eta'_h} \leq 2\beta'_{i'}.$$

Finally, we obtain $\beta_i + \beta_j < 2\beta'_{i'}$.

(h) Remark 4 implies $\beta_j \leq \beta_i$; therefore, we find that $2\beta_j \leq \beta_i + \beta_j < 2\beta'_{i'}$, i.e. $\beta_j < \beta'_{i'}$.

(i) If player i is a dictator in game G, then the other players in game G are dummies. These players are dummies in game G' too, see (a), (e) and (f). Hence, player i' is a dictator in game G'. As a result we obtain $\beta_j = 0$, $\beta_i = \beta'_{i'} = 1$ and $\beta_i + \beta_j = \beta'_{i'} = 1$.

The theorem is proven. □

Remark 5. Examples 1 and 2 and Theorem 5 imply that the following results are possible: $\beta_i + \beta_j < \beta'_{i'}$, $\beta_i + \beta_j = \beta'_{i'}$ or $\beta_i + \beta_j > \beta'_{i'}$. But Theorem 5 also gives us that $\beta_i + \beta_j < 2\beta'_{i'}$ when player i' is not a dummy in game G' and $\beta_i + \beta_j = \beta'_{i'}$ when player i' is a dummy in game G' or player i is a dictator in game G.

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МАНИПУЛИРАНЕ ЧРЕЗ СЛИВАНЕ В ТЕГЛОВНИ ИГРИ С ГЛАСУВАНЕ

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Проблемът за манипулация в гласуването е основен и получава голямо внимание в последните изследвания в теория на игрите. В тази статия ние разглеждаме един случай на манипулиране в тегловните игри с гласуване, направено чрез сливане на коалиции в единични играчи, погледнато от две гледни точки: на ефекта от колебанието на играчите и на ролята на индекса на Банцхаф. Ние доказваме една теорема за манипулация чрез сливане на двама играчи и показваме няколко атрактивни свойства в този процес.
Ключови думи: тегловни игри с гласуване, клатене, манипулация, сливане, индекс на Банцхаф