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USING THE UNIVERSAL MATH ENVIRONMENT "MATH-XPRESS" FOR MATH EDUCATION OF ENGINEERING STUDENTS

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We describe in the talk the main features of the Universal Math Environment "Math-XPress" and its use for training and assessment of engineering students studying Math at university level.
Math-Xpress includes linked modules of equation editor, 2D and 3D graph plotter, CAS evaluator and step-to-step solver, dynamic geometry (2D and 3D) and problem solving tutor.
Based on Math-Xpress platform, the sets of exercises in basic courses in Higher Math, such as: Calculus, Linear Algebra, Differential Equations, Statistics, Discrete Math and others have been developed and used at several colleges and universities in Israel regularly since 2007, involving thousands of students each academic year.
Using XPress-Problem Generator the courses in Linear Algebra and Discrete Mathematics have been adapted to the requirements of local curricula and used at the Technical University of Sofia.

1. Math-Xpress – the Universal Math Environment. Teachers need the ability to edit pages including math formulas, graphs and geometrical drawings, to demonstrate interactive graphs and drawings at class lessons, to check tests and exams and to develop new content. Students need mostly tutor and assessment tools, and in some cases graph plotter and geometrical explorer.

Several math tools and packages offer support for each one of the abovementioned abilities, like Math Type formulas editor [1], Derive for plotting graphs [2], Geo-Gebra for interactive geometry [3], Maple [4] and Wolfram [5] for computer algebra, and WebAssign [6] for test assessment.

None of them satisfies the main teacher's desire of common user interface, nor the inclusion of all the features in one application; the objects created in one program cannot be used in another.

The need of universal simple and yet powerful program is the main idea behind the development of Math-Xpress, first announced in 2001 [7].

Math-Xpress exhibits a multifunctional package offering to math teachers' and students' communities a tool covering the most of their needs and requirements.

Since then new modules have been added to the program, among them 3-D Graph plotter, Space geometry and Task editor, completing it to the multifunctional math package, having common user graphic interface.

The interconnected modules are based on proprietary Computer Algebra and Interactive Geometry algorithms, making it possible to link the modules sharing common objects.

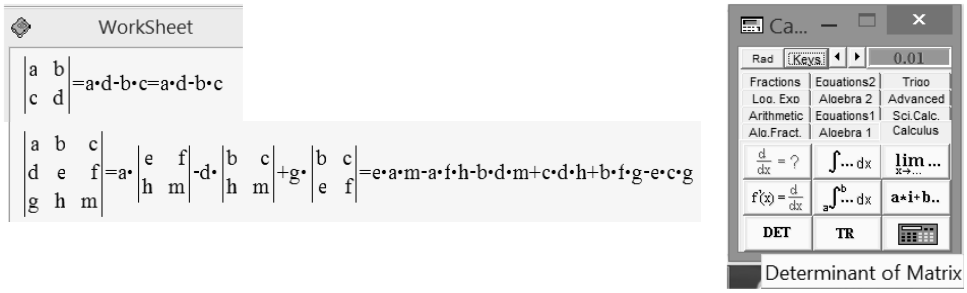
Math-Xpress includes a number of linked modules: *Equation editor, 2D and 3D graph plotter, CAS Expression Evaluator and step-by-step solver, Dynamic geometry (2D and 3D) and Problem solving Tutor.*

2. Using "Math-XPress" for interactive demonstrations of basic operations in Linear Algebra, Calculus and Discrete Mathematics. *XPress-evaluator* enables users to present mathematics in live and attractive form, by means of graphical representation of functions, equations, inequalities, complex numbers, vectors, etc., or detailed operations on Math objects.

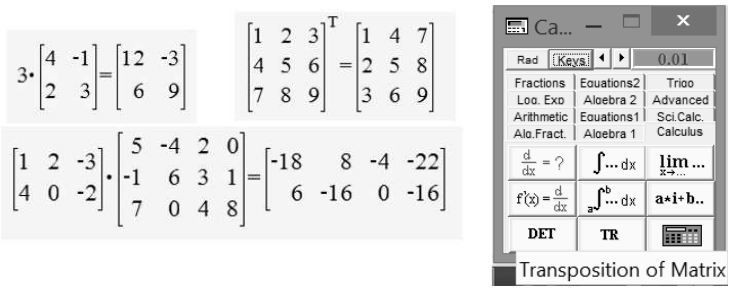
Such demonstrations are available due to CAS proprietary core enabling to interpret expressions and perform algebraic operations in 12 fields in Math, represented by 12 folders of XPress-Calculator panel.

Such operations in *Linear Algebra* include:

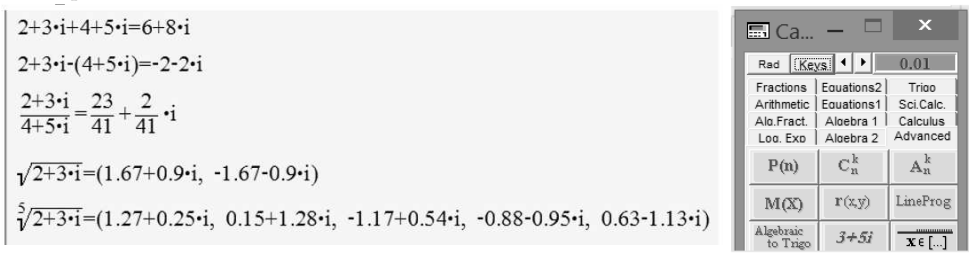
Calculation of determinants



Operations with matrices



Complex numbers



In *Calculus* such operations include:

Calculation of limits

$$\lim_{x\to\infty}\frac{x}{3\cdot x^2+\sqrt{x-5}+x^3}=\lim_{x\to\infty}\left(\frac{1}{x^1}\cdot\frac{1}{\frac{3}{x}+\sqrt{\frac{1}{x^5}-\frac{5}{x^6}}+1}\right)=0\cdot\frac{1}{0+0^{\frac{1}{2}}+1}=0$$

Calculation of derivatives

$$\frac{d\left(e^{2x\cdot\cos x}+\sqrt{\ln(x+5)}\right)}{dx}=e^{2\cdot\cos x\cdot x}\cdot 2(-\sin x\cdot x+\cos x)+\frac{1}{2}(\ln(x+5))^{\frac{1}{2}-1}\cdot\frac{1}{x+5}=$$

$$=2(\cos x-\sin x\cdot x)\cdot e^{2\cos x\cdot x}+\frac{1}{2}\cdot\sqrt{\ln(x+5)}\cdot(x+5)=2\cos x\cdot e^{2\cos x\cdot x}-2e^{2\cdot\cos x\cdot x}\cdot\sin x\cdot x+\frac{1}{2}\cdot\sqrt{\ln(x+5)}\cdot(x+5)$$

Calculation of indefinite integral

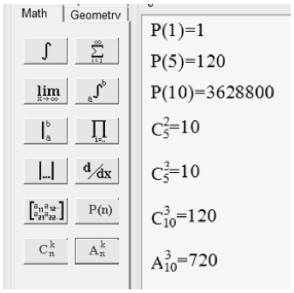
$$\int\left(3\cdot\sin(4\cdot x)-\frac{1}{x^5}\right)dx=3\cdot\left(\frac{1}{4}\cdot(-\cos(4\cdot x))\right)-\frac{x^{-5+1}}{1\cdot(-5+1)}+C=\frac{1}{4}\cdot x^{-4}-\frac{3}{4}\cdot\cos(4\cdot x)+C$$

Calculation of definite integral

$$\int_1^2\left(3\cdot x-\frac{1}{x^2}\right)dx=\left(\frac{3}{2}\cdot x^2+x^{-1}\right)\Big|_1^2=\frac{3}{2}\cdot 2^2+2^{-1}-\left(\frac{3}{2}\cdot 1^2+1^{-1}\right)=4$$

In ***Discrete Mathematics*** (the "Advanced" folder) such operations include:

Permutations
Permutations of n elements,
Binomial coefficients,
Permutations of n by k:
Statistics
Calculation of mean value,
Standard deviation,
Correlation



3. Using Math-XPress for assessment of homework in Higher Mathematics courses. All the abovementioned modules offer *tools* rather than content, therefore Math-Xpress can be used for different teaching methods, either individual, like recorded lessons [8], or work in class [9].

In each one of those Math-Xpress can be included and used as an additional tool for interactive demonstrations or for intermediate calculations, hence its usage does not depend on specific curriculum, nor on the language of teaching.

In contrast, the main objective of *XPress-Tutor* is student's Training and Assessment of homework for the courses taught according to the local curriculum in a native language.

An assessment of math tests exhibits several problems, which prevent a wide usage of an assessment system developed for non-math subjects.

The problem is that general-purpose assessment system cannot recognize math expressions written by the student and conclude whether they represent correct answers.

As a result, the general-purpose multi-choice tests are limited to a small number of sets of initial data (defined as several pre-written options), and they have no CAS ability to recognize and to evaluate student's input.

For that reason Moodle [7], so widely today is not used for assessments of tests in Higher Math.

Math-Xpress *Tutor* generates every problem using proprietary CAS algorithm and Task Development Language (TDL), enabling on one hand, description of the recommended ways of problem solution, and on the other, enabling the student to offer his/her solution and to decide whether it is correct.

Another difficulty that the developers of assessment systems encounter is the translation and localization of the content. The problem is not so critical when the system includes only the assessment mode (yet even in this case the text of problems should be translated), but in the case of complete system, including Training and Learning modes, such a translation turns to be very time consuming.

In order to solve those problems at least partially, Halomda developed Math-Xpress interface in four languages: English, Hebrew, Bulgarian and Russian (Fig. 1), and a tool, based on Google translator, enabling translation of all the content (the problems and their detailed solutions).

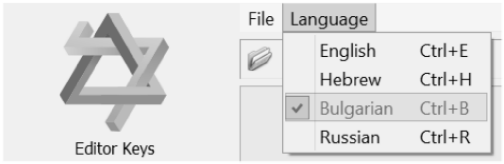


Fig. 1

Language settings of Math-XPress allow to choose the required one, and to get the tasks according to the curriculum and language of the country.

XPress-Tutor offers the student a series of problems, organized into weekly tasks, according to the course curriculum [2].

For the course in Linear Algebra taught at the Technical University of Sofia, the following tasks were developed:

Systems of linear equations, Basics of Determinants, Basics of Matrix, Vector Spaces Parts 1, 2, 3, 4, Complex Numbers, Determinants (Advanced), Matrix (Advanced) – more than 100 exercises, divided into 10 weekly tasks.

Each task is presented in three modes of operation: **Learning, Training** and **Test** (Fig. 2).

Fig. 2

During the *Learning mode*, the student obtains a series of problems; each problem includes random parameters, so that different runs exhibit different initial sets of the parameters.

The student can solve the problem in his/her way, by entering an answer using the Editor Keys. The program checks the input expression and responds either by question mark, if the answer does not coincide with any of the predicted answers, or by "Wrong", if it coincides with one of the predicted wrong answers (usually typical) (Fig. 3).

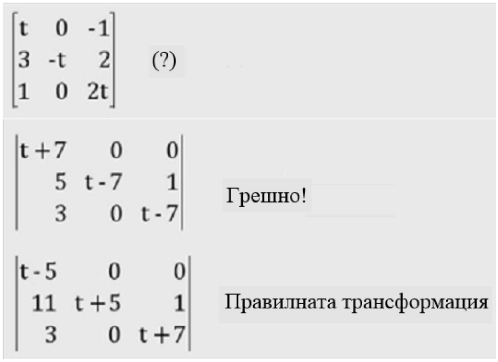


Fig. 3

The student can also ask for a Help that is given at three levels:

1) General Help, where a method of solution that is common to all the problems of a specific subject is described (Fig. 4);

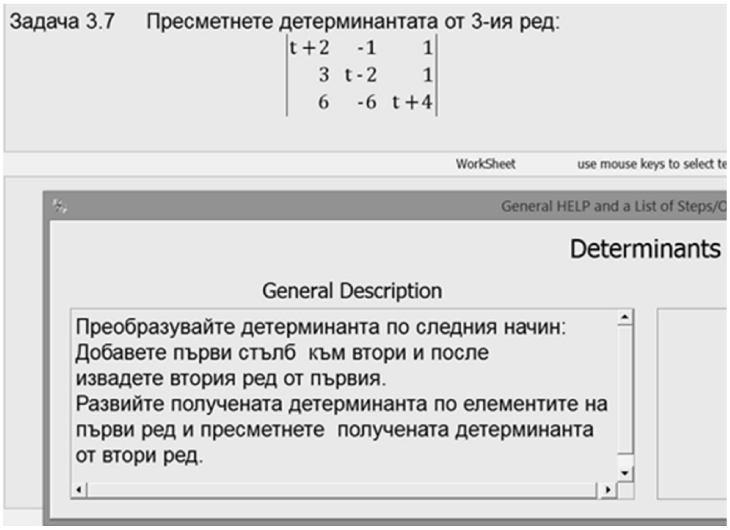


Fig. 4

2) A List of Steps of a problem's solution and the description of each step (Fig 5, 6 and 7):

At any moment the student can enter his/her proposal for the next step of the solution, and the program will compare it with the predicted answers (both correct and typical wrong answers), and respond correspondingly.

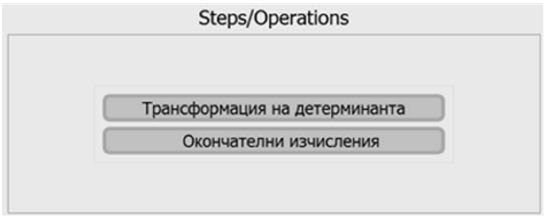


Fig. 5

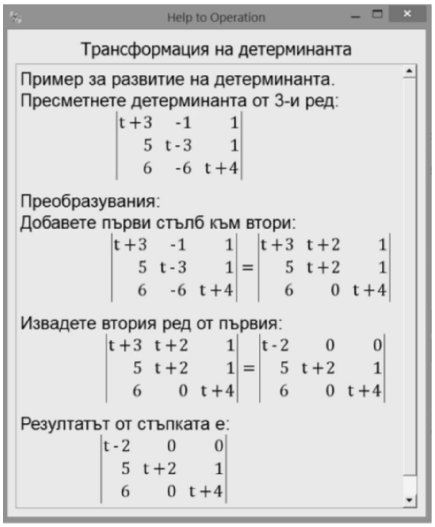


Fig. 6

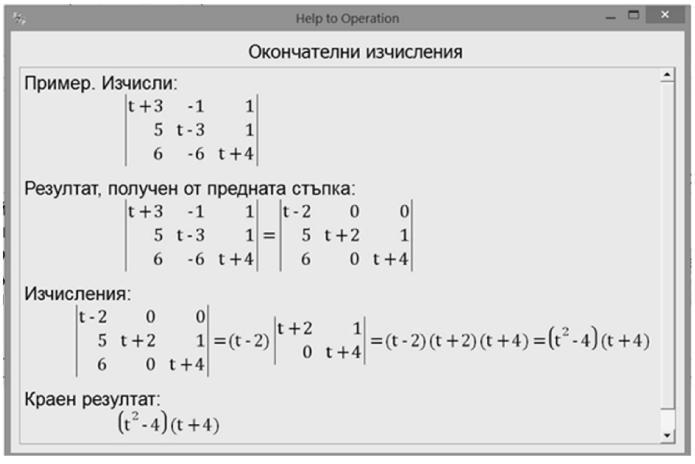


Fig. 7

If, however, he/she, while being in **Learning** mode, still couldn't suggest the correct result of one of the following steps (or the final answer), clicking on **Hint** button will demonstrate the correct result of the nearest unsolved step.

Working in **Training** mode, the student obtains 4 possible answers (one true and 3 wrong), and in order to proceed, he/she has to find the correct one; however the number of trials in training mode is unlimited.

In **Test** mode, Help and Hint are not available, and a student has only one attempt to find the correct answer (Fig, 8).

After finishing all the steps of the solution, in Learning and Training modes, the student can either move to the next problem, or repeat the current one with a new set of initial data.

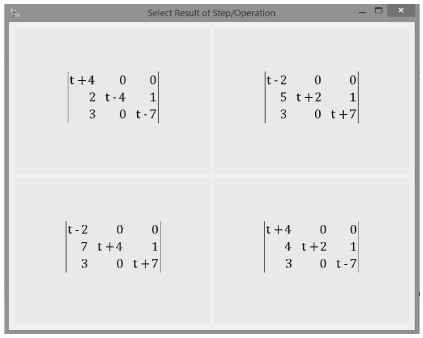


Fig. 8

In both Learning and Training modes **all the features of Math Xpress are available**, so that in a process of problem solving the student can explore the problem using different tools, such as *XPress-graph plotter* or *XPress-evaluator,* that can help him/her in better understanding of a solution. In typical student's activity scenario, he/she usually starts with Test mode, looking at the set of the problems and trying to realize whether he/she can solve them without help.

The system allows exiting the test, without reducing the marks. However, this is allowed only twice for each problem, otherwise the mark for that problem will be null.

While being out of the Test, the student can enter Learning or Training modes, and learn how to solve a problem similar to the one, in which he/she encountered difficulties: the difference between the problems presented in Test and other modes is usually in the values of the random parameters, defining the problem.

4. Development of a new content. The problems are developed using *XPress Problem Generator* – an external module, enabling compiling of new items by people unexperienced in programming [3], [4].

Using the proprietary *Task Translator* of Halomda the tasks can be translated and adapted to the required local curriculum.

Ten years ago, the course of Linear Algebra at the Math and Computing and Engineering faculties at Ariel University in Israel has been taught using the homework assignment system of Math-Xpress. The results of student's exams have shown essential improvements of about 20% [10].

Since then, during the last 10 academic years, all the courses on Higher Math at Ariel University are taught using this system, reaching more than 3000 students each year.

For the last two years, a course of Quantitative Thinking has been taught using Math-Xpress system for two groups of students at Talpiot teacher's college in Israel, also showing raise of interest and improvement of the test results of the students [11].

Using the translation tool of Math-Xpress, the course of Linear Algebra has been translated and adopted to the curriculum of the Technical University of Sofia [6], and the pilot of the system has been started at the beginning of the academic year 2017–2018 with a group of 28 students.

Apart the basic exercises on Matrices and Determinants, the course included problems on Linear Operators, for instance:

Задача 5.22 Нека T е линеен оператор от R^2 към R^3, дефиниран по следния начин:

$$T(-1,\ 0) = [-1\ \ 0\ \ -2],\quad T(1,\ 1) = [0\ \ 2\ \ 3]$$

Намерете матрицата на линейния оператор $T(a,\ b)$.

Задача 5.30 Линейният оператор T от R^3 към R^3 е дефиниран чрез формулата:

$$T(x,\ y,\ z) = [x - 2y + z\ \ \ x + 3y + 5z\ \ \ -2x - y + 5z]$$

Намерете матрицата на оператора T в базата S:

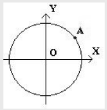
$$S = \{[-1\ \ 2\ \ 4],\ [-2\ \ -2\ \ -3],\ [2\ \ 1\ \ 1]\}$$

Other courses on Higher Math for engineering students include: Discrete Mathematics, Calculus, Analytic Geometry, Differential equations.

Here are some examples of exercises from those courses:

Analytic Geometry

Задача 4.1 Намерете уравнението на окръжността, която минава през точка $A(9,\ 12)$ и има за център началото на координатната система.



Calculus

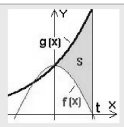
Задача 1.10 За функцията: $y(x) = \dfrac{\sin(6x + \alpha) - \sin\alpha}{20\sin x}$

да се пресметне границата: $\lim\limits_{x \to 0} y(x)$

Задача 3.4 Изчислете лицето на областта, ограничена от графиките на функциите

$$f(x) = 3\cos(4x),\quad g(x) = 3e^{7x}$$

и правата линия $x = t$, където $t = \dfrac{\pi}{8}$.



Differential equations

Задача 2.1 Намерете общото решение на диференциалното уравнение:

$$\left(2x^2+3x\right)y''+2(x+3)y'-2y=0,$$

ако е известно, че $y_1=\frac{1}{x}$ е негово частно решение.

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ИЗПОЛЗВАНЕ НА УНИВЕРСАЛНАТА МАТЕМАТИЧЕСКА СРЕДА "MATH-XPRESS" ЗА ОБУЧЕНИЕ ПО ВИСША МАТЕМАТИКА НА СТУДЕНТИТЕ – БЪДЕЩИ ИНЖЕНЕРИ

Мариана Дурчева, Филип Слободский

Описваме някои характеристики на универсалната математическа среда "Math-XPress" и използването ѝ при обучението и оценяването на студентите от инженерните специалности. Math-XPress включва редактор на уравнения, плотер за 2D и 3D графики, система за компютърна алгебра с постъпково решаване на задачите, динамична геометрия, и самоучител за решаване на задачи. На базата на платформата Math-Xpress са разработени редица упражнения по основните курсове от Висшата математика. Те се прилагат в редица колежи и университети в Израел от 2007 година, като всяка година в такива курсове се включват около 1000 студенти. Генераторът на задачи XPress-Problem Generator беше адаптиран към съответните учебни програми и успешно използван в курсовете по Линейна алгебра и Дискретна математика на ТУ–София.