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# WAYS TO INTRODUCE OPTIMIZATION ALGORITHMS AND MATHEMATICAL MODELING IN THE SCHOOL MATHEMATICS COURSE

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The main motive of this dissertation and the purpose of the study are to arouse students interest through the STEM classrooms, to show the ways of solving everyday problems (for example, in the economy, agriculture, tourism, etc.) with mathematical methods and models. The subject of the study is the educational cognitive activity of 9th- and 10th-grade students in an interactive educational environment during lessons focused on new knowledge and exercises in mathematics and the self-preparation afterwards. The first hypothesis is that teaching in STEM classrooms will increase students interest in learning mathematics. The second hypothesis is that the classic method of teaching students in the 9th and 10th grades in secondary school—that is, with chalk and a blackboard—remains more effective.
**Keywords:** optimization algorithms, mathematical modelling, STEM cabinets, mathematical tasks

# НАЧИНИ ЗА ВЪВЕЖДАНЕ НА ОПТИМИЗАЦИОННИ АЛГОРИТМИ И МАТЕМАТИЧЕСКО МОДЕЛИРАНЕ В УЧИЛИЩНИЯ КУРС ПО МАТЕМАТИКА

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Основният мотив на дисертацията и целта на изследването е да събуди интереса на учениците чрез СТЕМ класните стаи, да покаже начините за решаване на ежедневни проблеми (например в икономиката, селското стопанство, туризма и др.) с математически задачи и модели. **Предмет на изследването** е учебно-познавателната дейност на учениците от 9-и и 10-и клас в интерактивна образователна среда по време на уроците за нови знания и упражнения по математика и самоподготовката след това. Първата хипотеза е, че преподаването в СТЕМ класни стаи ще повиши интереса на учениците към изучаването на математика. Втората хипотеза е, че класическият пример за обучение на ученици в 9-и и 10-и клас в средното училище остава по-ефективен - тоест с тебешир и черна дъска.

**Ключови думи:** алгоритми за оптимизация, математическо моделиране, СТЕМ кабинети, математически задачи

# 1 Introduction

The existence of large, complex macrosystems necessitates the creation of a general theory for their management. So far, the solution of this problem is limited to the field of research of individual classes of such systems or their subsystems - industrial enterprises, branches, economy, military complex, etc.

In 1939, a brochure published at St. Petersburg University, authored by 27-year-old professor L. C. Kantorovich, was dedicated to "Mathematical methods for organization and production planning" [5]. This seemingly modest work laid the foundation for mathematical programming.

Mathematical programming (optimization) has as its subject the theoretical research and development of effective numerical methods for solving classes of extreme problems related to the optimization of decision-making in the economy, technology, and other spheres [1].

The essence of this research is the optimization process and mathematical modeling. Through these, it will be demonstrated how to solve everyday tasks with low transportation costs or create a large amount of production, with minimal costs. Optimization is a purposeful activity to obtain the best result. The object of the optimization is the human activity for a certain period - the time for conducting the research. Mathematical modeling —the research work —is not related to the research objects themselves, but to their models. The latter are divided into experimental and theoretical. The last are the mathematical models.

The study contains three important points. The first involves introducing students to the optimization problems to pique their interest. Many problems of our daily life (for example, in economics, agriculture, tourism, etc.) are solved with mathematical methods and models. This will be of great interest to students.

The second part aims to show how mathematical models are made to solve these real-life problems. This is not an easy task at all. It is important to show that many different problems from diverse fields lead to one or similar mathematical models.

The third part aims to show and justify different algorithms and methods for optimization tasks. For example, the transportation problem is solved with potential method and distribution method.

Tasks formulated at the set goal:

- Conduct a literature review of the advantages and disadvantages of secondary school education.
- Analyze the main components of teaching mathematics in secondary schools and for effectively conducting lessons focused on new knowledge and exercises.
- Analyze and compare the software products and technologies that could be used in the learning process for the relevant discipline.
- Model and implement an organizational model for mathematics education, in which interactive learning methods will be used during lessons focused on exercises and new knowledge, with specially selected software products and technologies supporting student self-training.
- Conduct an experiment to highlight the advantages and disadvantages of the organizational model.

# 2 Methods

The pedagogical research includes observation and surveying of students in experimental learning. The survey will be anonymous and all results will be analyzed, and the conclusions they lead to will be described. Sample questions in the survey are:

1. How would you compare traditional mathematics teaching in a classroom with teaching in a STEM classroom?
2. Do you think that the subject *Mathematics* will have a practical application in your further studies and in the future implementation on the labor market?
3. Is the knowledge from the subject *Mathematics* used in the other subjects?

The survey aims to explore students' opinions on the quality of mathematics education in a classical teaching style (chalk and blackboard) or with the help of multimedia information technologies. Determining which one is more effective, in their opinion, is the aim of the study.

The study will include an experimental group and a control group. The experimental group will consist of students who study math with new information technology and tools in a STEM classroom, and the control group will consist of students who study math with classic methods with blackboard and a chalk.

# 3 Related Work

This topic, *Ways to Introduce Optimization Algorithms and Mathematical Modeling into the School Math Course*, was developed by Julia Roberts, a math teacher at Cupertino High School in the Fremont Union High School District, in collaboration with Mikel Kochenderfer, Ph.D., professor of aeronautics and astronautics at Stanford University [3]. Their work, *Mathematical Optimization*, is a five-unit high school course comprising 56 lessons. The first three units are non-calculus based, requiring only algebra; the last two units require completion of Calculus AB. All units utilize the Julia programming language to teach students how to apply basic coding techniques to solve complex and relevant mathematical problems. These *Mathematical Optimization* lessons were written in 2014 by Julia Roberts, in conjunction with Dr. Mikel Kochenderfer, through a grant from the National Science Foundation. They were edited, updated, and converted into Julia notebooks in 2015 by Renee Trochet, a math teacher at Eastside College Preparatory in East Palo Alto, also in conjunction with Dr. Kochenderfer and funded by the NSF. The goals of these lessons are:

- to increase high school students' exposure to current mathematical topics, including optimization and programming;
- to increase the number and diversity of students interested in STEM fields;
- to better prepare students for future STEM studies.

# 4 Linear Optimization Problem

Allocation and transportation problems are types of linear optimization problems, and all concepts and results related to duality in linear optimization apply to them [2, 3, 4, 7].

These problems are a linear optimization problem that can be solved using the simplex method. However, specialized methods have been developed for it, which consider its specificity and are therefore much more efficient than the simplex method.

When implementing the various methods for solving these problems, rectangular tables and related concepts are used. Each rectangular table has rows and columns and consists of cells. Each cell is characterized by a pair of indices $(i, j)$: the index $i$ indicates the row number, and the index $j$ - the column number in which the cell is located [7].

# 5 Examples of Optimization Problems

Next, we will consider two examples of optimization problems and their mathematical models, which relevant to our study.

## 5.1 Example 1: Prepare feed mixtures using the lowest price criterion.

Develop a mathematical model for the task.

**Task.** The daily ration for animal feed consists of hay, silage, and concentrates and must contain at least 2000 g of protein, 120 g of calcium, and 40 g of vitamins. The nutrient content per kilogram of each feed is given in grams in the following table:

| Nutrients/ Products | Hay | Silage | Concentrates |
|---|---|---|---|
| Proteins | 50 | 20 | 180 |
| Calcium | 6 | 4 | 3 |
| Vitamins | 2 | 1 | 1 |

The ration should contain no more than 12 kg of hay, 20 kg of silage, and 16 kg of concentrates. The price per kilogram of each feed is as follows: hay – 3 BGN, silage – 2

BGN, and concentrates – 5 BGN. The goal is to compose a feed mixture that meets the daily ration requirements at the lowest cost.

**Mathematical model.** Let $x_1$, $x_2$, $x_3$ be respectively the quantities (in kilograms) of hay, silage and concentrates making up the feed mixture, $x = (x_1, x_2, x_3)$. Then the mathematical model is:

$$L(x) = 3x_1 + 2x_2 + 5x_3 \to \min .$$

Under conditions

$$\begin{gathered} 50x_1 + 20x_2 + 180x_3 \geq 2000, \\ 6x_1 + 4x_2 + 3x_3 \geq 120, \\ 2x_1 + x_2 + x_3 \geq 40, \\ 0 \leq x_1 \leq 12, \\ 0 \leq x_2 \leq 20, \\ 0 \leq x_3 \leq 16. \end{gathered}$$

## 5.2 Example 2: Allocation of sowing posts.

Develop a mathematical model for the task.

**Task.** Four types of agricultural crops ($K_1$, $K_2$, $K_3$ and $K_4$) are to be produced in quantities $a_1$, $a_2$, $a_3$ and $a_4$ tons respectively. Three plots of land ($P_1$, $P_2$ and $P_3$) with areas of $b_1$, $b_2$ and $b_3$ acres are available for sowing. The average yield of the $j$-th crop ($j = 1, 2, 3, 4$) planted on one hectare of the $i$-th plot ($i = 1, 2, 3$) is $a_{ij}$ tons:

| Plot/ Crop | $K_1$ | $K_2$ | $K_3$ | $K_4$ |
|---|---|---|---|---|
| $P_1$ | $a_{11}$ | $a_{12}$ | $a_{13}$ | $a_{14}$ |
| $P_2$ | $a_{21}$ | $a_{22}$ | $a_{23}$ | $a_{24}$ |
| $P_3$ | $a_{31}$ | $a_{32}$ | $a_{33}$ | $a_{34}$ |

The profit from the sale of one ton of crops $K_1$, $K_2$, $K_3$ and $K_4$ is respectively $c_1$, $c_2$, $c_3$ and $c_4$ BGN, respectively.

The objective is to determine how to sow the available land with the crops to meet the specified requirements and maximize profit.

**Mathematical model.** Let us denote by $x_{ij}$ that area of the plot $P_i$, $i = 1, 2, 3$, which should be sown with the crop $K_j$, $j = 1, 2, 3, 4$. Then the sowing plan $\{x\}$ is given by the following table (matrix):

| Plot/ Crop | $K_1$ | $K_2$ | $K_3$ | $K_4$ |
|---|---|---|---|---|
| $P_1$ | $x_{11}$ | $x_{12}$ | $x_{13}$ | $x_{14}$ |
| $P_2$ | $x_{21}$ | $x_{22}$ | $x_{23}$ | $x_{24}$ |
| $P_3$ | $x_{31}$ | $x_{32}$ | $x_{33}$ | $x_{34}$ |

$$L(x) = (a_{11}x_{11} + a_{21}x_{21} + a_{31}x_{31}).c_1 + (a_{12}x_{12} + a_{22}x_{22} + +a_{32}x_{32}).c_2+$$
$$+(a_{13}x_{13} + a_{23}x_{23} + a_{33}x_{33}).c_3 + (a_{14}x_{14} + a_{24}x_{24} + +a_{34}x_{34}).c_4 \to \max .$$

Under conditions [1]:

$$a_{11}x_{11} + a_{21}x_{21} + a_{31}x_{31} \geq a_1,$$

$$a_{12}x_{12} + a_{22}x_{22} + a_{32}x_{32} \geq a_2,$$
$$a_{13}x_{13} + a_{23}x_{23} + a_{33}x_{33} \geq a_3,$$
$$a_{14}x_{14} + a_{24}x_{24} + a_{34}x_{34} \geq a_4,$$
$$x_{11} + x_{12} + x_{13} + x_{14} \leq b_1,$$
$$x_{21} + x_{22} + x_{23} + x_{24} \leq b_2,$$
$$x_{31} + x_{32} + x_{33} + x_{34} \leq b_3,$$
$$x_{ij} \geq 0, \quad i = 1, 2, 3, \quad j = 1, 2, 3, 4.$$

**Remark.** These two examples, one from animal husbandry and the other from crop production, illustrate how diverse practical problems can be solved using the same mathematical model.

**Conclusion.** The transportation problem, possessing the property of yielding integer optimal solutions for integer production and demand values, serves as a powerful modeling tool. This characteristic allows diverse "non-transport" problems to be effectively represented by the transportation model [1]. As demonstrated by the feed mixture and crop planting examples, a wide range of real-world scenarios can be effectively modeled and optimized using linear programming techniques like those used in solving the transportation problem. This approach resonates with the pedagogical strategy employed in the educational resources developed by Roberts, Kochenderfer, and Trochet [6], which similarly emphasize the link between theoretical concepts and practical applications, equipping students with valuable tools for addressing optimization challenges across various disciplines.

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