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NEW OPTIMAL $(v, k, 1)$ OPTICAL ORTHOGONAL CODES WITH $k = 3, 5, 6$ [1]

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Abstract

The known classifications of optimal optical orthogonal codes with small k show that when the code length v grows, the number of nonequivalent codes increases very fast. This either makes the complete classification for relatively big values of v infeasible, or the number of the obtained codes is so big that it is very difficult to choose among them a code with the properties you need. That is why in the present work we do not obtain all the codes with the considered parameters. We construct sets of new optimal $(v, k, 1)$ optical orthogonal codes with $k = 3, 5$, and 6 for relatively small values of the code lengths v for which no classification results are known. The codes in each of these sets differ from one another in many codewords. They can be used in practical applications, as well as in further research on the topic.

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Key words: coding theory, optimal optical orthogonal code, binary cyclically permutable constant weight code, cyclic difference family, cyclic design.

1 Introduction.

1.1 Definitions and notations. Denote by $\mathbb{Z}_v$ the additive group of integers modulo v. Let M be a k-subset of $\mathbb{Z}_v$, where $M = \{m_1, m_2, \ldots, m_k\}$. Then $M + t$ denotes a t-translate of M, where $M + t = \{m_1 + t, m_2 + t, \ldots, m_k + t\}$, $t \in \mathbb{Z}_v$.

A $(v, k, 1)$ optical orthogonal code (OOC) can be defined as a collection

$$\mathcal{C} = \{C_1, \ldots, C_s\}$$

of k-subsets of $\mathbb{Z}_v$ (*codewords*), such that any two translates of a codeword share at most one element, and any two translates of two distinct codewords also share at most one element:

$$|C_i \cap (C_i + t)| \leq 1, \quad 1 \leq i \leq s, \quad 1 \leq t \leq v - 1, \tag{1}$$

$$|C_i \cap (C_j + t)| \leq 1, \quad 1 \leq i < j \leq s, \quad 0 \leq t \leq v - 1. \tag{2}$$

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Originally, OOCs were defined as a collection of $\{0,1\}$ sequences [18], but, for some purposes, it is easier to present the codewords as subsets of $\mathbb{Z}_v$, and that is why the above definition is widely used too [11]. Condition (1) is called the auto-correlation property and (2) — the cross-correlation property. A $(v,k,1)$ OOC is equivalent to a binary $(v,k,1)$ cyclically permutable constant weight (CPCW) code.

Two $(v,k,1)$ OOCs $\mathcal{C}$ and $\mathcal{C}'$ are *isomorphic* if there exists a permutation φ of $\mathbb{Z}_v$, which maps the collection of translates of each codeword of $\mathcal{C}$ to the collection of translates of a codeword of $\mathcal{C}'$. These two codes are *multiplier equivalent* if φ is an automorphism of $\mathbb{Z}_v$ (of the cyclic group of order v). An automorphism of a $(v,k,1)$ OOC $\mathcal{C}$ is a permutation α of $\mathbb{Z}_v$, which maps the collection of translates of each codeword of $\mathcal{C}$ to the collection of translates of a codeword of $\mathcal{C}$. If α is an automorphism of $\mathbb{Z}_v$, the automorphism of the OOC is called a multiplier automorphism.

Consider a codeword $C_i = \{c_1, c_2, \ldots, c_k\}$, $1 \leq i \leq s$. Denote by $\triangle C_i$ the multiset of the values of the differences $c_n - c_m,\ n \neq m,\ n,m = 1,\ 2,\ \ldots,\ k$. The auto-correlation property means that all the differences of a codeword of a $(v,k,1)$ OOC are distinct. For a $(v,k,1)$ OOC $\mathcal{C}$ the cross-correlation property means that $\triangle C_i \bigcap \triangle C_j = \emptyset$ for two distinct codewords $C_i, C_j \in \mathcal{C}$. If all possible $v-1$ nonzero differences are covered by the differences of the codewords, the $(v,k,1)$ OOC is called *perfect*.

OOCs are related to various other combinatorial structures [19]. We shall define some of them below.

Let $V = \{P_i\}_{i=1}^{v}$ be a finite set of *points*, and $\mathcal{B} = \{B_j\}_{j=1}^{b}$ a finite collection of k-element subsets of V, called *blocks*. $D = (V, \mathcal{B})$ is a *(partial) design* with parameters 2-(v,k,1) if any 2-subset of V is contained in (at most) exactly one block of $\mathcal{B}$. A 2-(v,k,1) design is also called a Steiner system and denoted by $S(2,k,v)$.

Two 2-$(v,k,1)$ (partial) designs D and D' are *isomorphic* if there exists a permutation of the points which maps each block of D to a block of D'. An automorphism of D is an isomorphism to itself.

A 2-(v,k,1) (partial) design is *cyclic* if it has an automorphism permuting its points in one cycle. A cyclic (partial) design is *strictly cyclic* if each block orbit under this automorphism is of length v (no short orbits). Two cyclic 2-$(v,k,1)$ (partial) designs D and D' are *multiplier equivalent* if there exists an automorphism of $\mathbb{Z}_v$ which maps each block of D to a block of D'. A multiplier automorphism of the (partial) design D is an automorphism of $\mathbb{Z}_v$ which maps each block of D to a block of D.

A k-element subset of $\mathbb{Z}_v$ is called *full* if the number of its distinct translates is v, and *short* if it is less than v.

A $(v,k,1)$ *cyclic difference family* can be defined as a set $D = \{B_1,\ B_2, \ldots, B_s\}$ of k-element subsets of $\mathbb{Z}_v$ (blocks), such that $B_n = \{b_{n1},\ b_{n2}, \ldots, b_{nk}\}$ and each nonzero element of $\mathbb{Z}_v$ either appears in the short block $\{0, v/k, 2v/k, \ldots, (k-1)v/k\}$ (possible if k divides v), or is obtained exactly once as a difference $b_{ni} - b_{nj}$ for $i,j = 1, \ldots, k$ and $i \neq j$.

Two $(v,k,1)$ cyclic difference families are *equivalent* if there is an automorphism α of $\mathbb{Z}_v$ which maps each k-element subset of the first family to a translate of a subset of the second family.

The size s of a $(v,k,1)$ OOC is the number of its codewords. It cannot exceed

$$\left\lfloor \frac{v-1}{k(k-1)} \right\rfloor .$$

OOCs which reach this bound are called *optimal.* If the size is exactly $(v-1)/k(k-1)$, the $(v,k,1)$ OOC is perfect. That is why optimal $(v,k,1)$ OOCs with $k = 3,5,6$ are perfect if and only if $v = 6n+1$, $v = 20n+1$, and $v = 30n+1$ respectively.

A perfect $(v,k,1)$ OOC corresponds to a cyclic $(v,k,1)$ difference family and to a strictly cyclic 2-$(v,k,1)$ design. An optimal $(v,k,1)$ OOC which is not perfect corresponds to a strictly cyclic partial 2-$(v,k,1)$ design. The blocks of this partial design correspond to the codewords and their translates. Each codeword and its translates correspond to one block orbit of length v under the cyclic automorphism of order v. Binary constant weight codes are related to $(v,k,1)$ OOCs too [32].

We illustrate the above definitions and relations by the following two examples.

Example 1.1. *A* $(61,5,1)$ *perfect OOC.*

optimal perfect $(61,5,1)$ *OOC = optimal perfect* $(61,5,1)$ *CPCW code =*
= cyclic $(61,5,1)$ *difference family = cyclic* 2-$(61,5,1)$ *design =*
= cyclic Steiner system $S(2,5,61)$

codeword	*covered nonzero differences — 60 of 60*
{0,1,3,13,34}	*1 2 3 10 12 13 21 27 28 30 31 33 34 40 48 49 51 58 59 60*
{0,4,9,23,45}	*4 5 9 14 16 19 20 22 23 25 36 38 39 41 42 45 47 52 56 57*
{0,6,17,24,32}	*6 7 8 11 15 17 18 24 26 29 32 35 37 43 44 46 50 53 54 55*

Example 1.2. *An optimal* $(44,5,1)$ *OOC.*

optimal $(44,5,1)$ *OOC = optimal* $(44,5,1)$ *CPCW code =*
= partial cyclic 2-$(44,5,1)$ *design*

codeword	*covered nonzero differences 40 of 43*
{0,1,3,28,40}	*1 2 3 4 5 7 12 16 17 19 25 27 28 32 37 39 40 41 42 43*
{0,6,14,24,35}	*6 8 9 10 11 14 15 18 20 21 23 24 26 29 30 33 34 35 36 38*

noncovered nonzero differences: $13, 22, 31.$

OOCs have various applications [8, 18], [21] and have been widely studied. A summary of the known results follows.

1.2 Known results and the present paper. Optimal $(v,k,1)$ OOCs are widely studied in many papers. Results obtained for other combinatorial objects like CPCW codes, cyclic difference families and cyclic combinatorial designs hold for the corresponding OOCs too.

The existence problem is completely solved for $(v,3,1)$ [10] and $(v,4,1)$ OOCs [38]. The complete investigation of the existence problem of optimal $(v,k,1)$ OOCs for $k \geq 5$ is still open. Existence of optimal OOCs with $k = 5$ and 6 for particular values of v are obtained in [1, 7, 14, 29, 30, 34]. Some existence results follow from results about cyclic difference families and combinatorial designs [11, 15, 16, 24, 28]. Constructions of optimal OOCs are presented in [9, 13, 23, 31, 32, 36, 35, 37]. The nonexistence of optimal $(v, 5, 1)$ OOCs with $v \equiv 22 \pmod{40}$ and of a $(62, 6, 1)$ OOC is proved in [12].

Classification results about optimal $(v, k, 1)$ OOCs or related to them combinatorial structures can be found in [2, 3, 4, 5, 6, 22, 20, 25, 26, 27], and [33].

The results obtained in these works show that when v grows, the number of nonequivalent codes becomes extremely big. This makes the classifications for the next bigger lengths practically unusable. In the present work we consider the first values of v where classification or nonexistence results are not known, but instead of complete classifications we find hundreds to thousands of multiplier nonequivalent optimal $(v, k, 1)$ OOCs such that each obtained code differs from the rest in many codewords. An access to these codes allows potential users to easily choose the most appropriate code for a given application with no need of any additional, sometimes complicated, mathematical computations. On the other hand, some of the constructed codes can be used with recursive constructions of OOCs of relatively big length which use OOCs with certain smaller parameters and possibly some additional properties [17, 23, 37]. It is also possible that the present results will reveal previously unnoticed dependencies that can possibly be used in the construction of new infinite families.

In the present paper we construct sets of multiplier nonequivalent optimal $(v, k, 1)$ OOCs with $k = 3, 5$, and 6 and $63 \leq v \leq 200$, $90 \leq v \leq 123$, and $106 \leq v \leq 121$ respectively. We do not consider $k = 4$ because this has been done in [6]. The investigation is computer-aided. Section 2 deals with the description of the implemented algorithm. A summary of the results is given in Section 3 and the codes themselves are available online at http://www.moi.math.bas.bg/ tsonka/(v,3,1)OOC.rar,
http://www.moi.math.bas.bg/ tsonka/(v,5,1)OOC.rar and
http://www.moi.math.bas.bg/ tsonka/(v,6,1)OOC.rar.

2 The construction algorithm. Our aim is to construct a set of optimal $(v, k, 1)$ OOCs with different properties which to be easily usable in possible applications or in future research. The classification algorithms from [2]-[6] obtain the codes in lexicographic order, and each code usually differs from the previous (next) one in only the last one or two codewords. Therefore it is not a good idea to just run the classification software for some time, and use the obtained codes for any possible purposes. That is why here we develop an algorithm A which is very similar to those used in [2]-[6], but instead of finding all OOCs with definite parameters it constructs a set of multiplier nonequivalent optimal OOCs which differ from one another in a relatively big number of codewords.

Similar to the classification algorithms, A is a back-track search algorithm implying minimality tests on some of the partial solutions. Before starting the search we construct an array L of all possibilities for codewords (namely all k-element subsets with the autocorrelation property) and consider them with respect to both lexicographic order and the action of the automorphisms of $\mathbb{Z}_v$. This makes it possible to define a lexicographic order on the solutions too, and to easily check by the minimality test if a solution can be mapped to a lexicographically smaller one by some of the automorphisms of $\mathbb{Z}_v$. If so, the solution is rejected. This way only multiplier nonequivalent codes are constructed.

Suppose that the algorithm A has chosen the first n codewords, and that there are m elements of the array L which can be added as $(n + 1)$-st codeword. A classification algorithm would extend the partial solution by one codeword in all possible m ways, but A extends it in at most e ways. For that purpose it uses only the lexicographically smallest e codewords because the minimality test will reject the code C if it is mapped by some automorphism of $\mathbb{Z}_v$ to a lexicographically smaller code C'. Therefore we have to be sure that C' is constructed. This may not be true if when there are more than e possibilities for the $n+1$-st codeword, only e out of them are chosen at random. That is why our algorithm extends the partial solution of n codewords using the lexicographically smallest e (out of all m) possibilities for the $(n+1)$-st codeword.

As a result at most e branches of the nodes of the search tree are considered for $n > 1$. Thus codes with all possibilities for the first and second codeword are constructed and each of the obtained codes differs in many codewords from most of the other codes.

3 Results. We use the algorithm A to construct sets of $(v, 3, 1)$ OOCs with $e = 1, 2$ or 3. Classification results for $k = 3$ are known for all $v \leq 61$ [4], and for $v = 67$ [25]. The number of the obtained multiplier nonequivalent $(v, 3, 1)$ OOCs is presented in Table 1 for all $v \leq 200$ for which neither classification [4, 25] nor nonexistence results [10] are known.

In the results available online the $(v, 3, 1)$ OOCs with length v are listed in the files E*e*oc_v_3_1_1.txt. For instance, the file E2oc_79_3_1_1.txt contains the $(79, 3, 1)$ OOCs obtained for $e = 2$. This is pointed out in the file too. It starts with the explanation: "These are NOT ALL codes with the upper parameters. The codes below are constructed by backtrack search, but only the first 2 possibilities for the n-th codeword $(n > 2)$ are considered".

For $k = 5$ and 6 we further restrict the search that the algorithm A performs by using only the first e_0 possibilities for the second codeword. The OOCs with length v and weight k are in the file Ee_0_eoc_v_k_1_1.txt. For instance the file E51_3oc_90_5_1_1.txt contains $(90, 5, 1)$ OOCs obtained for $e_0 = 51$ and $e = 3$.

Classification results for $k = 5$ are known for all $v \leq 89$ [4], and for $v = 101$ [25]. The number of the obtained multiplier nonequivalent $(v, 5, 1)$ OOCs is presented in Table 2 for all $v \leq 123$ for which neither classification [4, 25] nor nonexistence results [12] are known.

Classification results for $k = 6$ are known for all $v \leq 105$ [4], and for $v = 121$ [25]. The number of the obtained multiplier nonequivalent $(v, 6, 1)$ OOCs is presented in Table 3 for all $v \leq 121$ for which neither classification [4, 25] nor nonexistence results [12] are known.

Some of the constructed $(v, k, 1)$ OOCs are invariant under nontrivial automorphisms of $\mathbb{Z}_v$. In this case the corresponding partial designs have multiplier automorphisms. Information about them is presented in Table 4, where $Aut_{\mathbb{Z}_v}$ is the number of automorphisms of $\mathbb{Z}_v$, the total number of $(v, k, 1)$ OOCs with these parameters is given in column *codes* and the number of multiplier automorphisms is presented as $M : N$, where N is the number of OOCs which have M multiplier automorphisms.

4 Final remarks and open problems. The powerful parallel computers available nowadays allow to obtain classification results for lengths which have not been covered in previous works, but the extremely great number of codes makes these results difficult to use. That is why our aim in the present paper is to construct only a 'representative' part of the codes, namely codes which differ substantially from one another. We hope that the online availability of the constructed optimal $(v, k, 1)$ OOCs will make them usable

Table 1: Optimal $(v, 3, 1)$ OOCs with $63 \le v \le 200$

$e = 3$

length	size	codes
63	10	≥362252
64	10	≥530156
65	10	≥1019787
66	10	≥2491070
69	11	≥978887

length	size	codes
70	11	≥1632062
71	11	≥1587574
72	11	≥9434672
73	12	≥104647

length	size	codes
74	12	≥953899
75	12	≥5107667
76	12	≥3338259
77	12	≥10048011

$e = 2$

length	size	codes
78	12	≥889982
79	13	≥11470
80	13	≥193280
81	13	≥359508
82	13	≥456202
83	13	≥358501

length	size	codes
84	13	≥2613409
85	14	≥83589
87	14	≥864447
88	14	≥1938951
89	14	≥826409

length	size	codes
90	14	≥8446453
91	15	≥144426
93	15	≥1777834
94	15	≥1289924
95	15	≥3951297

$e = 1$

length	size	codes
96	15	≥1873
97	16	≥6
98	16	≥168
99	16	≥321
100	16	≥635
101	16	≥278
102	16	≥1216
103	17	≥7
104	17	≥210
105	17	≥685
106	17	≥498
107	17	≥246
108	17	≥2579
109	18	≥15
111	18	≥293
112	18	≥1124
113	18	≥270
114	18	≥1803
115	19	≥39
117	19	≥515
118	19	≥244
119	19	≥729
120	19	≥4714
121	20	≥27
122	20	≥102
123	20	≥395
124	20	≥330
125	20	≥1048
126	20	≥4816
127	21	≥15
128	21	≥183
129	21	≥280
130	21	≥1402

length	size	codes
131	21	≥297
132	21	≥2588
133	22	≥61
135	22	≥1060
136	22	≥1480
137	22	≥261
138	22	≥2858
139	23	≥12
141	23	≥448
142	23	≥296
143	23	≥1075
144	23	≥6924
145	24	≥22
146	24	≥85
147	24	≥737
148	24	≥360
149	24	≥249
150	24	≥6049
151	25	≥5
152	25	≥243
153	25	≥592
154	25	≥2220
155	25	≥1215
156	25	≥3458
157	26	≥7
159	26	≥398
160	26	≥2272
161	26	≥1360
162	26	≥7980
163	27	≥4
165	27	≥1655

length	size	codes
166	27	≥301
167	27	≥241
168	27	≥6792
169	28	≥23
170	28	≥443
171	28	≥576
172	28	≥562
173	28	≥224
174	28	≥3441
175	29	≥117
176	29	≥407
177	29	≥443
178	29	≥761
179	29	≥223
180	29	≥8280
181	30	≥4
183	30	≥622
184	30	≥2102
185	30	≥2206
186	30	≥4707
187	31	≥28
189	31	≥1556
190	31	≥1325
191	31	≥227
192	31	≥7820
193	32	≥3
194	32	≥110
195	32	≥2128
196	32	≥1476
197	32	≥166
198	32	≥8557
199	33	≥4
200	33	≥841

Table 2: Optimal $(v, 5, 1)$ OOCs with $90 \leq v \leq 123$ for $e_0 = 51$

$e = 3$

length	size	codes
90	4	≥21372
91	4	≥32407
92	4	≥46949
93	4	≥104572
94	4	≥113907
95	4	≥196781

length	size	codes
96	4	≥406483
97	4	≥302082
98	4	≥613765
99	4	≥916558
100	4	≥1241082

length	size	codes
103	5	≥72
104	5	≥278
105	5	≥1844
106	5	≥1260
107	5	≥1772

$e = 2$

length	size	codes
108	5	≥5632
109	5	≥6676
110	5	≥18672
111	5	≥33326
112	5	≥45744
113	5	≥52711

length	size	codes
114	5	≥181952
115	5	≥168511
116	5	≥256721
117	5	≥443632
118	5	≥506751

length	size	codes
119	5	≥612808
120	5	≥1762705
121	6	≥16
122	6	≥330
123	6	≥145

Table 3: Optimal $(v, 6, 1)$ OOCs with $106 \leq v \leq 120$ for $e_0 = 11$

$e = 3$

length	size	codes
106	3	≥10980
107	3	≥18664
108	3	≥47543
109	3	≥47604
110	3	≥106081

length	size	codes
111	3	≥172996
112	3	≥217699
113	3	≥236986
114	3	≥655131
115	3	≥583691

$e = 2$

length	size	codes
116	3	≥693968
117	3	≥1065289
118	3	≥1143590

length	size	codes
119	3	≥1218363
120	3	≥1358001

Table 4: $(v, k, 1)$ OOCs invariant under nontrivial automorphisms of $\mathbb{Z}_v$

length	weight	codes	$\mathrm{Aut}_{\mathbb{Z}_v}$	Number of automorphisms:Number of codes
63	3	362252	36	1: 361991 2: 147 3: 81 6: 33
65	3	1019787	48	1: 1019767 3: 20
67	3	34709	66	1: 34311 3: 395 11: 2 33: 1
69	3	978887	44	1: 978824 2: 61 11: 1 22: 1
72	3	9434672	24	1: 9434366 2: 306
73	3	104647	72	1: 104071 3: 570 9: 6
74	3	953899	36	1: 953832 3: 65 9: 2
75	3	5107667	40	1: 5107637 2: 5 4: 5 5: 16 10: 2 20: 2
76	3	3338259	36	1: 3338194 3: 65
79	3	11470	78	1: 11364 3: 105 39: 1
80	3	193280	32	1: 193198 2: 74 4: 8
87	3	864447	56	1: 864346 2: 96 4: 2 7: 1 28: 2
91	3	144426	72	1: 143226 2: 24 3: 1160 6: 4 9: 8 12: 4
93	3	1777834	60	1: 1777497 2: 178 3: 138 5: 6 6: 10 10: 1 15: 3 30: 1
97	3	6	96	1: 5 3: 1
103	3	7	102	1: 6 3: 1
105	3	685	48	1: 683 3: 2
109	3	15	108	1: 14 3: 1
111	3	293	72	1: 292 2: 1
123	3	395	80	1: 394 2: 1
127	3	15	126	1: 14 3: 1
129	3	280	84	1: 279 2: 1
139	3	12	138	1: 11 3: 1
141	3	448	92	1: 447 2: 1
151	3	5	150	1: 4 3: 1
157	3	7	156	1: 6 3: 1
159	3	398	104	1: 397 2: 1
163	3	4	162	1: 3 3: 1
177	3	443	116	1: 442 2: 1
181	3	4	180	1: 3 3: 1
183	3	622	120	1: 621 2: 1
193	3	3	192	1: 2 3: 1
199	3	4	198	1: 3 3: 1
90	5	21372	24	1:21366 2: 6
91	5	32407	72	1: 32367 2: 4 3: 36
93	5	104572	60	1: 104401 3: 171
95	5	196781	72	1: 196634 2: 8 3: 139
96	5	406483	32	1: 406440 2: 43
99	5	916558	60	1: 916517 2: 41
101	5	8	100	1: 6 5: 2
104	5	278	48	1: 254 3: 24
105	5	1844	48	1: 1841 2: 1 4: 2
114	5	181952	36	1: 181924 3: 28
115	5	168511	88	1: 168472 2: 27 4: 12
119	5	612808	96	1: 612802 2: 6
120	5	1762705	32	1: 1762531 2: 154 4: 20
122	5	330	60	1: 262 3: 16 5: 52
123	5	145	80	1: 130 5: 15
109	6	47604	108	1: 47475 3: 129
111	6	172996	72	1: 172886 3: 108 9: 2
112	6	217699	48	1: 217582 2: 117
114	6	655131	36	1: 655041 3: 86 6: 1 9: 2 18: 1
117	6	1065289	72	1: 1064783 2: 258 3: 232 4: 4 6: 8 12: 4
119	6	1218363	96	1: 1218266 2: 91 4: 6

both in applications and in future research. Another approach might be to construct only the codes with certain properties important for particular applications. The full classification of the considered $(v, k, 1)$ OOCs remains an open problem.

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НОВИ ОПТИМАЛНИ $(v, k, 1)$ ОПТИЧНИ ОРТОГОНАЛНИ КОДОВЕ С $k = 3, 5, 6$

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Абстракт

Известните класификации на оптимални оптични ортогонални кодове с малки k показват, че когато дължината на кода v нараства, броят на нееквивалентните кодове се увеличава много бързо. Това или прави пълната класификация за относително големи стойности на v невъзможна, или броят на получените кодове е толкова голям, че е много трудно да се избере сред тях код с необходимите свойства. Ето защо в настоящата работа не получаваме всички кодове с разглежданите параметри. Конструираме множества от нови оптимални $(v, k, 1)$ оптични ортогонални кодове с $k = 3, 5$ и 6 за относително малки стойности на дължините на кода v, за които не са известни резултати от класификацията. Кодовете във всеки от тези множества се различават един от друг по много кодови думи. Те могат да бъдат използвани в практически приложения, както и в по-нататъшни изследвания по темата.

Ключови думи: теория на кодирането, оптимален оптичен ортогонален код, двоичен константно тегловен циклично пермутационен код, циклични разностни фамилии, цикличен дизайн.