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ON THE MAXIMAL LENGTH OF NEAR-MDS CODES [1]

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Abstract

This paper summarizes the known results about near-MDS codes proved in the past thirty years. The main focus is put on the problem of determining the value of the function $m'(k, q)$ defined as the maximal length of a near-MDS code of dimension k over the field with q elements. For dimensions $k > q + 2$ we improve the upper bound $m'(k, q) \leq 2q+k-2$ which follows from the nonexistence of maximal arcs over fields of odd characteristic. In analogy with the main conjecture for MDS codes we formulate some conjectures on the exact value of $m'(k, q)$.

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Key words: near-MDS codes, MDS codes, elliptic curves, $(n, 3)$-arcs, n-tracks, projective geometries, designs.

1 Introduction. Near-MDS codes have been introduced in 1995 in [13]. They are defined by weakening some restrictions in the definition of the MDS codes. The most popular definition is via generalized Hamming weights. The r-th generalized Hamming weight $d_r(C)$ of a linear code C is defined as the size of the smallest support of an r-dimensional subcode of C. A linear $[n, k]_q$-code C is called a near-MDS code if

$$d_i(C) = n - k + i \text{ for } i = 2, \ldots, k, \quad d_1(C) = n - k.$$

Of course, it is enough to require $d_1(C) = n - k$ and $d_2(C) = n - k + 2$. The remaining conditions for $i = 3, \ldots, k$ are satisfied automatically. From the properties of the generalized Hamming weights one can easily deduce that the dual of a near-MDS code is again a near-MDS code. This follows immediately from the fact that for every linear code

$$\{d_i(C) \mid i = 1, \ldots, k\} \cup \{d_j(C^\perp) \mid j = 1, \ldots, n-k\} = \{1, \ldots, n\}.$$

The following propositions characterize near-MDS codes and can serve as alternative definitions. The proofs can be found in [13].

Proposition 1.1. *A linear $[n, k]_q$-code C is a near-MDS code if and only if any parity-check matrix H_C of C satisfies the conditions:*

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(1) any $n-k-1$ columns of H_C are linearly independent;

(2) there exist $n-k$ linearly dependent columns;

(3) any $n-k+1$ columns of H_C are of rank $n-k$.

Proposition 1.2. *A linear $[n,k]_q$-code C is near-MDS if and only if any generator matrix G_C of C satisfies the conditions:*

(1) any $k-1$ columns of G_C are linearly independent;

(2) there exist k linearly dependent columns;

(3) any $k+1$ columns of G_C are of rank k.

Proposition 1.3. *A linear $[n,k]_q$ code is a near-MDS code if and only if $d(C)+d(C^{\perp}) = n$.*

Closely related to near-MDS codes are the so-called almost-MDS codes introduced by de Boer [11, 12]. Almost-MDS codes are defined as $[n,k]_q$-codes with minimum distance $d = n-k$, or, in other words, as codes with Singleton defect 1. Not every almost-MDS code is near-MDS, which is demonstrated by the following example.

Consider a MDS-code C with parameters $[n,k]_q$ defined by a parity-check matrix H. Extend H by a row of weight less than $k-1$. Clearly, this row is not a linear combination of the rows of H. Denote the obtained matrix by H' and let C' be the code having H' as a parity=check matrix. By construction, the code C' has parameters $[n',k',d']$ where $n'=n$, $k'=k-1$, $d' \geq n'-k'$. The last inequality follows from the fact that every $d'-1 = n'-k'-1 = n-k$ columns of H' are linearly independent. Obviously, $d(C'^{\perp}) < k-1 = k'$ and C' is not an MDS-code. Hence $d(C') = n'-k'$ and C' is an $[n',k',n'-k']$-code which is not a near-MDS code.

However, for large n every almost-MDS code is also a near-MDS code.

Proposition 1.4. *If $n > k+q$ every $[n,k,n-k]_q$-code is a near-MDS code.*

The weight distribution of a near-MDS code can be determined up to a single parameter. In the theorem below this parameter is taken to be the number of words of minimal weight.

Theorem 1.5. *Let C be an $[n,k]_q$ near-MDS code. Let (A_i) and (A'_i) be the spectra of C and $C^{\perp}$, respectively. Then*

$$A_{n-k+s} = \binom{n}{k-s} \sum_{j=0}^{s-1} (-1)^j \binom{n-k+s}{j} (q^{s-j}-1) + (-1)^s \binom{k}{s} A_{n-k},$$

where $s = 1, \ldots, k$, and

$$A'_{k+s} = \binom{n}{k+s} \sum_{j=0}^{s-1} (-1)^j \binom{k+s}{j} (q^{s-j}-1) + (-1)^s \binom{n-k}{s} A'_{k},$$

where $s = 1, \ldots, n-k$.

For almost-MDS codes the situation is more complicated. The numbers of the parameters depends on the Singleton defect of the orthogonal code (cf. [18]). Theorem 1.5 gives a simple upper bound on the number of words of minimal weight.

Corollary 1.6. *For an* $[n, k]_q$ *near-MDS code*

$$A_{n-k} \leq \binom{n}{k-1} \frac{q-1}{k},$$

with equality if and only if $A_{n-k+1} = 0$*. By duality,*

$$A'_k \leq \binom{n}{k+1} \frac{q-1}{n-k},$$

with equality if and only if $A'_{k+1} = 0$*.*

It is interesting to find near-MDS codes with $A_{n-k+1} = 0$. In such a case it is easy to see that the existence of an $[n, k, n-k]$ near-MDS code is equivalent to that of a $(k-1)$-$(n, k, 1)$-design or a Steiner system $S(k-1, k, n)$. Apart from the ternary Golay code, there are just a few examples of such near-MDS codes. All of them are of dimension $k = 3$. It looks likely that for $k > 3$, $q > 3$, such near-MDS codes do not exist.

2 The geometric view at near-MDS codes. It is known that with every $[n, k, d]_q$-code of full length one can associate a multiset of points in $\mathrm{PG}(k-1, q)$ (possibly in a non-unique way) so that isomorphic codes are associated with projectively equivalent multisets (cf. [16]). This implies that the existence of an $[n, k]_q$ near-MDS code is equivalent to that of a set $\mathcal{K}$ of points in $\mathrm{PG}(k-1, q)$ with the following properties:

(1) every $k-1$ points from $\mathcal{K}$ are in general position (generate a hyperplane);

(2) there exist k points from $\mathcal{K}$ that lie in a hyperplane;

(3) every $k+1$ points from $\mathcal{K}$ generate $\mathrm{PG}(k-1, q)$.

In particular, if $k = 3$ a near-MDS code is equivalent to an $(n, 3)$-arc in $\mathrm{PG}(2, q)$. The nonexistence of maximal $(n, 3)$-arcs, i.e. arcs with $n = 2q + 3$ for $q > 3$ was ruled out originally by Thas [28]. This result is a part of a more general theorem about the nonexistence of maximal arcs in $\mathrm{PG}(2, q)$ for odd q proved by Ball, Blokhuis and Mazzocca [4, 5]. Since every $(2q + 2, 3)$-arc is extendable one gets that the size of an $(n, 3)$-arc is bounded by $n \leq 2q + 1$. This provides the best upper bound on the length of a near-MDS code (cf. Theorem 4.1(vi)).

This bound is attained only for $q = 4, 5, 7$. For $q = 4$ there exist three non-equivalent $(9, 3)$-arcs: the complement of a triangle, the complement of an oval and two secants, and the Hermitian curve, or, equivalently, a triangle without the vertices. For $q = 5$ there exist two non-equivalent $(11, 3)$-arcs: the complement of four lines in general position and two additional points (for which there are two non-equivalent choices) (cf. [22]). For $q = 7$ there exists exactly one $(15, 3)$-arc which is obtained as the intersection points of the ten lines in the Desarguesian configuration that are not from the configuration (cf. also [26]). For larger values of q we have strict inequality. It is conjectured that

q	4	5	7	8	9	11	13
n	9	11	15	15	17	21	23
q	17	19	23	25	27	29	31
n	28–33	31–39	37–47	38–51	42–55	44–59	46–63

Table 1: Lower and upper bounds on the size of an $(n, 3)$-arc in $\mathrm{PG}(2, q)$

$q = 4, 5, 7$ are the only values of q for which $(2q + 1, 3)$-arcs in $\mathrm{PG}(2, q)$ are known. The largest known sizes for $(n, 3)$-arcs in $\mathrm{PG}(2, q)$, as well as upper bounds are given in the table below (cf. also [7, 10]).

Almost-MDS codes are equivalent to so-called n-tracks. An n-track is a set of points in $\mathrm{PG}(r, q)$ such that every r of them are in general position. Tables containing exact values and bounds on the maximal size of an n-track are contained in [3, 11, 12, 21].

3 Near-MDS codes over small fields. With no loss of generality, we consider only codes with $k \leq n/2$. Near-MDS codes of dimension greater than $n/2$ are obtained as orthogonal to near-MDS codes with $k \leq n/2$. This implies that from now on we can assume that $k \leq 2q$ and $n \geq 2q$.

It is known that all binary MDS codes are trivial. For near-MDS codes we have some non-trivial examples, but in fact we can list all near-MDS codes. These are the extended Hamming $[8, 4, 4]$-code, the simplex $[7, 3, 4]$-code, the $[6, 3, 3]$-codes obtained by shortening the Hamming code of length 7, as well as, several trivial codes of dimensions one and two.

In the ternary case, we have one $[9, 3, 6]_3$-code associated with the affine plane $\mathrm{AG}(2, 3)$, one $[10, 4, 6]_3$-code, one $[11, 5, 6]_3$-code (the orthogonal to the Golay code) and one $[12, 6, 6]_3$-code (the extended ternary Golay code).

For codes over $\mathbb{F}_4$, there exist three non-isomorphic $[9, 3, 6]_4$-codes, associated with the three non-equivalent $(9, 3)$-arcs in $\mathrm{PG}(2, 4)$, two $[10, 4, 6]_4$-codes, exactly one $[11, 5, 6]_4$-code and exactly one $[12, 6, 6]_4$-code [14, 15]. It should be noted that the $[12, 6, 6]_4$ code was constructed by Dumer and Zinoviev in [17] as the first member of an infinite family of uniformly packed codes. Remarkably, this code yields a cascade representation of the extended binary Golay code.

There exist two non-isomorphic $[11, 3, 8]_5$ codes associated with the two $(11, 3)$-arcs in $\mathrm{PG}(2, 5)$. One of them extends to a $[12, 4, 8]_5$ code which cannot be further extended. A $[12, 6, 6]_5$-code does exist. It was constructed in [9] using a computer. Later on, Abatangelo and Larato [2] constructed six non-isomorphic codes with these parameters. They extended by two points the elliptic curve Γ_6 of degree 6 in $\mathrm{PG}(5, q)$ arising from a non-singular cubic curve of $\mathrm{PG}(2, q)$ via the canonical Veronese embedding

$$\nu : (X : Y : Z) \to (X^2 : XY : Y^2 : XZ : YZ : Z^2).$$

In [27] the authors determine the maximal length of $[n, 4, n - 4]_{11}$-code which turns out to be 20. For $[n, 3, n - 3]_{11}$ the maximal length is 21 and is equal to the maximal size of an $(n, 3)$-arc in $\mathrm{PG}(2, 11)$. In the same paper the authors investigate also NMDS codes with parameters

$$[13,5,8]_7,\ [14,5,9]_8,\ [15,4,11]_9.$$

4 Near-MDS codes of maximal length.

As with MDS-codes, the main problem for near-MDS codes is to determine the maximal length of such a code for a fixed dimension k and a fixed ground field $\mathbb{F}_q$. In analogy with MDS codes, we denote by $m'(k,q)$ the maximum possible length for which there exists a $[n,k]_q$ near-MDS code. The following theorem summarizes some straightforward observations about $m'(k,q)$.

Theorem 4.1. *Let k be a positive integer and let q be a prime power. Then*

(i) $m'(2,q) = 2q+2$*;*

(ii) $m'(k,q) \leq m'(k-\alpha,q)+\alpha$*, for every positive integer α with $\alpha \leq k$;*

(iii) $m'(k,q) = k+1$ *for $k > 2q$;*

(iv) $m'(2q,q) = 2q+2$*;*

(v) $m'(2q-1,q) = 2q+1$*;*

(vi) $m'(k,q) \leq 2q+k-2$*.*

Near-MDS codes with parameters $[n,k]_q$ can be constructed from elliptic curves over $\mathbb{F}_q$ having exactly n rational points [29] (cf. also [1, 2, 19, 20]). Such codes are referred to as elliptic codes. For every prime power $q = p^r$, p a prime, near-MDS codes exist for lengths up to $N_q(1)$, where $N_q(1)$ denotes the maximum number of $\mathbb{F}_q$-rational points an elliptic curve defined over $\mathbb{F}_q$ can have. By a result of Waterhouse [30], we know that for every $q = p^e$

$$N_q(1) = \begin{cases} q + \lfloor 2\sqrt{q} \rfloor & \text{for } p \mid \lceil 2\sqrt{q} \rceil \text{ and odd } e, \\ q + \lfloor 2\sqrt{q} \rfloor + 1 & \text{otherwise.} \end{cases}$$

This gives a lower bound on $m'(k,q)$: $m'(k,q) \geq N_q(1)$. Due to extensive computational work by Bartoli, Marcugini, Milani and Pambianco [8, 15, 25, 27], the exact values of $m'(k,q)$ were determined for all fields of order $q \leq 9$, as well as lower and upper bounds on the size of the longest near-MDS code for some larger fields. In particular, in [25] the authors determine the three exact values

$$m'(5,8) = 15,\ \ m'(4,9) = 16, m'(5,9) = 16.$$

By duality, they also determine the values of $m'(k,8)$ for $k = 10, 11, 12$, and of $m'(k,9)$ for $k = 12, 13, 14$. Even more results were obtained for the case of dimension three, which corresponds to the problem of the maximal size of an $(n,3)$-arc in $\mathrm{PG}(2,q)$ (cf. [10, 23, 26, 24]). These results are summarized in the table below.

q	2	3	4	5	7	8	9	11	13
k									
2	6	8	10	12	16	18	20	24	28
3	7	9	9	11	15	15	17	21	23
4	8	10	10	12	14	16	16	20-21	21-24
5		11	11	11	13	15	16	18-22	21-25
6		12	12	12	13	14	16	18-23	21-26
7			9	11	14	15	17	18-24	21-27
8			10	12	13	16	18	18-25	21-28
9				11	13	14	19	19-26	21-29
10				12	14	15	20	20-27	21-30
11					14	15	16	18-28	21-31
12					15	16	16	18-29	21-32
13					15	15	16	18-30	21-33
14					16	16	17	18-31	21-34
15						17	17	18-32	21-35
16						18	18	18-33	21-36

Table 2: Exact values and bounds on $m'(k,q)$ for small fields $\mathbb{F}_q$, $q \leq 13$

5 An upper bound on the maximal length of a near-MDS code. According to Theorem 4.1(vi), we have $m'(k,q) \leq 2q+k-2$. It can be seen from the table above that equality is achieved just for several pairs (k,q). The following theorem gives an improvement over Theorem 4.1(vi) for sufficiently large dimensions.

Theorem 5.1. *There exist no $[2q+k-2,k]_q$ near-MDS codes for $k \geq q+2$ and $q \geq 7$.*

Proof. We are going to use the geometric interpretation of near-MDS codes. Fix an integer $k > q+2$. Let $\mathcal{K}$ be an arc with $2q+k-2$ points in $\mathrm{PG}(k-1,q)$, associated with a $[2q+k-2,k]_q$ near-MDS code. Furthermore, let $P_1,\dots,P_{k-2}$ be points from $\mathcal{K}$. By the properties of the arcs associated with near-MDS codes, these $k-2$ points are in general position. Without loss of generality we can take $P_i = (0,\dots,1,0,\dots,0)$, where the only unit is in position i. Set

$$\begin{aligned} S &= \langle P_1, P_2, \dots, P_{k-2} \rangle, \\ S_i &= \langle P_1, \dots, P_{i-1}, P_{i+1}, \dots, P_{k-2} \rangle, \quad i = 1,\dots,k-2. \end{aligned}$$

Clearly S is the hyperline defined by $x_{k-1} = x_k = 0$, and S_i are the subspaces defined by $x_i = x_{k-1} = x_k = 0$, $i = 1,\dots,k-2$. Obviously, $\dim S = k-3$, and $\dim S_i = k-4$. There exists a point Q that is not contained in any of the subspaces S_i, for example $Q = (1,\dots,1,0,0)$.

There exists a plane π which meets S only in the point Q, i.e. $\pi \cap S = \{Q\}$ (in fact, there exist many such planes). It is clear that $\pi \cap S_i = \varnothing$. Note that from the properties of the near-MDS codes and the arcs associated with them it follows that $|\mathcal{K} \cap \pi| \leq 3$.

Now consider projections φ_i, $i = 1, \ldots, k-2$, from each S_i to π given by

$$\varphi_i \colon \begin{cases} \mathcal{P} \setminus S_i & \to \quad \pi \\ P & \to \quad \langle S_i, P \rangle \cap \pi, \end{cases}$$

where $\mathcal{P}$ is the set of points of $\mathrm{PG}(k-1, q)$. Obviously, all induced arcs $\mathcal{K}^{\varphi_i}$ are plane arcs with parameters $(2q+1, 3)$.

The arcs $\mathcal{K}^{\varphi_i}$ are projective; otherwise, we easily get a contradiction since there exists a subspace of projective dimension $k-3$ (a hyperline) containing $k-1$ points.

If $P \in \mathcal{K} \cap \pi$, then P is contained in all induced arcs. Obviously $Q = S \cap \pi$ is also contained in all arcs $\mathcal{K}^{\varphi_i}$. Note that there are at most three such points. If R is a point from π which is contained neither in $\mathcal{K}$ nor in S, then it is contained in at most two of $\mathcal{K}^{\varphi_i}$ (since a hyperplane contains at most k points from $\mathcal{K}$). Counting the pairs $(P, \mathcal{K}^{\varphi_i})$ with $P \in \mathcal{K}^{\varphi_i}$ in two possible ways, we get

$$(k-2)(2q+1) \leq 2(q^2+q-3) + 4(k-2),$$

whence $k \leq q+4$, a contradiction. Now it remains to use Theorem 4.1(ii) to obtain the nonexistence for all dimensions $k > q+4$. ☐

Let us note that the proof of the nonexistence of $(2q+1, 3)$-arcs would immediately imply the nonexistence of $[2q+k-2, k]_q$ near-MDS codes. All numerical evidence suggests that this is true for all $q \geq 8$, but no proof of the nonexistence of $(2q+1, 3)$-arcs in $\mathrm{PG}(2, q)$ for large q seems to be known for the time being. There is a problem for $(n, 3)$-arcs in $\mathrm{PG}(2, q)$ suggested by A. Blokhuis asking to determine a constant c such that $n/q < c < 2$ for q large enough, or a construction where $n/q > c > 1$ [6].

We finish with two conjectures for near-MDS codes that are similar to the famous Main Conjecture for MDS codes.

Conjecture 5.2 (Weak Main Conjecture for NMDS codes)**.** *For all positive integers k and all prime powers $q \geq 5$ it holds that $m'(k, q) \leq 2(q+1)$.*

Conjecture 5.3 (Strong Main Conjecture for NMDS codes)**.** *There exists a universal constant c (not depending on q) such that $m'(k, q) \leq N_q(1) + c$.*

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ВЪРХУ МАКСИМАЛНАТА ДЪЛЖИНА НА ПОЧТИ-МДР КОДОВЕ

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Абстракт

В тази статия правим обзор на резултатите за почти-МДР кодове, получени през последните 30 години. Фокусът е поставен върху задачата за определяне на точната стойност на функцията $m'(k,q)$, дефинирана като максималната дължина на почти-МДР код с размерност k над поле с q елемента. За размерности $k > q+2$ ние подобряваме горната граница $m'(k,q) \leq 2q+k-2$, която следва от несъществуването на максимални арки над полета с нечетна характеристика. Подобно на основната хипотеза за МДР кодове, ние формулираме няколко хипотези за точната стойност на функцията $m'(k,q)$.

Ключови думи: почти-МДР кодове, МДР кодове, елиптични криви, $(n,3)$-арка, проективни геометрии, дизайни.