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ON THE STRUCTURE OF CYCLIC STEINER TRIPLE SYSTEMS OF SMALL ORDER [1]

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Abstract

An n-configuration is a collection of n blocks of a particular STS(v). An n-configuration is full if none of its points occur in just one block. Of special interest for understanding the structure of an STS(v) is the number of its full configurations with no more than $n+2$ points. An STS(v) is n-sparse if every set of i blocks covers more than $i+2$ points, $4 \leq i \leq n$. The first examples of 6-sparse STS(v)s were presented by Forbes, Grannell and Griggs in 2007. The smallest known 6-sparse STS(v) is of order 139. Recently the properties of all STS(19)s were analyzed by Colbourn and coauthors, 2010 and of STS(21)s with nontrivial automorphisms by Erskine, Griggs, 2024 and 6-sparse STS(v)s were not found among them. It is not known if there exists a 6-sparse STS(v) for some order smaller than 139. In an attempt to answer this question we count the number of the full 6-configurations with 8 points in cyclic STS(v)s of small order. There are no 6-sparse ones but we find some examples of STS(v)s, $25 \leq v \leq 63$ with interesting structure.

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Key words: combinatorial design; Steiner Triple system; configuration.

1 Introduction. Let V be a finite set of v *points*, and $\mathcal{B} = \{B_j\}_{j=1}^{b}$ a finite collection of k-element subsets of V, called *blocks*. $D = (V, \mathcal{B})$ is a 2-(v,k,λ) *design* if any 2-element subset of V is contained in exactly λ blocks of $\mathcal{B}$. Designs with $k = 3$ and $\lambda = 1$ are usually called *Steiner triple systems* and denoted by STS(v). Two designs $D_1 = (V_1, \mathcal{B}_1)$ and $D_2 = (V_2, \mathcal{B}_2)$ are *isomorphic* if there is a bijection from V_1 to V_2 which maps each block of D_1 to a block of D_2. A permutation of the design points which maps each block to a block of the same design is an *automorphism*. All the automorphisms of a design form its *full automorphism group* G under composition of permutations. If the automorphism group G contains a cycle of length v the design is *cyclic*. In what follows we denote cyclic STS(v)s by CSTS(v)s. CSTS(v)s have been classified up to order 69 in a series of papers [2, 6, 20]. More details on STS(v)s and CSTS(v)s can be found in [4, 7].

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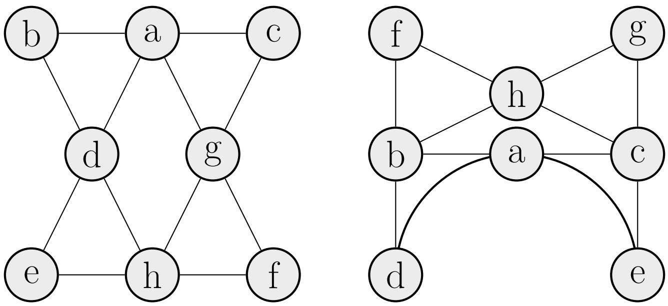
An n-*configuration* is a collection of n blocks of a particular STS(v), $n < b$. Next we only use the term *configuration* if the number of the involved blocks is not important. The configurations in an STS(v) can be viewed as partial STS(v)s. The *degree* of a configuration point is the number of blocks which contain it. A configuration is *full* if the degree of each of its points is at least two. Two n-configurations are *isomorphic* if there exists a bijection on their point sets which maps each block of the first configuration to a block of the second one. Nonisomorphic full n-configurations are enumerated up to $n \leq 13$ [10, 19].

Next we are interested in the number of occurrences of some configurations in a particular CSTS(v) in connection with their applicability. Of special interest for understanding the structure of an STS(v) is the frequency of appearance of the full configurations with no more than $n + 2$ points. These with small number of blocks are well-known in the literature under special names. The unique up to isomorphism full 4-configuration covers 6 points and is known as *Pasch configuration* (quadrilateral) [3, 16, 25]. The full 5-configuration with $5 + 2 = 7$ points is called the *mitre configuration*, and is unique too [3, 6]. Up to isomorphism there are five full 6-configurations. They are first presented in [21]. Only two of them have $6 + 2 = 8$ points. Two of their points are of degree 3, the other 6 points are of degree 2. If the points of degree 3 are on two disjoint blocks, the *hexagon* configuration is obtained (Fig. 1a). In the other case the two points of degree 3 are in one and the same block and the *crown* configuration arises (Fig. 1b).



a) hexagon configuration b) crown configuration

Figure 1: Full 6-configurations with 8 points

The STS(v)s with lack of Pasch or mitre configurations are intensively studied. An STS(v) is *anti-Pasch* (*anti-mitre*) if it comprises no Pasch (mitre) configurations respectively. In the same manner we can call the STS(v)s missing the hexagon (crown) configuration *anti-hexagon* (*anti-crown*). An STS(v) is n-*sparse* if every set of i blocks covers more than $i + 2$ points, $4 \leq i \leq n$. Every n-sparse STS(v) is also $(n - 1)$-sparse. Hence the anti-Pasch STS(v)s are *4-sparse*. Anti-Pasch and simultaneously anti-mitre STS(v)s are *5-sparse*. For an STS(v) to be *6-sparse* it is enough besides Pasch and mitre configurations not to contain hexagon and crown configurations.

Next in our investigations we will denote by P the number of occurrences of Pasch configurations and by $P_{max}(v)$ the maximal number of Pasch configurations for the CSTS(v) of a particular order. We use respectively M and $M_{max}(v)$ for the number of mitre configurations; H and $H_{max}(v)$ for the hexagon configurations and C and $C_{max}(v)$ for the crown configurations. Thus to each CSTS(v) we can associate a *configuration vector*. It comprises 4 nonnegative integers, namely (P, M, H, C). A 6-sparse STS(v) should be characterized with an all zero vector. The results for P, $P_{max}(v)$, M and $M_{max}(v)$, $v \leq 61$ are known in advance [3, 6, 25, 26]. In this research we add the results for CSTS(63) and count the frequency of occurrences of hexagon and crown configurations in CSTS(v), $25 \leq v \leq 63$ seeking 6-sparse CSTS(v)s.

For more details on configurations in STS(v)s and on n-sparse STS(v)s we refer to [7, 14, 16, Chapter 13].

2 Preliminary results and motivation. The research on configurations in STS(v)s and on n-sparse STS(v)s was started long ago. There is a conjecture of Erdös [8] that there exists an n-sparse STS(v) for every integer $n \geq 4$ and $v > v_0(n)$, where v_0 is some order of an STS(v). The existence question of 4-sparse (anti-Pasch) STS(v)s is completely settled [17]. They exist for every $v \equiv 1, 3 \pmod 6$, $v \notin \{7, 13\}$.

The 5-sparse STS(v)s have been intensively studied too. The results show that 5-sparse STS(v)s exist for almost all possible orders [13, 28, 29]. Examples of 5-sparse CSTS(v)s up to order $v \leq 97$ are provided in [6, Table 3]. The presented there 5-sparse cyclic STS(19) is unique. The properties of all STS(19)s are studied in [5] by Colbourn and coauthors, and the absence of 6-sparse STS(19)s is reported. Recently all 4-sparse STS(21)s have been cataloged by Kokkala and Östergård [22]. There are three 5-sparse STS(21)s and none of them is 6-sparse. The next 5-sparse CSTS(v)s occur for $v = 33$ and hence give the possibility of searching for 6-sparse CSTS(v)s. But we start our investigations from CSTS(25)s to find out what their structure looks like in connection with 6-configurations.

The first examples of 6-sparse STS(v)s are presented by Forbes, Grannell and Griggs in 2007 [11]. The same authors in their next paper [12] give a construction of 6-sparse STS(v)s for $v = 3p$, $p \equiv 3 \pmod 4$ and large enough. The smallest known 6-sparse STS(v) is of order 139. It is not known if there exists a 6-sparse STS(v) for some smaller order. In [15, Corollary 2.4] Fujiwara shows that a cyclic n-sparse STS(v) does not exist for $n \geq 10$. However no n-sparse STS(v) is known for $n \geq 7$ up to now.

The sparse property of STS(v)s is valuable for their applications in coding theory. An incidence matrix of an STS(v) can be considered as a parity-check matrix of a regular LDPC code [1, 27]. The LDPC codes constructed from CSTS(v)s are especially advantageous. They are well structured and therefore suitable for a hardware implementation with simple shift registers. The CSTS(v)s can ensure low-complexity encoding and good performance with iterative decoding of the associated LDPC codes [18]. It is known that short cycles affect significantly the decoding performance of the LDPC codes regarding the iterative decoding [23]. In [24] the authors propose a modification of the incidence matrix of a CSTS(v) to be used as parity-check matrix of an LDPC code in order to avoid cycles of length 6 (full 6-configurations with 8 points). High sparseness of an STS(v) results in an LDPC code without short cycles and thus of better performance.

Next we will test the CSTS(v)s, $25 \leq v \leq 63$ for 6-sparseness. We count the number of hexagon and crown configurations in CSTS(v)s, $25 \leq v \leq 63$. We do not find 6-sparse CSTS(v)s, but some examples of CSTS(v)s with interesting structure follow.

3 Counting hexagon and crown configurations. We consider the nonisomorphic CSTS(v)s with point set $V = \{1, 2, \ldots, v\}$. They are presented by sets of base blocks.

The collection of blocks of the CSTS(v) can be obtained by the action of the mapping $j \to j+1$, $1 \leq j < v$ and $v \to 1$ on the points of the given base blocks. Next we use the design blocks in a lexicographic order defined by the numbers of the points they comprise. This gives us the opportunity to consider the configuration blocks in lexicographic order too. We count only the number of configurations which comprise different blocks by searching over the design blocks. In [19] the authors use a different algorithm for counting the number of n-configurations based on search over the points.

We are interested in the number of copies of two full 6-configurations on 8 points. For both of them two of the points have degree 3. These configurations differ by the place of their degree three points. The hexagon configuration is the unique one for which these two points are in two disjoint blocks. An example is presented graphically in Fig. 1a. It is defined by the following 6 blocks:

$$\{a,b,c\},\{a,d,e\},\{a,f,g\},\{b,d,h\},\{c,g,h\},\{e,f,h\}.$$

The points a and h are of degree 3 and the blocks $\{a,b,c\}$ and $\{e,f,h\}$ are disjoint. To count the number of all copies of the hexagon configuration in a CSTS(v) we consider all pairs of disjoint blocks.

The crown configuration is the unique one for which the two degree three points are in one and the same design block. Fig. 1b illustrates graphically an example of the crown configuration. It comprises the next six blocks:

$$\{a,b,c\},\{a,d,e\},\{b,d,f\},\{b,g,h\},\{c,e,g\},\{c,f,h\}.$$

The points of degree 3 are c and b, and are together in block $\{a,b,c\}$. In order to count the number of all copies of the crown configuration in a CSTS(v) we check all pairs of blocks with a common point as $\{a,b,c\}$ and $\{a,e,d\}$.

4 Results. The most studied configuration in the STS(v)s is the Pasch configuration. It is the smallest one. In comparison with the number of other configurations in consideration, it seems that the number of Pasch configurations is the smallest one and the smallest number of different values for P appears in a CSTS(v) for particular v . There are upper and lower bounds on the maximal number of Pasch configurations [25] and on the number of mitre configurations in an STS(v) [6]. The mitre and especially hexagon and crown configurations are less investigated. In Table 1 we present the maximal values for the number of hexagon and crown configurations in CSTS(v)s, $25 \leq v \leq 63$. Almost all CSTS(v)s with H_{max} or C_{max} are unique. All the data obtained as a result of this research is available upon request.

v	25	27	31	33	37	39	43	45	49	51	55	57	61	63
H_{max}	100	189	837	330	1443	3159	2322	990	3675	1479	1430	10260	7137	2646
C_{max}	425	486	930	825	1332	1794	2021	2070	2646	2856	3410	4788	4758	5733

Table 1: The values of H_{max} and C_{max} in CSTS(v)s, $25 \leq v \leq 63$

4.1 Anti-hexagon and anti-crown CSTS(v)s. The number of CSTS(v)s, $25 \leq v \leq 63$ with lack of hexagon and crown configurations is presented in Table 2. Just for comparison, the number of all CSTS(v)s obtained in [2, 6, 20] is presented in the second row and the number of 5-sparse CSTS(v)s [6, 26] in the last row of Table 2. One of the new results in this paper is the number of 5-sparse CSTS(63)s (7730). Also we found that 2700317 CSTS(63)s are anti-Pasch and 191313 CSTS(63)s are anti-mitre.

The number of CSTS(v)s without any crown configuration is less than 1% except for CSTS(31)s. The CSTS(27)s are the only ones without any hexagon configuration. The STS(v)s avoiding simultaneously some of these configurations can be of interest too. There are a few examples of STS(v)s with lack of three of the configurations in consideration. We use a generalized configurations vector for a set of CSTS(v)s with the same value for three of the four indicators P, M, H or C. For example $(*, 0, 0, 0)$ denotes the set of all CSTS(v)s with $M = H = C = 0$. The value of P for these CSTS(v)s is different hence we choose to denote it by *. The number of CSTS(v)s with the corresponding generalized configurations vector can be seen in the 5th, 6th and 7th row of Table 2. A vector $(0, *, 0, 0)$ is not mentioned because such CSTS(v)s are not found.

v	25	27	31	33	37	39	43	45	49	51	55	57	61	63
|CSTS(v)|	12	8	80	84	820	798	9508	11616	157340	139828	3027456	2353310	42373196	49526744
anti-hexagon	2	0	12	8	44	22	301	204	2576	1433	29845	14617	245539	190914
anti-crown	0	0	2	0	1	0	1	0	16	1	0	8	5	5
$(*, 0, 0, 0)$	0	0	1	0	0	0	0	0	0	0	0	0	0	5
$(0, 0, *, 0)$	0	0	0	0	0	0	1	0	0	0	0	0	0	0
$(0, 0, 0, *)$	0	0	0	1	0	0	2	0	2	0	7	4	36	11
5-sparse	0	0	0	1	4	1	28	13	155	104	1197	843	9309	**7730**

Table 2: The number of anti-hexagon and anti-crown CSTS(v)s, $25 \leq v \leq 63$

4.2 CSTS(v)s without a crown configuration. Cyclic STS(v)s without crown configurations seem to be very rare (Table 2). Six of them are included in the generalized configurations vector $(*, 0, 0, 0)$ (5th row in Table 2) and appear for two of the considered CSTS(v) orders only. The one is a CSTS(31) isomorphic to the unique design derived from PG$(4, 2)$. Its configurations vector is $(1085, 0, 0, 0)$. The other five are CSTS(63)s (Table 3) one of which is isomorphic to the point-line design of PG$(5, 2)$.

P	2079	3717	4347	6237	9765
|STS(63)|	1	1	1	1	1

Table 3: Properties of STS(63)s with $(*, 0, 0, 0)$ vectors

The upper bound on the number of Pasch configurations in an STS(v) is settled in [25, Theorem 3.1] $P_{max}(v) = v(v-1)(v-3)/24$. It is attained for the point-line STS(v) of the projective space PG$(n, 2)$, $n \geq 2$, $v = 2^{n+1} - 1$. Following this bound $P_{max}(31) = 1085$. The result which we obtain by our program for CSTS(63) is $P_{max}(63) = 9765$ and it coincides with the value calculated by the theorem. We also check the configuration vector of the CSTS(15) connected to PG$(3, 2)$ and get $(105, 0, 0, 0)$. It can be conjectured that the STS(v) corresponding to PG$(n, 2)$ must have the $(P_{max}(v), 0, 0, 0)$ configurations vector.

Despite the relatively large number of CSTS(v)s with lack of hexagon configurations there is only one example of CSTS(v), $v \leq 63$ with $(0, 0, *, 0)$ (6th row in Table 2). It is a CSTS(43) with $H = 1806$. The properties of the remaining 32 STS(v)s with $C = 0$ are presented in Table 4. For all of them $H \neq 0$, nine are with generalized configurations vector $(0, *, *, 0)$, 14 with $(*, 0, *, 0)$ and 9 with $(*, *, *, 0)$. In the sets with $M \neq 0$ there are CSTS(v)s for which the maximal value of mitre configurations is reached, namely $M_{max}(49) = 1764$ and $M_{max}(57) = 5130$.

v	31	37	49	51	57	61
$(0, *, *, 0)$	0	0	0	1	8	0
$(*, 0, *, 0)$	1	1	8	0	0	4
$(*, *, *, 0)$	0	0	0	0	8	1

Table 4: Number of CSTS(v)s with $C = 0$ and at most one other missing configuration

4.3 CSTS(v)s with crown configurations only. Almost all the CSTS(v)s of the considered orders contain many crown configurations and for them the number of the other investigated configurations is not zero. Only 63 CSTS(v)s, $33 \leq v \leq 63$ have generalized configurations vector $(0, 0, 0, *)$ (7th row in Table 2). Next in Tables 5, 6 and 7 these 63 CSTS(v)s are presented depending on their order. In Table 5 the number of crown configurations for each CSTS(v), $33 \leq v \leq 57$ is given. For each instance of the CSTS(v)s in the table, C is different and less than $C_{max}(v)$ except for CSTS(33) where $C_{max}(33) = 825$ appears.

v	33	43	49	55	57
$(0, 0, 0, *)$	1	2	2	7	4
C	**825**	1419 1677	1323 1764	1540 1650 1870 1925 2035 2145 2255	1539 1653 1881 2223

Table 5: Properties of CSTS(v)s with $(0, 0, 0, *)$ vector, $33 \leq v \leq 57$

The properties of CSTS(v)s of the next two orders are presented in Tables 6 and 7 in detail. They comprise 3/4 of all investigated CSTS(v)s with the $(0, 0, 0, *)$ generalized configurations vector. Here CSTS(v)s with equal values of C occur.

C	1464	1525	1647	1830	1891	1952	2013	2074	2135	2196	2257	2318	2379	2440	2501	2562	2623
$\lvert STS(61)\rvert$	3	1	1	2	3	2	5	1	1	4	3	1	3	2	1	2	1

Table 6: Properties of CSTS(61)s with the $(0, 0, 0, *)$ vector

4.4 Conclusion remarks. Before the present work the number of crown and hexagon configurations was known only for STS(19)s [5], and for STS(21)s with nontrivial automorphism groups [9]. Now we know a bit more about the structure of cyclic STS(v)s.

C	1575	1890	1953	2016	2079	2457
|STS(63)|	1	3	1	2	2	2

Table 7: Properties of CSTS(63)s with $(0,0,0,*)$ vector

This is especially important in connection with their applications in Coding Theory. The question of the existence of a 6-sparse STS(v) of order less than 139 remains open. Recently the CSTS(v)s with $v = \{67, 69\}$ have been classified [20], but their number is more than 20 times bigger than the number of the nonisomorphic CSTS(63)s. To investigate their structure one should use parallel algorithms.

Our results are computer-aided and that is why mistakes are always possible. We partially verify them in the following ways. We obtain the number of crown and hexagon configurations in a particular CSTS(v) by two different software implementations of both authors. We obtain the same results for the examples of STS(21) presented by Erskine, Griggs [9]:

- three 5-sparse STS(21);
- an STS(21) with no hexagons;
- a cyclic STS(21) with no mitres, no crowns and $H_{max}(21) = 441$ with automorphism group 882;
- two STS(21) with $M_{max}(21) = 252$, each with $H = 252$ and $C = 0$, they are cyclic, with automorphism groups 504 and 1008;
- an STS(21) with $C_{max}(21) = 396$.

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ЗА СТРУКТУРАТА НА ЦИКЛИЧНИТЕ ЩАЙНЕРОВИ СИСТЕМИ ОТ ТРОЙКИ МАЛЪК РЕД

Антоанета Цветанова и Стела Железова

Абстракт

Колекция от n блока в Щайнерова система от тройки от ред v (STS(v)) се нарича n-конфигурация. Една n-конфигурация е пълна, ако никоя от точките ѝ не участва само в един блок. От специален интерес за разбиране на структурата на STS(v) е броят на пълните конфигурации с не повече от $n+2$ точки, които тя притежава. STS(v) е n-sparse, ако всяко множество от i блока е инцидентно с повече от $i+2$ точки, $4 \leq i \leq n$. Първите примери на 6-sparse STS(v) са публикувани от Forbes, Grannell и Griggs през 2007. Най-малката известна 6-sparse STS(v) е от ред 139. Свойствата на всички STS(19) са анализирани през 2010 от Colbourn и съавтори и на STS(21) с нетривиални групи от автоморфизми от Erskine, Griggs, 2024. Не са намерени 6-sparse STS(v) между тях. Не се знае съществуват ли 6-sparse STS(v) от редове по-малки от 139. В опит да отговорим на този въпрос, изследваме броя на пълните 6-конфигурации с 8 точки в цикличните STS(v) от малък ред. Не намираме 6-sparse циклични STS(v), $25 \leq v \leq 63$, но представяме някои примери с интересна структура.

Ключови думи: комбинаторен дизайн; Щайнерова система от тройки; конфигурация.