Problem. Prove that there are infinitely many positive integers n that couldn't be written as a sum of two positive integers that all of their prime divisors are less than 2020.

Problem. Let a_n be a sequence of distinct positive integers satisfying $a_n \leq cn^k$ for some positive integers n, k, prove that set of primes dividing $a_1, \ldots$ is infinite.

Problem. Prove that:

$$\varphi(2) + \cdots + \varphi(n) \geq 1 + \frac{n(n-1)}{4}.$$

Problem. Let $p \equiv 3 \pmod 4$, be a prime number and q be an integer that is not divisible by p. Prove that for each positive integer a, we have:

$$\sum_{k=1}^{p} \left\lfloor \frac{ak^2 + a}{p} \right\rfloor = 1 + 2a + \sum_{k=1}^{p} \left\lfloor \frac{ak^2 - a - 1}{p} \right\rfloor.$$

Problem- let $p, \frac{p-1}{2}$ being prime, prove that for $p \nmid abc$ there is at most $1 + \sqrt{2p}$ positive integer $n < p$, such that $p | a^n + b^n + c^n$.

Problem. Find all triples of integers (a, b, c) such that $a \neq 0, P(x) = ax^2 + bx + c$, satisfies the following condition; for each positive integer n, there exists a positive integer m such that:

$$P(n).P(n+1) = P(m).$$

Problem. Let k be a positive integer, we call a positive integer n, good if among $\binom{n}{0}, \ldots, \binom{n}{n}$, there are at least $0.99n$ numbers that are divisible by k. Prove that there is a positive integer N such that among $1, 2, \ldots, N$, there are at least $0.99N$ good numbers.

Problem- Let p being odd $p \equiv 1$ (mod 4) , $\frac{\varphi(p-1)}{p-1} > \frac{1}{3}$ prove that there is $g \leq p - 1$ which is primitive root mod p, $\gcd(g, p-1) = 1$.

Problem. Let $k > 1$ be an integer. Prove that for each positive integers x, y there exist (a, b) of integers such that $x^k + y^k \not\equiv b \pmod a$.

Problem- Find all m such that $a^{2015} + b^{2015}$ has at least $\frac{m}{5}$ different residues modulo m.

Problem- Let a, b be tow integers and n be a positive integer such that set

$$\mathbb{Z} - \{ax^n + by^n | x, y \in \mathbb{Z}\}$$

Is finite, prove $n = 1$.

Problem- Let p be a prime, find the number of $0 \leq a \leq p^2 - 1$, for which the equation $x^p + ay^p \equiv N \pmod{p^2}$ has solution.for all N.

Problem- Does there exist an infinite, non-constant arithmetic progressions, each term of which is of the form a^b where a and b are positive integers with $b \geq 2$?

Problem- Let p be a prime number, determine all m as a function of p such that, there are integers $a_1, \dots, a_p$ such that

$$a_1^p + \cdots + a_p^p \equiv m \pmod{p}.$$

Problem- Let $a > 1$ be an integer and $P(x)$ be a polynomial with integer coefficients and positive leading ceofficients. Lets define the set S as set of positive integers n such that $a^{P(n)} - 1$ divisible by n, prove that $\lim_{n\to\infty} \frac{|S \cap \{1,2,\dots,n\}|}{n} = 0$.

Florian Luca

Problem- Is the set of positive integers n such that $n! + 1$ divides $(2017n)!$ finite or infinite?

Fedor Petrov

Problem- Given a positive integer n, a sequence of integers $a_1, \dots, a_r$, where $0 \leq a_i \leq k$ for all $1 \leq i \leq r$, is said to be "*k-rep*" of n if there existe an integer c such that,

$$\sum_{i=1}^{r} a_i = \sum_{i=1}^{r} a_i k^{c-i} = n$$

Prove that every positive integer n has a *k-rep,* and that the *k-rep* is unique iff 0 does not appear in the base-k reperesentation of $n - 1$.

Problem- Let p be a prime positive integer. Define a mod $- p$ recurrence of degree n to be a sequence $\{a_k\}, k \geq 0$ of numbers modulo p satisfying a relation of the form $a_{i+n} = c_{n-1}a_{i+n-1} + \ldots + c_1 a_{i+1} + c_0 a_i$ for all $i \geq 0$, where $c_0, c_1, ..$are integers and $c_0 \not\equiv 0 (\text{mod } p)$. Compute the number of distinct linear recurrences of degree at most n in terms of p and n.

Problem- Let $t(n)$ denote the sum of digits in the binary reperesentation of a positive integer n, and let $k \geq 2$ be an integer.

i. Show that there is a sequence a_n of integers such that $a_m \geq 3$ is an integer and $t(a_1 a_2 \ldots a_m) = k$ for all m.
ii. Show that there is an integer N such that $t\big(3.5 \ldots . (2m+1)\big) > k$ for all $m \geq N$.

Problem- Is there a 2016-digit number, which by permuting the numbers of which you can get 2016 different 2016-digit full squares? (MA Evdokimov)

Problem- Does there exist an increasing sequence a_n of positive integers such that each positive integer occurs exactly once among the numbers $a_{n+1} - a_n$ and each positive integer not less than some number occurs exactly once among the numbers $a_{n+2} - a_n$?

Problem- Let $S = \{(a,b) | 0 < 2a < 2b < p, p | a^2 + b^2\}$, where $p \equiv 1 \ (\text{mod } 4)$. Prove that,

$$2 \sum_{(a,b) \in S} a = \sum_{(a,b) \in S} b$$

Problem- Let $S = \{0,1,2,..,n-1\}$ and $|S| > \frac{3n}{4}$, prove that there are a, b, c such that $a, b, c, a+b. a+c, c+a, a+b+c \ (\text{mod } n)$ belongs to S.

Problem- Let $\boldsymbol{Z}^+$ denote the set of all positive integers and $\boldsymbol{P}$ denote the set of all prime numbers. For subsets A and $S$of $\boldsymbol{Z}^+$, A is called $S -$ proper if there exists a positive integer N such that for all $a \in A$ and integer b with $0 \leq b < a$ there exist not necessarily distinct elements

$s_1, s_2, \ldots, s_n$ of S satisfying the conditions $b \equiv s_1 + s_2 + \ldots + s_n \ (mod\ a)$ and $1 \le n \le N$. Find a subset S of $\boldsymbol{Z}^+$ for which $\boldsymbol{P}$ is S - proper but $\boldsymbol{Z}^+$ is not.

Problem- Prove thre is a vertical needele of lentgh l, which, one can transfer it horizontally by passing no more than 2018 lattice points.

Problem- Let $n > 1$, which has k distinct prime divisores, prove that there is an integer $1 < a < 1 + \frac{n}{k}$ such that $a^2 - a$ is divisible by n.

Problem- Let $f(n)$ be number of divisors of $n^2 + n + 1$, prove that there are infinitely many n, such that $f(n) \ge f(n+1)$.

Problem- We call positive integers n, k similar if both of them being divisible by the square of same primes, let $f(n) = |\{1 \le k \le n,\ n, k \text{ are similar}\}|$. For example $f(16) = 4$. Find all positive integers n, such that $\frac{n}{f(n)}$ be an integer.

Problem- For a positive integer n, let $f(n)$ be the smallest natural number such that $nf(n)$ be square(for example $f(12) = 3$). Prove that for all k in the interval $[10^{2011}. 10^{2011} + 10^{1000}]$ all the values of $f(k)$ are different.

Problem- For a sequence $a_1, a_2, \ldots, a_n$ of positive integers that for allfifferent k, m we have

$$\gcd(|a_k - a_m|, |k - m|) < 2017$$

Find the maximal value of n.

Problem- A positive integer number N is *abundant* if the sum of its positive divisors (1 and N inclusive) exceeds $2N$. Given any positive integer number n, show that there are n consecutive abundant numbers.

Problem- Let $p \equiv 1 \pmod 4$, prove there is a prime number $q < \sqrt{p}$ such that

$$q^{p-1} \not\equiv 1(\text{mod } p^2)$$

Problem- Prove there is a real number C such that for any point (x, y) in Cartesian plane, there are $\gcd(m, n) = 1$ such that

$$\sqrt{(x-m)^2 + (y-m)^2} < C.\log(x^2 + y^2 + 2)$$

Problem- Find number of 100 —tuples $(x_1, .., x_{100})$ of $\{1,2, \dots ,2017\}$ such that

$$x_1 + .. + x_{100} \equiv x_1^2 + \cdots + x_{100}^2 \equiv 0 \ (\text{mod } 2017)$$

Problem- Let $m = 2^k t$ where t is an odd number, we define $f(m) = t^{1-k}$. Prove for all $a \leq n$, $f(1)f(2) \dots . f(n)$ is divisible by a.

Problem- For a given positive integer n and prime number p, find the minimum value of positive integer m that satisfies the following property: for any polynomial $Q(x) = (x + a_1) \dots (x + a_n)$ where $a_1, a_2, \dots a_n$ are positive integers, and for any non-negative integer k, there exists a non-negative integer k' such that

$$v_p\big(Q(k)\big) \leq v_p\big(Q(k')\big) \leq m + v_p\big(Q(k)\big)$$

Hint: $m = v_p(n!)$.

Problem- Let n be fixed positive integer, prove for any a, b, c of positive integers such that $a, b, c \leq 4n + 3n^2$, there are integers x, y, z such that $|x|, |y|, |z| < 2n$ and not all of them are zero and $ax + by + cz = 0$.

Problem- let $S \subseteq \{1,2, \dots , p-1\}$and $N_s$be number of solutions of the congruence $\sum_{i=1}^{q} x_i \equiv 0 \pmod p$Where $x_i \in S$and q be the other prime different from p , prove that $N_s$is divisible by q.

Problem- Let $p \geq 5$ be a prime number, prove there exist positive integers m, n where $m + n \leq \frac{p-1}{2}$ and $2^m 3^n - 1$ is divisible by p.

Problem- Let $m_1, \dots, m_{2017} > 1$ be 2017 pairwise relatively prime positive integers and $A_1, \dots, A_{2017}$ be 2017(possibly empty) sets with $A_i \subseteq \{1,2,..,m_i - 1\}$. Prove there is a positive integer N such that

$$N \le (1 + 2|A_1|) \dots (1 + 22|A_{2017}|)$$

And for all $1 \le i \le 2017$, there does not exist $a \in A_i$ such that $m_i | N - a$.

Problem- Let $p = 1 + 2^{2^k}$ for some positive integer k, be a prime number. Find number of pairs (m, n) of integers, where $0 \le m < n \le p - 1$. That,

$$2^n + 3^n \equiv 2^m + 3^m \pmod p$$

Variant:

Let $p > 13$ be a prime number of the form $2q + 1$ for some prime q. Find number of pairs (m, n) of integers, where $0 \le m < n \le p - 1$. That,

$$(-12)^n + 3^n \equiv (-12)^m + 3^m \pmod p$$