

Exam 01

Time: 3 hours.

Problem 1. Let $n \geq 1$ be a positive integer and $A = \{1, 2, \dots, n\}$; if B is a subset of A such that the greatest common divisor of elements of B is d . Find the smallest positive integer k such that for each subset C of A with exactly k elements for which $B \subseteq C$, we have the greatest common divisors of elements of C , is equal to 1.

Problem 2. Each of the squares in a $2 * 2018$ grid of a squares is to be colored black or white such that in any $2 * 2$ block, at least one of the 4 squares is white. Let P be the number of ways coloring the grid. Find the largest k so that 3^k divides P .

Problem 3. Unit square $ABCD$ is divided into 10^{12} smaller squares(not necessarily equal). Prove that the sum of perimeters of all the smaller squares having common points with diagonal AC does not exceed 1500.