

## Exam 02

**Time: 3 hours.**

**Problem 1.** A convex hexagon is given. Let  $s$  be the sum of the lengths of the three segments connecting the midpoints of its opposite sides. Prove that there is a point in the hexagon such that the sum of its distances to the lines containing the sides of hexagon does not exceed  $s$ .

**Problem 2.** Find all non-decreasing functions from  $\mathbb{R}$  onto  $\mathbb{R}$  such that:

$$f(x + f(y)) = f(x) + f(f(y)).$$

**Problem 3.** A group of persons is called good if its members can be distributed to several rooms so that nobody is acquainted with any person in the same room but it is possible to choose person from each room so that all the chosen persons are acquainted with each other. A group called perfect if it is good and every set of its members is also good. A perfect group planned a party. However one of its members- Lisaveta- brought her acquaintance -Sergey-, who was not originally expected, and introduced him to all of her other acquaintances. Prove that the new group is also perfect.