Digits and etc

Problem-1. Solve the following equations:

1. $n + S(n) + S\big(S(n)\big) = 1993$
2. $n + S(n) + S\big(S(n)\big) + S\left(S\big(S(n)\big)\right) = 1993$

Moskow-1993

Problem-2: Solve the equation: $n(S(n) - 1) = 2010$ Tzaloa-4-2010-p:68

Problem-3. Prove that for all positive integer , there exist, integer k such that $S(k) = n, S(k^2) = n^2$

Tzaloa-3-2009-p:37

Problem-4. Prove that for all positive integer , there exist, integer k such that $S(k) = n, S(k^2) = n^2, S(k^3) = n^3$.

Problem-5: Let $P(n)$ being product of nonzero digits of number n, which of the sequences $\frac{P(n)}{P(n^2)}, \frac{P(n^2)}{P(n)}$ could be bounded? (Ukrainian Math Olympiads)

Problem-6. Let $P(n)$ being product of all digits of an integer n, prove that the sequence $a_1 = a\ , a_{n+1} = a_n + P(a_n)$ is bounded. USM-43

Problem-7. Does there exist positive integers a, b such that the inequality $S(an) < S(bn)$ holds for all n? USM-78

Problem-8. Call the positive integer self-divisible if it is divisible by each sum of its consecutive digits, in particular it is divisible by each of its digits. Prove that the set of self-divisible integers is finite. USM-198

Problem-9. Prove that there exist 10 irrational numbers such that each of them is a root of quadratic equation with integer coefficients and for every $n \geq 1$ their n-th digits after decimal point are pairwise distinct.

USM-205

Problem-10. Denote by s(n) and k(n) the sum and the number of decimal digits of n. Find all the integers n such that $S(n^2).K(n) = n + 1$

USM-223

Problem-10. Denote by S(n) the sum of digits of integer n. The sequence (an) is defined as follows: $a_1 = 1, a_{n+1} = S(a_n + S(a_n))$, find the value of $a_1 + S(a_2) + S\big(S(a_3)\big) + \cdots. + S(S\big(\ldots.(a_{2015})\big)..)$, the number of last parenthesis is 2014.

USM-232

Problem-11: is the sequence $\frac{S(n)}{S(n^2)}$,is bounded?

USM-261

Problem-12. Find the minimum possible ratio of 5-digit number to the sum of its digits.

USM-322

Problem-13. A positive integer N is 999...99(number of 9digits is k) times greater than the sum of its digits. Find all possible values of k and give an example of such a number for each of these values.

Moskow-2001-B-5

Problem-14. The square of the sum of the digits of a number A is equal to the sum of the digits of A^2 . Find all two-digit numbers A with this property.

Moskow-2002-A-2

Problem-15; Find a ten-digit number without zero digits such that after adding the product of its digits to it, a number with the same product of digits is obtained.

Moskow-2003-A-2

Problem-16. 3. Prove that for any positive integer d there exists a positive integer n divisible by d such that it is possible to strike out a certain nonzero digit from its decimal notation so that the number thus obtained will also be divisible by d.

Moskow-2004-d-3

Problem-17. Dropping one of the digits of a certain positive integer leaves another number that divides the first. The dropped digit is not the leftmost one. What is the highest possible value for the first number, assuming it does not end in 0?

Moskow-94-b-5

Problem-18. Prove that there exists a positive integer that yields a com posite number whenever any three adjacent digits are replaced by arbitrary digits. Is there a 1997-digit number with this property?

Moskow-97-a-4

Problem-19. For all $n \geq 2$, define minimal k, by $f(n)$, such that there exist an n −element set S, such that for all nonempty subset X of S, sum of digits of elements of X, are equal to k, prove that there exist c_1, c_2 such that $c_1 logn \leq f(n) \leq c_2 logn$ (USAMO)

Problem-20.is the sequence $S(2^n)$ is bounded? (Komal)

Problem-21. We call n, nice if it hasn't any zero digits and sum of squares of its digits being square , prove that for all n, we have an n −digit nice number.

Problem-22. For a given positive integer *k* let *S*(*k*) denote the sum of all numbers from the set *{*1*,* 2*, . . . , k}* relatively prime to *k*. Let *m* be a positive integer and *n* an odd positive integer. Prove that there exist positive integers *x* and *y* such that *m* divides *x* and $2S(x) = y^n$

Croatia-2012

Problem-23. Prove that $\frac{S(2n)}{2} \leq S(n) \leq 5S(2n)$. Tzaloa-2-2011-35