Some Exemplify-Study Problems

Problem-1 . Prove that $n!$ has the property that sum of any two proper divisors of it , again are divisors of $n!$.

(St-Petersburg Math Olympiads)

Problem-2. If for all natural number m, n we know $am^2 + bn^2$ is square prove that $ab = 0$. (a,b are integers) (Russian Math Olympiads-2011)

Problem-3 find all integers x, y such that $x^6 + x^3y = y^3 + 2y^2$ (Belaurus-99)

Problem-4. Solve the equation $x^4 - 2x^3 + x = y^4 + 3y^2 + y$ (OM-66-1-P:4)

Problem-5. Prove that there exist an integer a, such that $\frac{a^{29}-1}{a-1}$ has more than, 2015 prime divisors. (Adapted After Brazilian Math Olympiads)

Problem-6. Is it possible to break all natural number greater than 2015 in two infinite set such that if x, y being in the same group then $x^2 + y$ is so. (St-Petersburg Math Olympiads)

Problem-7. Let a, b, c being integers such that $(b + c)^2 = (a + b)(a + c)$, prove that $(b - c)^2 > 8(b + c)$

(St-Petersburg Math Olympiads)

Problem-8.let for distinct integers $a_1, \dots, a_n$, which all is greater than unit we have $a_1 \dots . a_n | a$, prove that $(a + a_1 - 1) \dots . (a + a_n - 1)$ don't divide $a^{n+1} + a - 1$. (Romanian's Masters in Mathematics Shortlisted Problems)

Problem-9. Is there exist an n- element Set of integers that for all a, b in S , we have$(a - b)^2 | ab$?

(USA-Math Olympiads)

Problem=10. Find all integers x, y, z such that $\sqrt{\frac{2015}{x+y}} + \sqrt{\frac{2015}{y+z}} + \sqrt{\frac{2015}{z+x}}$ being integer. (Bulgarian Math Olympiads)

Problem-11. Doers there exist an Arithmetic Progression such that for all n , we have

$$a_n + a_{n+1} + \cdots + a_{n+9} | a_1 \dots . a_{n+9} \qquad \text{(Russia)}$$

Problem-12. We say that n, is good if absolute value of n wasn't square, find all integers m such that m could be represented in finitely many way as sum of three good integers such that theirs products is square of an odd number. (MOSP)

Problem-13. Prove that the equation $x^2 + y^2 + z^4 = 1$ has infinitely many rational solutions. (Kolmogorov Math Competitions)

Problem-14: let a being an integer , prove that there exist infinitely many prime p such that $n^2 + 3$ and $m^3 - a$ being divisible by p , for some integers m, n. (Czech-Polish –Slovak)

Problem-15. We know about the sequence a_n such that $a_1 = 1$ and $a_i | a_{i+1}$, prove that for any positive integer N, there exist integers $k_1, \dots . k_s$, where $0 \le k_j \le \frac{a_{j+1}}{a_j} - 1$ and also $k_s > 0$, such that N is uniquely represented as $\sum_{j=1}^{s} k_j a_j$. (Chinese Journal-High School Math)

Problem-16. in set of consecutive integers there exist exactly 100perfect cubes and 10 fourth power of integer, prove there are at least 200 squares. (Tuymada Olympiad)

Problem-17. Find all primes p, q such that $2p - 1, 2q - 1, 2pq - 1$ are all square. (St-Petersburg Olympiads)

Problem-18. Find all integers x, y such that $y(x^2 + 36) + x(y^2 - 36) + x^2(y - 12) = 0$ (Belaueus)

Problem-19. Find the least possible value of k such that there exist 2010 distinct natural numbers such that product of any k numbers chosen is divisible by product of $2010 - k$ others. – Ukr-09-10-p24

Problem-20. Does there exist an increasing sequence of integers $a_0 = 0 < a_1 < \cdots$ such that every natural number could be represented in the form $a_i + a_j$ and (for some i, j which may be equal) and $a_n \ge \frac{n^2}{16}$. Ukr-12-13.p:45

Problem-21. i.Are there exist positive integers $a_1, \dots, a_{2015}$, such that any two of them being coprime and $a_1 \dots . a_{2015} - 1$ being the product of two consecutive odd numbers?

ii. determine whether there exist positive integers $a_1, \dots, a_{2015}$, such that any two of them being coprime and $a_1 \dots . a_{2015} - 1$ being the product of two consecutive even numbers? Ukr-2015-p:48

Problem-22. Are there integers *a* and *b* such that both *a* + *b* and *ab* −1 are perfect squares?

Croatia-2012

Problem-23.We know about primes p, q such that $\frac{p}{p+1} + \frac{q+1}{q} = \frac{2n}{n+2}$, find all possible value of $q - p$.

Tzaloa-4-2012-40

Problem-24. Prove that for any positive integer *k*, there exists an arithmetic sequence$\frac{a_1}{b_1}, \dots \frac{a_k}{b_k}$, of rational numbers, where $a_i, b_i$are relatively prime positive integers for each *i* = 1, 2, . . . , *k*,
such that the positive integers $a_1, b_1, a_2, b_2, \dots, a_k, b_k$ are all distinct.

APMO-2009

Problem-25- we say a positive integer n, is good if any of its positive divisors add up to unit , being divisor of $n + 1$, find all good numbers.

Russia-14-11.5

Problem-26. Find all integers k, such that for all odd $n > 100$, the number $20^n + 13^n$ being divisible by k.

Russia-2013. P48

Problem-27. Prove that for all positive integer a, b with different parity and difference greater than unit , there exist a positive integer , such that $c + ab, c + a, c + b$ being square.

Petersburg-94-88

Problem-28. Does there exist an infinite sequence of positive integers such that for every positive integer k, the sum of every k consecutive terms of this sequence is divisible by k + 1?

Russia-2015-10.1