Problem-1: Let m be a positive integer and let p be a prime divisor of m. Suppose that the complex polynomial $a_0 + \cdots. + a_n x^n$ with $n < \frac{p}{p-1}\varphi(m), a_n \neq 0$ is divisible by the cyclotomic polynomial $\phi_m(x)$ Prove that there are at least p nonzero coefficients a_i . Vjmc-25

Problem-2: Let $n \geq 1$ be a fixed integer. Calculate the distance

$$\mathrm{Inf}_{\mathrm{P,f}} \max_{0 \leq x \leq 1} |\mathrm{f(x)} - \mathrm{P(x)}|$$

where p runs over polynomials of degree less than n with real coefficients and f runs over functions $f(x) = \sum_{k=n}^{\infty} c_k x^k$, defined on the closed interval $[0, 1]$, where $c_k \geq 0$, $\sum_{k=n}^{\infty} c_k = 1$. Miklows Schwitzer-2014

Problem-3: For $n \geq 1$ write $f(x) = x^n + x^{n-1} + \cdots. + x^2 + x + 1$ For which positive integer n can one find polynomials $g(x), h(x)$ with real coefficients and degree less than n such that $f(x) = g(h(x))$ holds? Miklows Schwitzer-2015

Problem-4: Let d be a fixed integer, at least 2. If $P(x)$ is a polynomial in x, let $\lceil P(x) \rceil$ be the polynomial obtained by rounding up each exponent of x to the nearest multiple of d, so that $\lceil P(x) \rceil$ is a polynomial inx^d. For example, if $d = 3$ then:

$$\lceil 2 + 5x^2 + 4x^3 + x^4 \rceil = 2 + 5x^3 + 4x^3 + x^6 = 2 + 9x^3 + x^6$$

Suppose that all we know about $P(x)$ is that it has nonnegative real coefficients. Show that if we are given all of the polynomials $\lceil P(x) \rceil, \lceil P(x)^3 \rceil, \lceil P(x)^4 \rceil, \ldots,$ we can determine $P(x)$. Sydney-2015

Problem-5: Find a monic polynomial $f(x)$ with integer coefficients, and degree at most three, such that there exist non-constant polynomials $g(x)$ and $h(x)$ with integer coefficients for which:

$$f(x)^3 - 2 = g(x).h(x)$$

Problem-6: Let $m > 1$ be an integer an $P(x)$ be a polynomial of degree d , prove that there exist an integer n such that $P(0), \ldots., P(n-1)$ could be divided in m sets , which sum of each set are equal.

Based On Ukrainian TST

Problem-7: Let $P(x)$ be a polynomial of degree at most , such that for all $0 < x \leq 1$, we have $|P(x)| \leq \frac{1}{\sqrt{x}}$, prove that $|P(0)| \leq 2d + 1$. Komal

Problem-8: find all polynomials $P(x)$ such that $P(x^3 - 2) = P(x)^3 - 2$. CIIM-2015

Problem-9: We say polynomial $x^3 + px + q$ with integer coefficients irrational if it has three different real but irrational roots $\alpha_1, \alpha_2, \alpha_3$, find all irrational polynomials such that the value of $|\alpha_1| + |\alpha_2| + |\alpha_3|$ be Minimal. Blr-2013

Problem-10: Let z denote a complex number of absolute value 1. Prove that there exists polynomial of degree d, all the coefficients of which are +1 or -1, such that $|P(z)| \leq 4$. Komal N.81-1997-1-p:19

Problem-11: Show that there are infinitely many natural numbers n or which .there exits a polynomial of degree n with the following propertles: Its coefficients are integers, its leading coefficients are less than 3^n and it has n distinct roots in the interval $(0,1)$. Komal N.82

Problem-12: Prove that for every positive integer n, there exists a polynomial $P(x)$ of degree not greater than $100n$ satisfying the inequality : $|P(0)| > |P(1)| + \cdots. + |P(n^2)|$ Komal N.91- 1997-2 p15

Problem-13: A polynomial $P(x)$ has degree at most $n - 10\sqrt{n}$. Prove that:

$$|P(0)| \leq \frac{1}{10}\left(\binom{n}{1}|P(1)| + \cdots. + \binom{n}{n}|P(n)|\right)$$ Komal. A190-1999-5

Problem-14: Prove that, for any sufficiently large integer n, there exists a polynomial $P(x)$ of degree at most $n - \frac{1}{10}\sqrt{n}$, such that : Ibid

$$|P(0)| > \frac{1}{10}\left(\binom{n}{1}|P(1)| + \cdots. + \binom{n}{n}|P(n)|\right)$$

Problem-15: The coefficients of a polynomial p are integers whose absolute values are not greater than 2014. Given that $P(2016)$ is a prime number, prove that p cannot be written as a product of two polynomials, each of positive degree and with integer coefficients. Komal. N.161 .PP77

Problem-16: For polynomial $f(x)$ with real coefficients of degree 2012 and positive leading coefficient , consider the region composed of all points with coordinates (x,y) for which $y \geq f(x)$. Could plane be covered by a finite number of such regions? Bulgarian TST-2012

Problem-17: Let f, g being polynomials with rational coefficients such that $(\mathbb{Q}) = g(\mathbb{Q})$, prove that there are rationals $a \neq 0$, b such that $f(x) = g(ax + b)$.

Problem-18: Let $k \geq 10$ be an integer such that among any $k$consecutive positive integers there exists a number relatively prime to the others. Prove that the product of$k$ consecutive positive integers can not be accurate m-th degree with $m \geq k + 2$ Russian TST

Problem-19: We know about polynomial $f(x)$ of degree , such that for all integers $0 \leq a < b \leq n$, the expression $\frac{f(a)-f(b)}{a-b}$ is an integer , prove that it is integer for all integers a, b. Fedor Petrov

Problem-20: Given a polynomial $p\ (x)$ with integer coefficients and a complex number w such that $lwl\ =\ 1$. Let $p\ (w)\ =\ c$, where $c$is a real number. Prove that there exists polynomial $q\ (x)$ with integer coefficients such That $c = q(w + \frac{1}{w})$ Russian Festival for young mathematicians

Problem-21: Prove that there is a nonzero multiple of $(x-1)^n$ of degree n^2 , with integer coefficients whose absolute values does not exceed n^2. Russian Festival for young mathematicians

see also komal 97-1. P:18