Gerd Faltings: From the Mordell Conjecture to the Abel Prize

Alexandros Konstantinou

ICMS, Bulgarian Academy of Sciences

National Mathematics Colloquium

June 12, 2026

Who is Gerd Faltings?

1986 Fields Medal

2026 Abel Prize

Who is Gerd Faltings?

"A towering figure in arithmetic geometry." – Abel Prize Committee

1986 Fields Medal

2026 Abel Prize

Who is Gerd Faltings?

> *"A towering figure in arithmetic geometry."* – Abel Prize Committee

- German mathematician, born in 1954.

1986 Fields Medal

2026 Abel Prize

Who is Gerd Faltings?

> *"A towering figure in arithmetic geometry."* – Abel Prize Committee

- German mathematician, born in 1954.
- PhD from the University of Münster in 1978.

1986 Fields Medal

2026 Abel Prize

Who is Gerd Faltings?

> *"A towering figure in arithmetic geometry."* – Abel Prize Committee

- German mathematician, born in 1954.
- PhD from the University of Münster in 1978.
- Became professor at Wuppertal at age 28, later worked at Princeton, and then became Director at the Max Planck Institute for Mathematics in Bonn.

1986 Fields Medal

2026 Abel Prize

Who is Gerd Faltings?

> *"A towering figure in arithmetic geometry."* – Abel Prize Committee

- German mathematician, born in 1954.
- PhD from the University of Münster in 1978.
- Became professor at Wuppertal at age 28, later worked at Princeton, and then became Director at the Max Planck Institute for Mathematics in Bonn.
- In 1983, proved the Mordell conjecture, one of the major open problems in number theory.

1986 Fields Medal

2026 Abel Prize

Who is Gerd Faltings?

> *"A towering figure in arithmetic geometry."* – Abel Prize Committee

- German mathematician, born in 1954.
- PhD from the University of Münster in 1978.
- Became professor at Wuppertal at age 28, later worked at Princeton, and then became Director at the Max Planck Institute for Mathematics in Bonn.
- In 1983, proved the Mordell conjecture, one of the major open problems in number theory.
- Awarded the **Fields Medal** in 1986 and the **Abel Prize** in 2026.

1986 Fields Medal

2026 Abel Prize

Who is Gerd Faltings?

> *"A towering figure in arithmetic geometry."* – Abel Prize Committee

- German mathematician, born in 1954.
- PhD from the University of Münster in 1978.
- Became professor at Wuppertal at age 28, later worked at Princeton, and then became Director at the Max Planck Institute for Mathematics in Bonn.
- In 1983, proved the Mordell conjecture, one of the major open problems in number theory.
- Awarded the **Fields Medal** in 1986 and the **Abel Prize** in 2026.

Mordell's conjecture (1922); Faltings' theorem (1983)

Let C be a smooth projective curve over $\mathbb{Q}$ of genus at least 2. Then $C(\mathbb{Q})$ is finite.

1986 Fields Medal

2026 Abel Prize

Who is Gerd Faltings?

> *"A towering figure in arithmetic geometry."* – Abel Prize Committee

- German mathematician, born in 1954.
- PhD from the University of Münster in 1978.
- Became professor at Wuppertal at age 28, later worked at Princeton, and then became Director at the Max Planck Institute for Mathematics in Bonn.
- In 1983, proved the Mordell conjecture, one of the major open problems in number theory.
- Awarded the **Fields Medal** in 1986 and the **Abel Prize** in 2026.

Mordell's conjecture (1922); Faltings' theorem (1983)

Let C be a smooth projective curve over $\mathbb{Q}$ of genus at least 2. Then $C(\mathbb{Q})$ is finite.

- Away from mathematics, he is known to enjoy opera, gardening, and collecting fine wines.

1986 Fields Medal

2026 Abel Prize

Who is Gerd Faltings?

> *"A towering figure in arithmetic geometry."* – Abel Prize Committee

- German mathematician, born in 1954.
- PhD from the University of Münster in 1978.
- Became professor at Wuppertal at age 28, later worked at Princeton, and then became Director at the Max Planck Institute for Mathematics in Bonn.
- In 1983, proved the Mordell conjecture, one of the major open problems in number theory.
- Awarded the **Fields Medal** in 1986 and the **Abel Prize** in 2026.

Mordell's conjecture (1922); Faltings' theorem (1983)

Let C be a smooth projective curve over $\mathbb{Q}$ of genus at least 2. Then $C(\mathbb{Q})$ is finite.

- Away from mathematics, he is known to enjoy opera, gardening, and collecting fine wines.

1986 Fields Medal

2026 Abel Prize

Areas of work

Arithmetic geometry,
Diophantine geometry,
abelian varieties,
moduli spaces,
p-adic Hodge theory.

Diophantine equations viewed geometrically

Faltings' work lies mainly in **arithmetic geometry**.

Diophantine equations viewed geometrically

Faltings' work lies mainly in **arithmetic geometry**.

> **Geometry:** *The area of mathematics relating to the study of space and the relationships between points, lines and curves.* -Cambridge Dictionary

Diophantine equations viewed geometrically

Faltings' work lies mainly in **arithmetic geometry**.

> **Geometry:** *The area of mathematics relating to the study of space and the relationships between points, lines and curves.* -Cambridge Dictionary

In **arithmetic geometry**, the geometric objects come from polynomial equations:

$$p(x_1, \ldots, x_n) = 0$$

Diophantine equations viewed geometrically

Faltings' work lies mainly in **arithmetic geometry**.

> **Geometry:** *The area of mathematics relating to the study of space and the relationships between points, lines and curves.* -Cambridge Dictionary

In **arithmetic geometry**, the geometric objects come from polynomial equations:

$$p(x_1, \ldots, x_n) = 0$$

and the arithmetic questions concern their **integer** or **rational** points.

Diophantine equations viewed geometrically

Faltings' work lies mainly in **arithmetic geometry**.

> **Geometry:** *The area of mathematics relating to the study of space and the relationships between points, lines and curves.* -Cambridge Dictionary

In **arithmetic geometry**, the geometric objects come from polynomial equations:

$$p(x_1, \ldots, x_n) = 0$$

and the arithmetic questions concern their **integer** or **rational** points.

Guiding questions

(1) Are there integer or rational solutions?

Diophantine equations viewed geometrically

Faltings' work lies mainly in **arithmetic geometry**.

> **Geometry:** *The area of mathematics relating to the study of space and the relationships between points, lines and curves.* -Cambridge Dictionary

In **arithmetic geometry**, the geometric objects come from polynomial equations:

$$p(x_1, \ldots, x_n) = 0$$

and the arithmetic questions concern their **integer** or **rational** points.

Guiding questions

(1) Are there integer or rational solutions? **(2)** How many are there?

Diophantine equations viewed geometrically

Faltings' work lies mainly in **arithmetic geometry**.

> **Geometry:** *The area of mathematics relating to the study of space and the relationships between points, lines and curves.* -Cambridge Dictionary

In **arithmetic geometry**, the geometric objects come from polynomial equations:

$$p(x_1, \ldots, x_n) = 0$$

and the arithmetic questions concern their **integer** or **rational** points.

Guiding questions

(1) Are there integer or rational solutions? **(2)** How many are there? **(3)** What are they?

Diophantine equations viewed geometrically

Faltings' work lies mainly in **arithmetic geometry**.

> **Geometry:** *The area of mathematics relating to the study of space and the relationships between points, lines and curves.* -Cambridge Dictionary

In **arithmetic geometry**, the geometric objects come from polynomial equations:

$$p(x_1, \ldots, x_n) = 0$$

and the arithmetic questions concern their **integer** or **rational** points.

Guiding questions

(1) Are there integer or rational solutions? **(2)** How many are there? **(3)** What are they?

Examples

1. $3x^3 - 2xy - y^2z - 7 = 0$ has the integral solution $(1, 2, -2)$.
2. $x^2 + y^2 + 1 = 0$ has no integral solution.

Diophantine equations viewed geometrically

Faltings' work lies mainly in **arithmetic geometry**.

> **Geometry:** *The area of mathematics relating to the study of space and the relationships between points, lines and curves.* -Cambridge Dictionary

In **arithmetic geometry**, the geometric objects come from polynomial equations:

$$p(x_1, \ldots, x_n) = 0$$

and the arithmetic questions concern their **integer** or **rational** points.

Guiding questions

(1) Are there integer or rational solutions? **(2)** How many are there? **(3)** What are they?

Examples

1. $3x^3 - 2xy - y^2z - 7 = 0$ has the integral solution $(1, 2, -2)$.
2. $x^2 + y^2 + 1 = 0$ has no integral solution.

A fundamental difficulty: Hilbert's tenth problem

By work of DPRM (1970), there is no algorithm which decides, for an arbitrary polynomial $p(x_1, \ldots, x_n) \in \mathbb{Z}[x_1, \ldots, x_n]$, whether the equation $p(x_1, \ldots, x_n) = 0$ has a solution in $\mathbb{Z}^n$.

The geometry of a curve

The general Diophantine problem is undecidable, so one restricts to special classes of equations.

The geometry of a curve

The general Diophantine problem is undecidable, so one restricts to special classes of equations.

The first major case is that of **curves**

The geometry of a curve

The general Diophantine problem is undecidable, so one restricts to special classes of equations.

The first major case is that of **curves**: roughly speaking, equations defining one-dimensional geometric objects.

The geometry of a curve

The general Diophantine problem is undecidable, so one restricts to special classes of equations.

The first major case is that of **curves**: roughly speaking, equations defining one-dimensional geometric objects.

Over the complex numbers, a smooth projective curve can be viewed as a closed orientable surface.

The geometry of a curve

The general Diophantine problem is undecidable, so one restricts to special classes of equations.

The first major case is that of **curves**: roughly speaking, equations defining one-dimensional geometric objects.

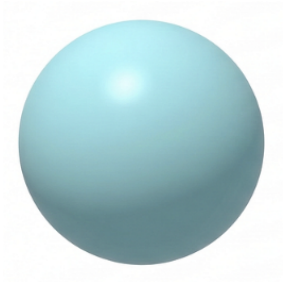
Over the complex numbers, a smooth projective curve can be viewed as a closed orientable surface. Its basic topological invariant is its **genus**: the number of holes.

The geometry of a curve

The general Diophantine problem is undecidable, so one restricts to special classes of equations.

The first major case is that of **curves**: roughly speaking, equations defining one-dimensional geometric objects.

Over the complex numbers, a smooth projective curve can be viewed as a closed orientable surface. Its basic topological invariant is its **genus**: the number of holes.



genus 0
conics

genus 1
elliptic curves

genus 2
hyperelliptic curves

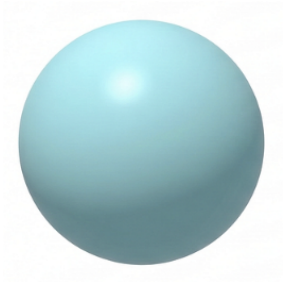
genus 3
higher genus

The geometry of a curve

The general Diophantine problem is undecidable, so one restricts to special classes of equations.

The first major case is that of **curves**: roughly speaking, equations defining one-dimensional geometric objects.

Over the complex numbers, a smooth projective curve can be viewed as a closed orientable surface. Its basic topological invariant is its **genus**: the number of holes.



genus 0
conics

genus 1
elliptic curves

genus 2
hyperelliptic curves

genus 3
higher genus

Guiding principle

For curves, the arithmetic of rational points is strongly governed by the genus.

Genus zero curves: conics

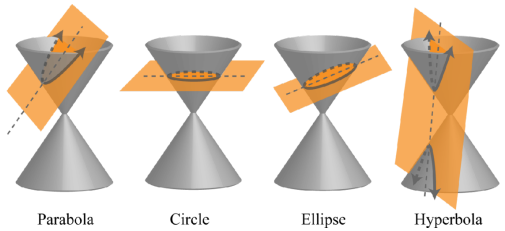
The first case is genus 0.

Genus zero curves: conics

The first case is genus 0. These curves are **smooth conics**.

Genus zero curves: conics

The first case is genus 0. These curves are **smooth conics**.



Conic sections

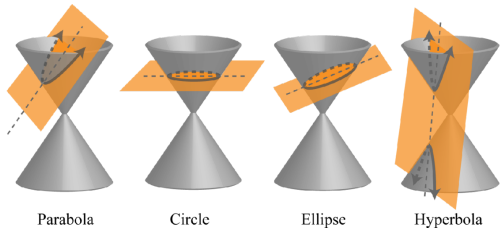
Genus zero curves: conics

The first case is genus 0. These curves are **smooth conics**.

These are given by a homogeneous equation of degree 2:

$$C: \ Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz = 0 \subset \mathbb{P}^2,$$

with $A, B, C, D, E, F \in \mathbb{Q}$.



Conic sections

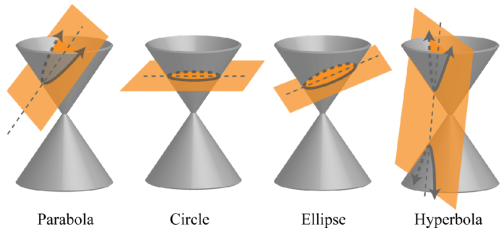
Genus zero curves: conics

The first case is genus 0. These curves are **smooth conics**.

These are given by a homogeneous equation of degree 2:

$$C: \ Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz = 0 \subset \mathbb{P}^2,$$

with $A, B, C, D, E, F \in \mathbb{Q}$.



Conic sections

A classical example

The Pythagorean equation $x^2 + y^2 = z^2$ defines a genus 0 curve.

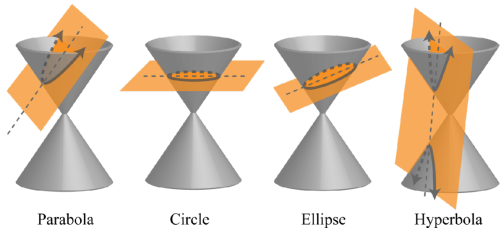
Genus zero curves: conics

The first case is genus 0. These curves are **smooth conics**.

These are given by a homogeneous equation of degree 2:

$$C: \ Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz = 0 \subset \mathbb{P}^2,$$

with $A, B, C, D, E, F \in \mathbb{Q}$.



Conic sections

A classical example

The Pythagorean equation $x^2 + y^2 = z^2$ defines a genus 0 curve. It has infinitely many rational points, corresponding to infinitely many Pythagorean triples.

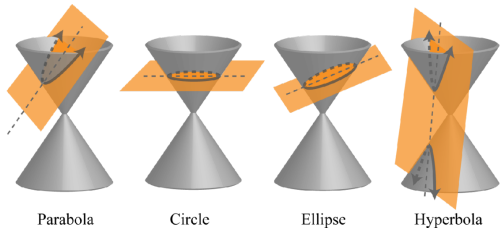
Genus zero curves: conics

The first case is genus 0. These curves are **smooth conics**.

These are given by a homogeneous equation of degree 2:

$$C: \ Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz = 0 \subset \mathbb{P}^2,$$

with $A, B, C, D, E, F \in \mathbb{Q}$.



Conic sections

A classical example

The Pythagorean equation $x^2 + y^2 = z^2$ defines a genus 0 curve. It has infinitely many rational points, corresponding to infinitely many Pythagorean triples.

The decisive point

We write $C(\mathbb{Q})$ to denote the set of rational points on a smooth conic C.

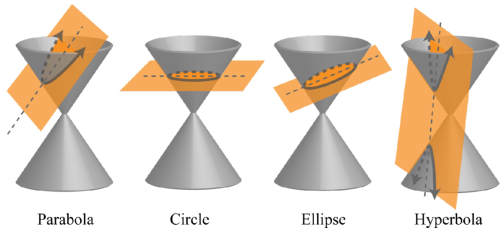
Genus zero curves: conics

The first case is genus 0. These curves are **smooth conics**.

These are given by a homogeneous equation of degree 2:

$$C: \ Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz = 0 \subset \mathbb{P}^2,$$

with $A, B, C, D, E, F \in \mathbb{Q}$.



Conic sections

A classical example

The Pythagorean equation $x^2 + y^2 = z^2$ defines a genus 0 curve. It has infinitely many rational points, corresponding to infinitely many Pythagorean triples.

The decisive point

We write $C(\mathbb{Q})$ to denote the set of rational points on a smooth conic C. This satisfies

$$C(\mathbb{Q}) = \emptyset \quad \text{or} \quad C(\mathbb{Q}) \text{ is infinite.}$$

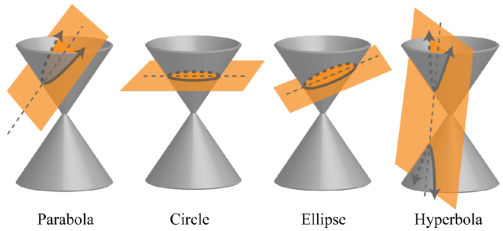
Genus zero curves: conics

The first case is genus 0. These curves are **smooth conics**.

These are given by a homogeneous equation of degree 2:

$$C: \ Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz = 0 \subset \mathbb{P}^2,$$

with $A, B, C, D, E, F \in \mathbb{Q}$.



Conic sections

A classical example

The Pythagorean equation $x^2 + y^2 = z^2$ defines a genus 0 curve. It has infinitely many rational points, corresponding to infinitely many Pythagorean triples.

The decisive point

We write $C(\mathbb{Q})$ to denote the set of rational points on a smooth conic C. This satisfies

$$C(\mathbb{Q}) = \emptyset \quad \text{or} \quad C(\mathbb{Q}) \text{ is infinite.}$$

Once we know one rational point on C, every other rational point can be obtained by drawing rational lines through it.

Conics: parametrisation

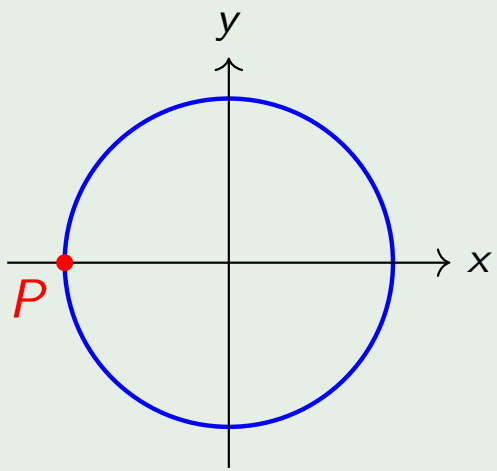
One rational point gives a parametrisation on a genus zero curve.

Conics: parametrisation

One rational point gives a parametrisation on a genus zero curve.

Example

For $x^2 + y^2 = 1$, take $P = (-1, 0)$ and $\ell_t : y = t(x + 1)$.

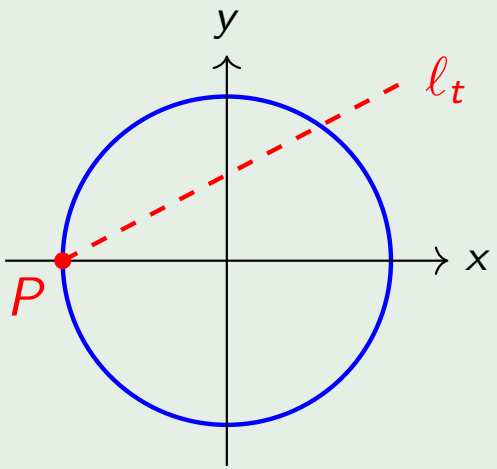


Conics: parametrisation

One rational point gives a parametrisation on a genus zero curve.

Example

For $x^2 + y^2 = 1$, take $P = (-1, 0)$ and $\ell_t : y = t(x + 1)$.

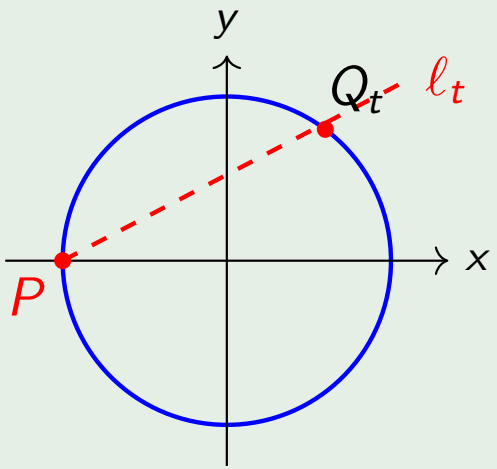


Conics: parametrisation

One rational point gives a parametrisation on a genus zero curve.

Example

For $x^2 + y^2 = 1$, take $P = (-1, 0)$ and $\ell_t : y = t(x + 1)$.

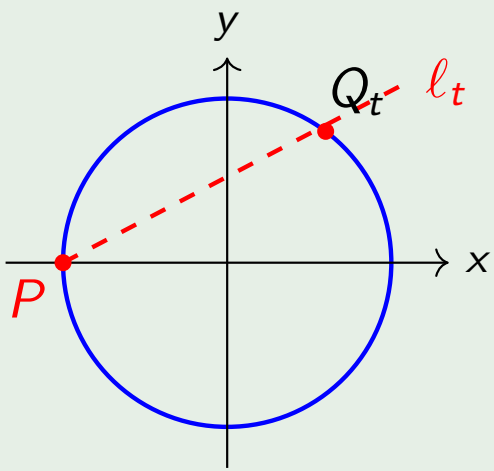


Conics: parametrisation

One rational point gives a parametrisation on a genus zero curve.

Example

For $x^2 + y^2 = 1$, take $P = (-1, 0)$ and $\ell_t : y = t(x + 1)$.



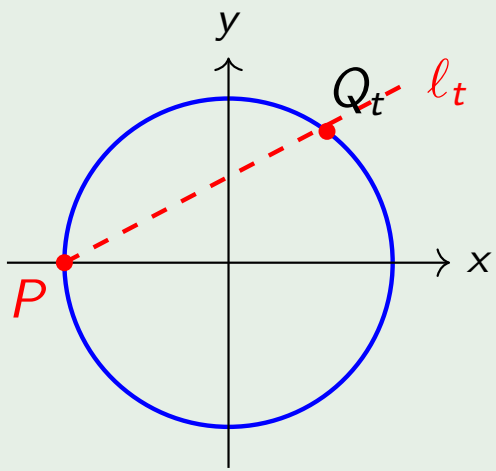
$$Q_t = \left(\frac{1 - t^2}{1 + t^2}, \frac{2t}{1 + t^2} \right)$$

Conics: parametrisation

One rational point gives a parametrisation on a genus zero curve.

Example

For $x^2 + y^2 = 1$, take $P = (-1, 0)$ and $\ell_t : y = t(x + 1)$.



$$Q_t = \left(\frac{1 - t^2}{1 + t^2}, \frac{2t}{1 + t^2}\right)$$

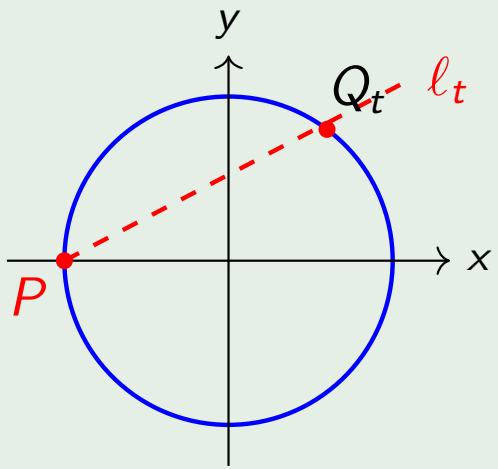
$$Q_0 = (1, 0), \quad Q_1 = (0, 1)$$

Conics: parametrisation

One rational point gives a parametrisation on a genus zero curve.

Example

For $x^2 + y^2 = 1$, take $P = (-1, 0)$ and $\ell_t : y = t(x + 1)$.



$$Q_t = \left(\frac{1 - t^2}{1 + t^2}, \frac{2t}{1 + t^2}\right)$$

$$Q_0 = (1, 0), \quad Q_1 = (0, 1)$$

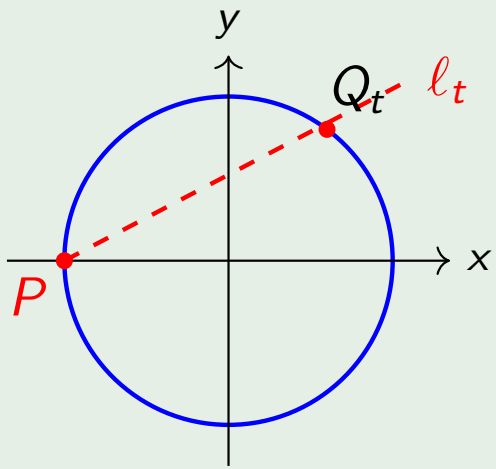
$$Q_{5/12} = \left(\frac{119}{169}, \frac{120}{169}\right)$$

Conics: parametrisation

One rational point gives a parametrisation on a genus zero curve.

Example

For $x^2 + y^2 = 1$, take $P = (-1, 0)$ and $\ell_t : y = t(x + 1)$.



$$Q_t = \left(\frac{1 - t^2}{1 + t^2}, \frac{2t}{1 + t^2}\right)$$
$$Q_0 = (1, 0), \quad Q_1 = (0, 1)$$
$$Q_{5/12} = \left(\frac{119}{169}, \frac{120}{169}\right)$$

Existence of the first point (Hasse–Minkowski)

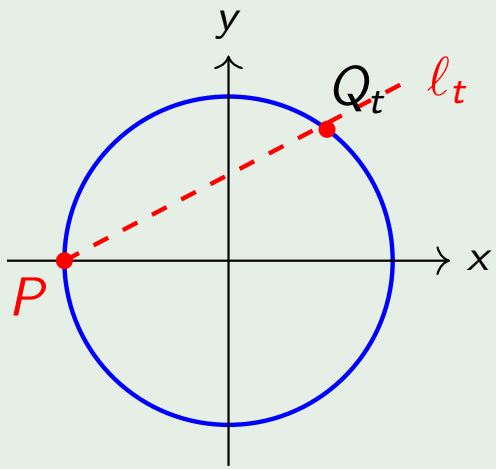
$C(\mathbb{Q}) \neq \emptyset \iff C(\mathbb{R}) \neq \emptyset$ and $C(\mathbb{Q}_p) \neq \emptyset$ for all primes p.

Conics: parametrisation

One rational point gives a parametrisation on a genus zero curve.

Example

For $x^2 + y^2 = 1$, take $P = (-1, 0)$ and $\ell_t : y = t(x + 1)$.



$$Q_t = \left(\frac{1 - t^2}{1 + t^2}, \frac{2t}{1 + t^2}\right)$$
$$Q_0 = (1, 0), \quad Q_1 = (0, 1)$$
$$Q_{5/12} = \left(\frac{119}{169}, \frac{120}{169}\right)$$

Existence of the first point (Hasse–Minkowski)

$$C(\mathbb{Q}) \neq \emptyset \iff C(\mathbb{R}) \neq \emptyset \text{ and } C(\mathbb{Q}_p) \neq \emptyset \text{ for all primes } p.$$

In practice, only finitely many primes need to be checked.

Genus one curves: elliptic curves

A genus one curve with a rational point is called an **elliptic curve**.

Genus one curves: elliptic curves

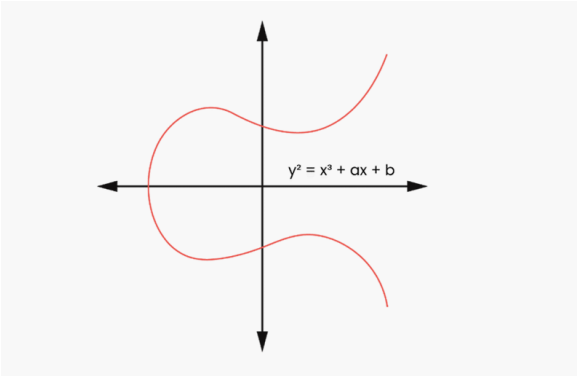
A genus one curve with a rational point is called an **elliptic curve**. It can be written in the form

$$E: \ y^2 = x^3 + Ax + B, \qquad A, B \in \mathbb{Q}.$$

Genus one curves: elliptic curves

A genus one curve with a rational point is called an **elliptic curve**. It can be written in the form

$$E: \ y^2 = x^3 + Ax + B, \qquad A, B \in \mathbb{Q}.$$

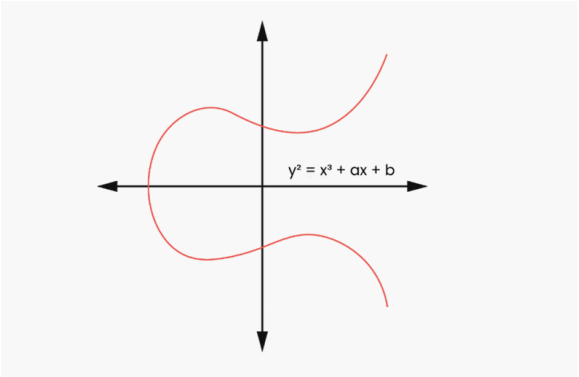


Genus one curves: elliptic curves

A genus one curve with a rational point is called an **elliptic curve**. It can be written in the form

$$E: \ y^2 = x^3 + Ax + B, \qquad A, B \in \mathbb{Q}.$$

Unlike conics, rational points on elliptic curves are not parametrised by one parameter.

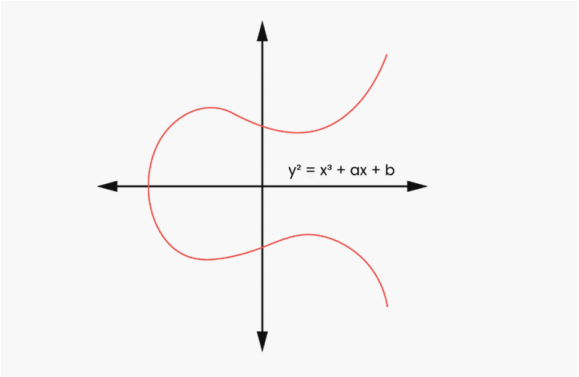


Genus one curves: elliptic curves

A genus one curve with a rational point is called an **elliptic curve**. It can be written in the form

$$E: \ y^2 = x^3 + Ax + B, \qquad A, B \in \mathbb{Q}.$$

Unlike conics, rational points on elliptic curves are not parametrised by one parameter.



Poincaré (1901)

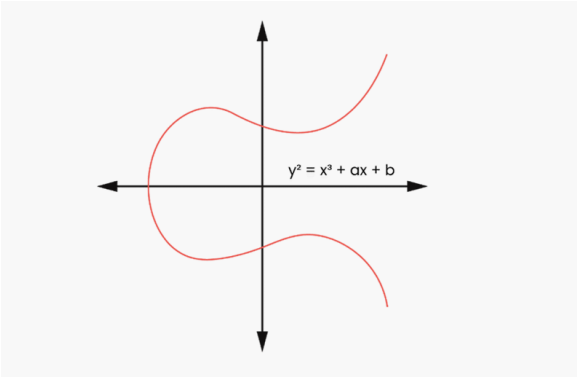
The rational points $E(\mathbb{Q})$ form an abelian group.

Genus one curves: elliptic curves

A genus one curve with a rational point is called an **elliptic curve**. It can be written in the form

$$E: \ y^2 = x^3 + Ax + B, \qquad A, B \in \mathbb{Q}.$$

Unlike conics, rational points on elliptic curves are not parametrised by one parameter.



Poincaré (1901)

The rational points $E(\mathbb{Q})$ form an abelian group.

Mordell's theorem (1922)

The group $E(\mathbb{Q})$ is finitely generated:

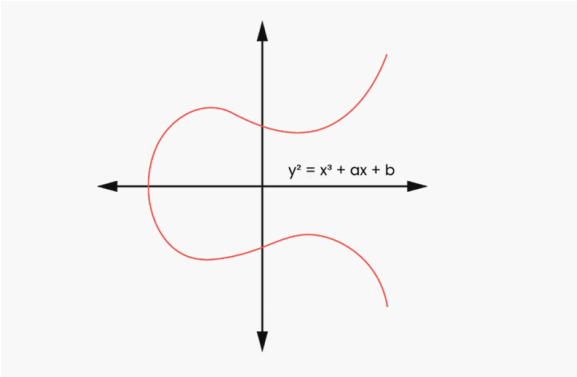
$$E(\mathbb{Q}) \simeq \mathbb{Z}^r \times (\text{finite group})$$

Genus one curves: elliptic curves

A genus one curve with a rational point is called an **elliptic curve**. It can be written in the form

$$E: \ y^2 = x^3 + Ax + B, \qquad A, B \in \mathbb{Q}.$$

Unlike conics, rational points on elliptic curves are not parametrised by one parameter.



Poincaré (1901)

The rational points $E(\mathbb{Q})$ form an abelian group.

Mordell's theorem (1922)

The group $E(\mathbb{Q})$ is finitely generated:

$$E(\mathbb{Q}) \simeq \mathbb{Z}^r \times (\text{finite group})$$

The new difficulty

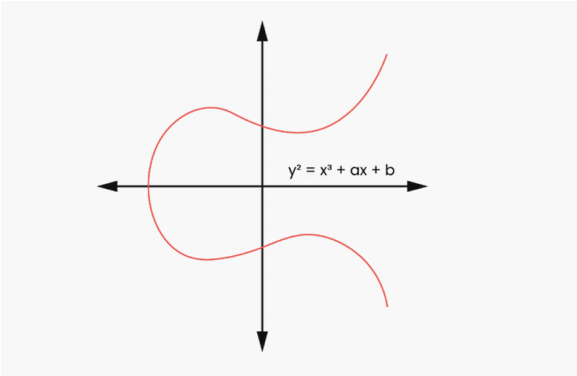
The integer r is the **rank**. It measures the size of the non-torsion part of $E(\mathbb{Q})$.

Genus one curves: elliptic curves

A genus one curve with a rational point is called an **elliptic curve**. It can be written in the form

$$E: \ y^2 = x^3 + Ax + B, \qquad A, B \in \mathbb{Q}.$$

Unlike conics, rational points on elliptic curves are not parametrised by one parameter.



Poincaré (1901)

The rational points $E(\mathbb{Q})$ form an abelian group.

Mordell's theorem (1922)

The group $E(\mathbb{Q})$ is finitely generated:

$$E(\mathbb{Q}) \simeq \mathbb{Z}^r \times (\text{finite group})$$

The new difficulty

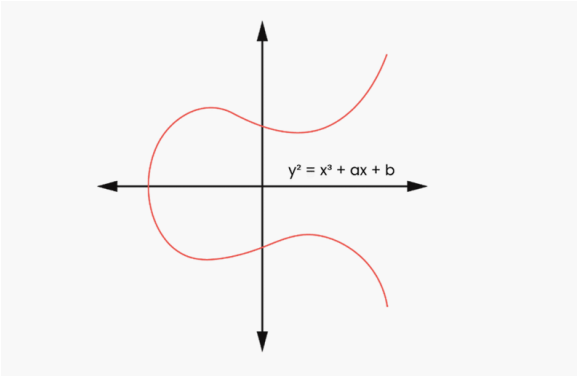
The integer r is the **rank**. It measures the size of the non-torsion part of $E(\mathbb{Q})$.
It is notoriously difficult to determine.

Genus one curves: elliptic curves

A genus one curve with a rational point is called an **elliptic curve**. It can be written in the form

$$E: \ y^2 = x^3 + Ax + B, \qquad A, B \in \mathbb{Q}.$$

Unlike conics, rational points on elliptic curves are not parametrised by one parameter.



Poincaré (1901)

The rational points $E(\mathbb{Q})$ form an abelian group.

Mordell's theorem (1922)

The group $E(\mathbb{Q})$ is finitely generated:

$$E(\mathbb{Q}) \simeq \mathbb{Z}^r \times (\text{finite group})$$

The new difficulty

The integer r is the **rank**. It measures the size of the non-torsion part of $E(\mathbb{Q})$.
It is notoriously difficult to determine.

Birch and Swinnerton-Dyer (1960s)

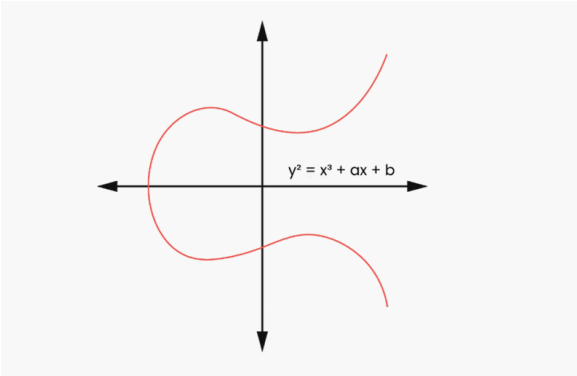
This leads to one of the Millennium Prize Problems, the Birch and Swinnerton-Dyer conjecture:

Genus one curves: elliptic curves

A genus one curve with a rational point is called an **elliptic curve**. It can be written in the form

$$E: \ y^2 = x^3 + Ax + B, \qquad A, B \in \mathbb{Q}.$$

Unlike conics, rational points on elliptic curves are not parametrised by one parameter.



Poincaré (1901)

The rational points $E(\mathbb{Q})$ form an abelian group.

Mordell's theorem (1922)

The group $E(\mathbb{Q})$ is finitely generated:

$$E(\mathbb{Q}) \simeq \mathbb{Z}^r \times (\text{finite group})$$

The new difficulty

The integer r is the **rank**. It measures the size of the non-torsion part of $E(\mathbb{Q})$.
It is notoriously difficult to determine.

Birch and Swinnerton-Dyer (1960s)

This leads to one of the Millennium Prize Problems, the Birch and Swinnerton-Dyer conjecture:

$$r = \operatorname{ord}_{s=1} L(E, s).$$

Higher genus curves: Faltings' theorem

The pattern by genus suggests that rational points become increasingly constrained.

Higher genus curves: Faltings' theorem

The pattern by genus suggests that rational points become increasingly constrained.

Genus 0

$C(\mathbb{Q}) = \emptyset$ or $C(\mathbb{Q})$ is infinite.

one point gives a parametrisation

Higher genus curves: Faltings' theorem

The pattern by genus suggests that rational points become increasingly constrained.

Genus 0

$C(\mathbb{Q}) = \emptyset$ or $C(\mathbb{Q})$ is infinite.

one point gives a parametrisation

Genus 1

$$E(\mathbb{Q}) \simeq \mathbb{Z}^r \times T$$

where T is finite

rational points form a group

Higher genus curves: Faltings' theorem

The pattern by genus suggests that rational points become increasingly constrained.

Genus 0

$C(\mathbb{Q}) = \emptyset$ or $C(\mathbb{Q})$ is infinite.

one point gives a parametrisation

Genus 1

$E(\mathbb{Q}) \simeq \mathbb{Z}^r \times T$

where T is finite

rational points form a group

Genus ≥ 2

$C(\mathbb{Q})$ is finite.

Mordell conjecture

Faltings' theorem

Higher genus curves: Faltings' theorem

The pattern by genus suggests that rational points become increasingly constrained.

Genus 0	Genus 1	Genus ≥ 2
$C(\mathbb{Q}) = \emptyset$ or $C(\mathbb{Q})$ is infinite.	$E(\mathbb{Q}) \simeq \mathbb{Z}^r \times T$	$C(\mathbb{Q})$ is finite.
one point gives a parametrisation	where T is finite	Mordell conjecture
	rational points form a group	Faltings' theorem

Mordell conjecture, proved by Faltings in 1983

If C is a curve over $\mathbb{Q}$ of genus at least 2, then $C(\mathbb{Q})$ is finite.

Higher genus curves: Faltings' theorem

The pattern by genus suggests that rational points become increasingly constrained.

Genus 0	Genus 1	Genus ≥ 2
$C(\mathbb{Q}) = \emptyset$ or $C(\mathbb{Q})$ is infinite.	$E(\mathbb{Q}) \simeq \mathbb{Z}^r \times T$	$C(\mathbb{Q})$ is finite.
one point gives a parametrisation	where T is finite	Mordell conjecture
	rational points form a group	Faltings' theorem

Mordell conjecture, proved by Faltings in 1983

If C is a curve over $\mathbb{Q}$ of genus at least 2, then $C(\mathbb{Q})$ is finite.

Why finiteness is reasonable

In **genus** 0, one rational point produces infinitely many.

Higher genus curves: Faltings' theorem

The pattern by genus suggests that rational points become increasingly constrained.

Genus 0

$C(\mathbb{Q}) = \emptyset$ or $C(\mathbb{Q})$ is infinite.

one point gives a parametrisation

Genus 1

$E(\mathbb{Q}) \simeq \mathbb{Z}^r \times T$

where T is finite

rational points form a group

Genus ≥ 2

$C(\mathbb{Q})$ is finite.

Mordell conjecture

Faltings' theorem

Mordell conjecture, proved by Faltings in 1983

If C is a curve over $\mathbb{Q}$ of genus at least 2, then $C(\mathbb{Q})$ is finite.

Why finiteness is reasonable

In **genus** 0, one rational point produces infinitely many.
In **genus** 1, rational points can be added.

Higher genus curves: Faltings' theorem

The pattern by genus suggests that rational points become increasingly constrained.

Genus 0	Genus 1	Genus ≥ 2
$C(\mathbb{Q}) = \emptyset$ or $C(\mathbb{Q})$ is infinite.	$E(\mathbb{Q}) \simeq \mathbb{Z}^r \times T$	$C(\mathbb{Q})$ is finite.
one point gives a parametrisation	where T is finite	Mordell conjecture
	rational points form a group	Faltings' theorem

Mordell conjecture, proved by Faltings in 1983

If C is a curve over $\mathbb{Q}$ of genus at least 2, then $C(\mathbb{Q})$ is finite.

Why finiteness is reasonable

In **genus** 0, one rational point produces infinitely many.
In **genus** 1, rational points can be added. In particular, 17 rational points already force infinitely many.

Higher genus curves: Faltings' theorem

The pattern by genus suggests that rational points become increasingly constrained.

Genus 0	Genus 1	Genus ≥ 2
$C(\mathbb{Q}) = \emptyset$ or $C(\mathbb{Q})$ is infinite.	$E(\mathbb{Q}) \simeq \mathbb{Z}^r \times T$	$C(\mathbb{Q})$ is finite.
one point gives a parametrisation	where T is finite	Mordell conjecture
	rational points form a group	Faltings' theorem

Mordell conjecture, proved by Faltings in 1983

If C is a curve over $\mathbb{Q}$ of genus at least 2, then $C(\mathbb{Q})$ is finite.

Why finiteness is reasonable

In **genus** 0, one rational point produces infinitely many.
In **genus** 1, rational points can be added. In particular, 17 rational points already force infinitely many.
In **genus** ≥ 2, there is no comparable mechanism on the curve itself: many rational points do not automatically produce one more.

From curves to Jacobians

Mordell's conjecture (1922); Faltings' theorem (1983)

Let C be a smooth projective curve defined over $\mathbb{Q}$. If $\text{genus}(C) \geq 2$, then $C(\mathbb{Q})$ is finite.

From curves to Jacobians

Mordell's conjecture (1922); Faltings' theorem (1983)

Let C be a smooth projective curve defined over $\mathbb{Q}$. If genus$(C) \geq 2$, then $C(\mathbb{Q})$ is finite.

For curves of genus at least 2, there is no group law on $C(\mathbb{Q})$ itself.

From curves to Jacobians

Mordell's conjecture (1922); Faltings' theorem (1983)

Let C be a smooth projective curve defined over $\mathbb{Q}$. If genus$(C) \geq 2$, then $C(\mathbb{Q})$ is finite.

For curves of genus at least 2, there is no group law on $C(\mathbb{Q})$ itself.

Jacobians

The **Jacobian** is a way of turning points on C into an abelian group.

From curves to Jacobians

Mordell's conjecture (1922); Faltings' theorem (1983)

Let C be a smooth projective curve defined over $\mathbb{Q}$. If genus$(C) \geq 2$, then $C(\mathbb{Q})$ is finite.

For curves of genus at least 2, there is no group law on $C(\mathbb{Q})$ itself.

Jacobians

The **Jacobian** is a way of turning points on C into an abelian group.

Instead of adding points directly on C, we consider formal sums $\sum_i n_i P_i$,

From curves to Jacobians

Mordell's conjecture (1922); Faltings' theorem (1983)

Let C be a smooth projective curve defined over $\mathbb{Q}$. If genus$(C) \geq 2$, then $C(\mathbb{Q})$ is finite.

For curves of genus at least 2, there is no group law on $C(\mathbb{Q})$ itself.

Jacobians

The **Jacobian** is a way of turning points on C into an abelian group.

Instead of adding points directly on C, we consider formal sums $\sum_i n_i P_i$, where $n_i \in \mathbb{Z}$ and P_i are points on C.

From curves to Jacobians

Mordell's conjecture (1922); Faltings' theorem (1983)

Let C be a smooth projective curve defined over $\mathbb{Q}$. If genus$(C) \geq 2$, then $C(\mathbb{Q})$ is finite.

For curves of genus at least 2, there is no group law on $C(\mathbb{Q})$ itself.

Jacobians

The **Jacobian** is a way of turning points on C into an abelian group.

Instead of adding points directly on C, we consider formal sums $\sum_i n_i P_i$, where $n_i \in \mathbb{Z}$ and P_i are points on C.

The degree zero classes form the **Jacobian**: $\mathrm{Jac}(C) = \mathrm{Pic}^0(C)$.

From curves to Jacobians

Mordell's conjecture (1922); Faltings' theorem (1983)

Let C be a smooth projective curve defined over $\mathbb{Q}$. If $\text{genus}(C) \geq 2$, then $C(\mathbb{Q})$ is finite.

For curves of genus at least 2, there is no group law on $C(\mathbb{Q})$ itself.

Jacobians

The **Jacobian** is a way of turning points on C into an abelian group.

Instead of adding points directly on C, we consider formal sums $\sum_i n_i P_i$, where $n_i \in \mathbb{Z}$ and P_i are points on C.

The degree zero classes form the **Jacobian**: $\text{Jac}(C) = \text{Pic}^0(C)$.

Choosing $P_0 \in C(\mathbb{Q})$ gives the Abel–Jacobi embedding:

From curves to Jacobians

Mordell's conjecture (1922); Faltings' theorem (1983)

Let C be a smooth projective curve defined over $\mathbb{Q}$. If $\text{genus}(C) \geq 2$, then $C(\mathbb{Q})$ is finite.

For curves of genus at least 2, there is no group law on $C(\mathbb{Q})$ itself.

Jacobians

The **Jacobian** is a way of turning points on C into an abelian group.

Instead of adding points directly on C, we consider formal sums $\sum_i n_i P_i$, where $n_i \in \mathbb{Z}$ and P_i are points on C.

The degree zero classes form the **Jacobian**: $\text{Jac}(C) = \text{Pic}^0(C)$.

Choosing $P_0 \in C(\mathbb{Q})$ gives the Abel–Jacobi embedding:

$$C(\mathbb{Q}) \quad \subset \quad \text{Jac}(C)(\mathbb{Q}).$$

From curves to Jacobians

Mordell's conjecture (1922); Faltings' theorem (1983)

Let C be a smooth projective curve defined over $\mathbb{Q}$. If $\text{genus}(C) \geq 2$, then $C(\mathbb{Q})$ is finite.

For curves of genus at least 2, there is no group law on $C(\mathbb{Q})$ itself.

Jacobians

The **Jacobian** is a way of turning points on C into an abelian group.

Instead of adding points directly on C, we consider formal sums $\sum_i n_i P_i$, where $n_i \in \mathbb{Z}$ and P_i are points on C.

The degree zero classes form the **Jacobian**: $\text{Jac}(C) = \text{Pic}^0(C)$.

Choosing $P_0 \in C(\mathbb{Q})$ gives the Abel–Jacobi embedding:

$$C(\mathbb{Q}) \quad \subset \quad \text{Jac}(C)(\mathbb{Q}).$$

Mordell–Weil for Jacobians (1928)

$\text{Jac}(C)(\mathbb{Q}) \simeq \mathbb{Z}^r \times \text{finite}$

From curves to Jacobians

Mordell's conjecture (1922); Faltings' theorem (1983)

Let C be a smooth projective curve defined over $\mathbb{Q}$. If $\text{genus}(C) \geq 2$, then $C(\mathbb{Q})$ is finite.

For curves of genus at least 2, there is no group law on $C(\mathbb{Q})$ itself.

Jacobians

The **Jacobian** is a way of turning points on C into an abelian group.

Instead of adding points directly on C, we consider formal sums $\sum_i n_i P_i$, where $n_i \in \mathbb{Z}$ and P_i are points on C.

The degree zero classes form the **Jacobian**: $\text{Jac}(C) = \text{Pic}^0(C)$.

Choosing $P_0 \in C(\mathbb{Q})$ gives the Abel–Jacobi embedding:

$$C(\mathbb{Q}) \quad \subset \quad \text{Jac}(C)(\mathbb{Q}).$$

Mordell–Weil for Jacobians (1928)

$\text{Jac}(C)(\mathbb{Q}) \simeq \mathbb{Z}^r \times \text{finite}$

Chabauty's theorem (1941)

Let C be a curve of genus $g \geq 2$. If $r < g$, then $C(\mathbb{Q})$ is finite.

Faltings' proof: the broad strategy

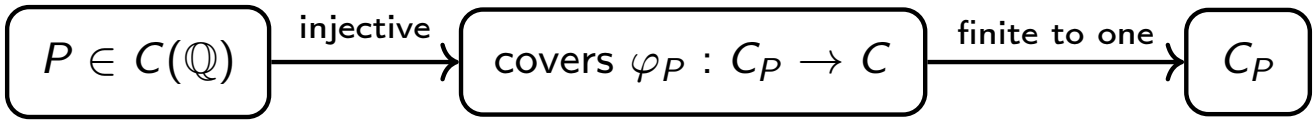
From points to auxiliary curves

Parshin's trick associates to each rational point $P \in C(\mathbb{Q})$ an auxiliary curve C_P.

Faltings' proof: the broad strategy

From points to auxiliary curves

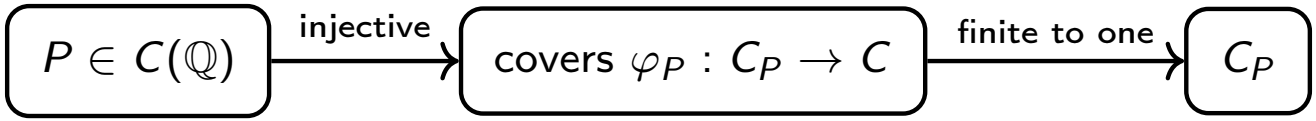
Parshin's trick associates to each rational point $P \in C(\mathbb{Q})$ an auxiliary curve C_P.



Faltings' proof: the broad strategy

From points to auxiliary curves

Parshin's trick associates to each rational point $P \in C(\mathbb{Q})$ an auxiliary curve C_P.

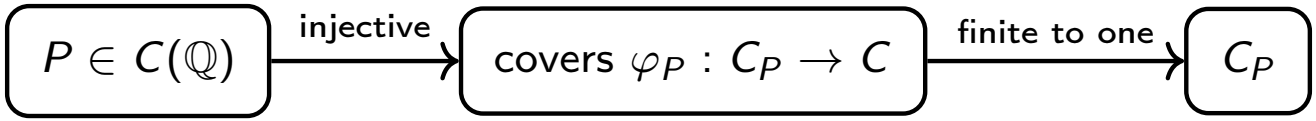


Parshin's trick constructs curves C_P of bounded genus
with **good reduction outside a fixed finite set of primes**.

Faltings' proof: the broad strategy

From points to auxiliary curves

Parshin's trick associates to each rational point $P \in C(\mathbb{Q})$ an auxiliary curve C_P.



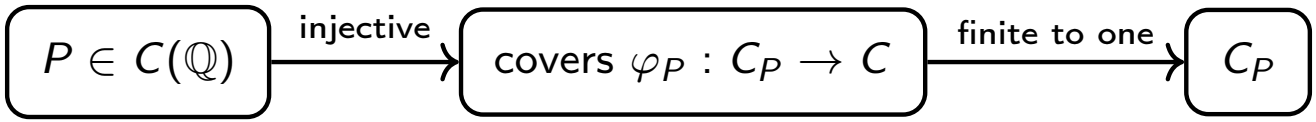
Parshin's trick constructs curves C_P of bounded genus
with **good reduction outside a fixed finite set of primes**.

Conclusion: If $\{C_P\}/\simeq$ is finite, then $C(\mathbb{Q})$ is finite.

Faltings' proof: the broad strategy

From points to auxiliary curves

Parshin's trick associates to each rational point $P \in C(\mathbb{Q})$ an auxiliary curve C_P.



Parshin's trick constructs curves C_P of bounded genus
with **good reduction outside a fixed finite set of primes**.

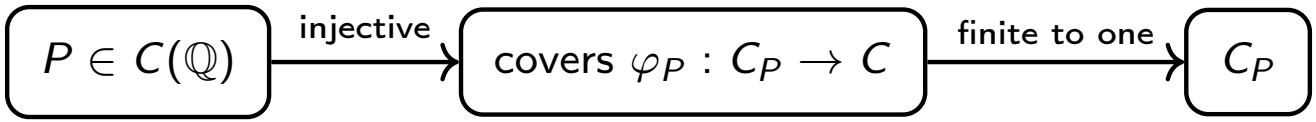
Conclusion: If $\{C_P\}/\simeq$ is finite, then $C(\mathbb{Q})$ is finite.

Finiteness input: Shafarevich conjecture for Jacobians

Faltings' proof: the broad strategy

From points to auxiliary curves

Parshin's trick associates to each rational point $P \in C(\mathbb{Q})$ an auxiliary curve C_P.



Parshin's trick constructs curves C_P of bounded genus
with **good reduction outside a fixed finite set of primes**.

Conclusion: If $\{C_P\}/\simeq$ is finite, then $C(\mathbb{Q})$ is finite.

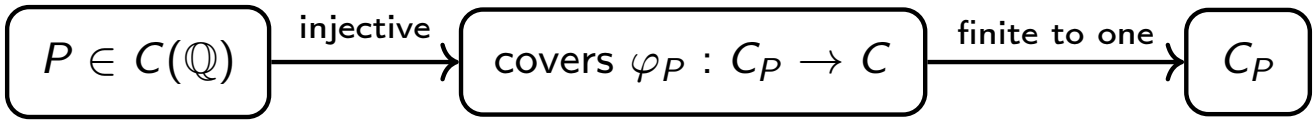
Finiteness input: Shafarevich conjecture for Jacobians

Fix an integer $g \geq 1$ and a finite set S of primes.

Faltings' proof: the broad strategy

From points to auxiliary curves

Parshin's trick associates to each rational point $P \in C(\mathbb{Q})$ an auxiliary curve C_P.



Parshin's trick constructs curves C_P of bounded genus
with **good reduction outside a fixed finite set of primes**.

Conclusion: If $\{C_P\}/\simeq$ is finite, then $C(\mathbb{Q})$ is finite.

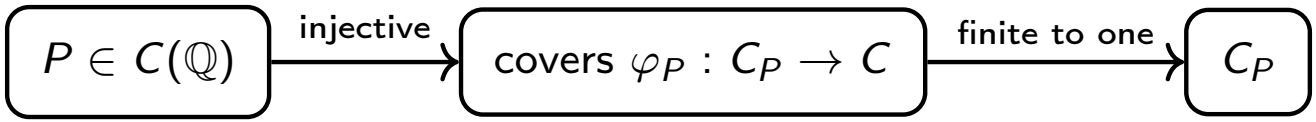
Finiteness input: Shafarevich conjecture for Jacobians

Fix an integer $g \geq 1$ and a finite set S of primes.
There are only finitely many isomorphism classes of Jacobians $J/\mathbb{Q}$ of dimension g with good reduction outside S.

Faltings' proof: the broad strategy

From points to auxiliary curves

Parshin's trick associates to each rational point $P \in C(\mathbb{Q})$ an auxiliary curve C_P.



Parshin's trick constructs curves C_P of bounded genus
with **good reduction outside a fixed finite set of primes**.

Conclusion: If $\{C_P\}/\simeq$ is finite, then $C(\mathbb{Q})$ is finite.

Finiteness input: Shafarevich conjecture for Jacobians

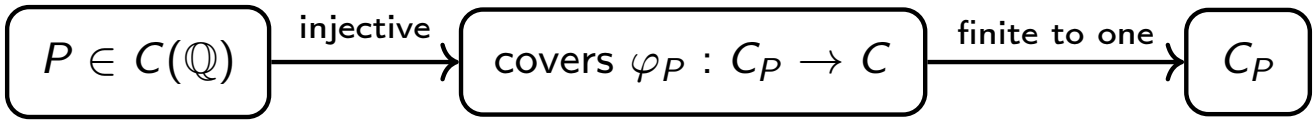
Fix an integer $g \geq 1$ and a finite set S of primes.
There are only finitely many isomorphism classes of Jacobians $J/\mathbb{Q}$ of dimension g with good reduction outside S.

Faltings proved this conjecture in 1983.

Faltings' proof: the broad strategy

From points to auxiliary curves

Parshin's trick associates to each rational point $P \in C(\mathbb{Q})$ an auxiliary curve C_P.



Parshin's trick constructs curves C_P of bounded genus
with **good reduction outside a fixed finite set of primes**.

Conclusion: If $\{C_P\}/\simeq$ is finite, then $C(\mathbb{Q})$ is finite.

Finiteness input: Shafarevich conjecture for Jacobians

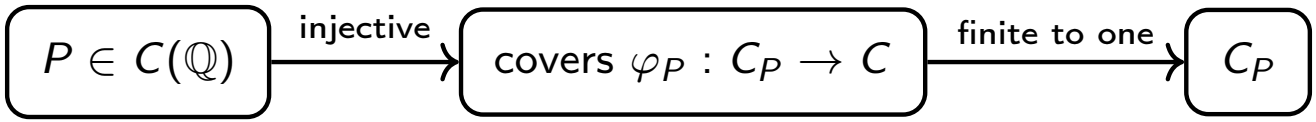
Fix an integer $g \geq 1$ and a finite set S of primes.
There are only finitely many isomorphism classes of Jacobians $J/\mathbb{Q}$ of dimension g with good reduction outside S.

Faltings proved this conjecture in 1983.
Applied to the Jacobians $\mathrm{Jac}(C_P)$, it gives only finitely many possible curves C_P,

Faltings' proof: the broad strategy

From points to auxiliary curves

Parshin's trick associates to each rational point $P \in C(\mathbb{Q})$ an auxiliary curve C_P.



Parshin's trick constructs curves C_P of bounded genus
with **good reduction outside a fixed finite set of primes**.

Conclusion: If $\{C_P\}/\simeq$ is finite, then $C(\mathbb{Q})$ is finite.

Finiteness input: Shafarevich conjecture for Jacobians

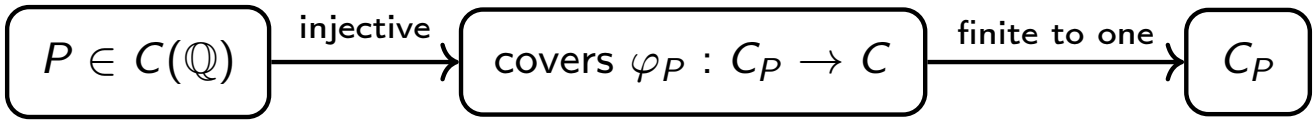
Fix an integer $g \geq 1$ and a finite set S of primes.
There are only finitely many isomorphism classes of Jacobians $J/\mathbb{Q}$ of dimension g with good reduction outside S.

Faltings proved this conjecture in 1983.
Applied to the Jacobians $\mathrm{Jac}(C_P)$, it gives only finitely many possible curves C_P,
and hence only finitely many points in $C(\mathbb{Q})$.

Faltings' proof: the broad strategy

From points to auxiliary curves

Parshin's trick associates to each rational point $P \in C(\mathbb{Q})$ an auxiliary curve C_P.



Parshin's trick constructs curves C_P of bounded genus
with **good reduction outside a fixed finite set of primes**.

Conclusion: If $\{C_P\}/\simeq$ is finite, then $C(\mathbb{Q})$ is finite.

Finiteness input: Shafarevich conjecture for Jacobians

Fix an integer $g \geq 1$ and a finite set S of primes.
There are only finitely many isomorphism classes of Jacobians $J/\mathbb{Q}$ of dimension g with good reduction outside S.

Faltings proved this conjecture in 1983.
Applied to the Jacobians $\mathrm{Jac}(C_P)$, it gives only finitely many possible curves C_P, and hence only finitely many points in $C(\mathbb{Q})$.

Mordell's conjecture is proved.

For this work, Faltings received the Fields Medal in 1986.

After Faltings: new directions

After Faltings' proof of the Mordell conjecture, two major directions emerged.

After Faltings: new directions

After Faltings' proof of the Mordell conjecture, two major directions emerged.

1. New proofs and stronger results

After Faltings: new directions

After Faltings' proof of the Mordell conjecture, two major directions emerged.

1. New proofs and stronger results

Faltings (1991): proved the Mordell–Lang conjecture.

After Faltings: new directions

After Faltings' proof of the Mordell conjecture, two major directions emerged.

1. New proofs and stronger results

Faltings (1991): proved the Mordell–Lang conjecture.

Vojta (1991): gave a new proof of the Mordell conjecture using Diophantine approximation.

After Faltings: new directions

After Faltings' proof of the Mordell conjecture, two major directions emerged.

1. New proofs and stronger results

Faltings (1991): proved the Mordell–Lang conjecture.

Vojta (1991): gave a new proof of the Mordell conjecture using Diophantine approximation.

Lawrence–Venkatesh (2020): gave a new proof using p-adic Hodge theory.

After Faltings: new directions

After Faltings' proof of the Mordell conjecture, two major directions emerged.

1. New proofs and stronger results

Faltings (1991): proved the Mordell–Lang conjecture.

Vojta (1991): gave a new proof of the Mordell conjecture using Diophantine approximation.

Lawrence–Venkatesh (2020): gave a new proof using p-adic Hodge theory.

Mazur's uniform upper bounds (1986)

Can one bound $\#C(\mathbb{Q})$ in terms of the genus of C and the rank of its Jacobian?

Can such a bound be made explicit?

After Faltings: new directions

After Faltings' proof of the Mordell conjecture, two major directions emerged.

1. New proofs and stronger results

Faltings (1991): proved the Mordell–Lang conjecture.

Vojta (1991): gave a new proof of the Mordell conjecture using Diophantine approximation.

Lawrence–Venkatesh (2020): gave a new proof using p-adic Hodge theory.

Mazur's uniform upper bounds (1986)

Can one bound $\#C(\mathbb{Q})$ in terms of the genus of C and the rank of its Jacobian?

Can such a bound be made explicit?

A substantial literature

This question motivated work of **Bombieri** (1990), **Silverman** (1993), **de Diego** (1997), **Rémond** (2000), **David–Philippon** (2002), **Katz–Rabinoff–Zureick-Brown** (2016), and **Stoll** (2019).

After Faltings: new directions

After Faltings' proof of the Mordell conjecture, two major directions emerged.

1. New proofs and stronger results

Faltings (1991): proved the Mordell–Lang conjecture.

Vojta (1991): gave a new proof of the Mordell conjecture using Diophantine approximation.

Lawrence–Venkatesh (2020): gave a new proof using p-adic Hodge theory.

Mazur's uniform upper bounds (1986)

Can one bound $\#C(\mathbb{Q})$ in terms of the genus of C and the rank of its Jacobian?

Can such a bound be made explicit?

A substantial literature

This question motivated work of **Bombieri** (1990), **Silverman** (1993), **de Diego** (1997), **Rémond** (2000), **David–Philippon** (2002), **Katz–Rabinoff–Zureick-Brown** (2016), and **Stoll** (2019).

Dimitrov–Gao–Habegger and Kühne (2021)

After Faltings: new directions

After Faltings' proof of the Mordell conjecture, two major directions emerged.

1. New proofs and stronger results

Faltings (1991): proved the Mordell–Lang conjecture.

Vojta (1991): gave a new proof of the Mordell conjecture using Diophantine approximation.

Lawrence–Venkatesh (2020): gave a new proof using p-adic Hodge theory.

Mazur's uniform upper bounds (1986)

Can one bound $\#C(\mathbb{Q})$ in terms of the genus of C and the rank of its Jacobian?

Can such a bound be made explicit?

A substantial literature

This question motivated work of **Bombieri** (1990), **Silverman** (1993), **de Diego** (1997), **Rémond** (2000), **David–Philippon** (2002), **Katz–Rabinoff–Zureick-Brown** (2016), and **Stoll** (2019).

Dimitrov–Gao–Habegger and Kühne (2021)

There exists a constant $c(g)$, depending only on g, such that $\#C(\mathbb{Q}) \leq c(g)^{1+r}$.

After Faltings: new directions

After Faltings' proof of the Mordell conjecture, two major directions emerged.

1. New proofs and stronger results

Faltings (1991): proved the Mordell–Lang conjecture.

Vojta (1991): gave a new proof of the Mordell conjecture using Diophantine approximation.

Lawrence–Venkatesh (2020): gave a new proof using p-adic Hodge theory.

Mazur's uniform upper bounds (1986)

Can one bound $\#C(\mathbb{Q})$ in terms of the genus of C and the rank of its Jacobian?

Can such a bound be made explicit?

A substantial literature

This question motivated work of **Bombieri** (1990), **Silverman** (1993), **de Diego** (1997), **Rémond** (2000), **David–Philippon** (2002), **Katz–Rabinoff–Zureick-Brown** (2016), and **Stoll** (2019).

Dimitrov–Gao–Habegger and Kühne (2021)

There exists a constant $c(g)$, depending only on g, such that $\#C(\mathbb{Q}) \leq c(g)^{1+r}$.

Yu–Yuan–Zhou (2026)

$$\#C(\mathbb{Q}) \leq 10^{13} g^8 \cdot \min\left\{1 + \frac{5}{4\sqrt{g}}, 1 + \frac{3\log g}{g}\right\}^r.$$

After Faltings: finding the points

$C(\mathbb{Q})$ finite	Upper bound for $\#C(\mathbb{Q})$	$\#C(\mathbb{Q}) \leq f(g,r)$	Effectively computing $C(\mathbb{Q})$
Faltings (1983)	***Vojta (1990)***	***DGH; Kühne (2021)***	
✓	✓	✓	?

After Faltings: finding the points

$C(\mathbb{Q})$ finite	Upper bound for $\#C(\mathbb{Q})$	$\#C(\mathbb{Q}) \leq f(g,r)$	Effectively computing $C(\mathbb{Q})$
Faltings (1983)	***Vojta (1990)***	***DGH; Kühne (2021)***	
✓	✓	✓	?

In many arithmetic problems, one needs to determine the rational points **explicitly**.

After Faltings: finding the points

$C(\mathbb{Q})$ finite	Upper bound for $\#C(\mathbb{Q})$	$\#C(\mathbb{Q}) \leq f(g,r)$	Effectively computing $C(\mathbb{Q})$
Faltings (1983)	***Vojta (1990)***	***DGH; Kühne (2021)***	
✓	✓	✓	?

In many arithmetic problems, one needs to determine the rational points **explicitly**.

Fermat's Last Theorem: Wiles; Taylor–Wiles (1995)

After Faltings: finding the points

$C(\mathbb{Q})$ finite	Upper bound for $\#C(\mathbb{Q})$	$\#C(\mathbb{Q}) \leq f(g,r)$	Effectively computing $C(\mathbb{Q})$
Faltings (1983)	***Vojta (1990)***	***DGH; Kühne (2021)***	
✓	✓	✓	?

In many arithmetic problems, one needs to determine the rational points **explicitly**.

Fermat's Last Theorem: Wiles; Taylor–Wiles (1995)

If $x, y \in \mathbb{Q}$ satisfy $x^n + y^n = 1$ with $n \geq 3$,

After Faltings: finding the points

$C(\mathbb{Q})$ finite	Upper bound for $\#C(\mathbb{Q})$	$\#C(\mathbb{Q}) \leq f(g,r)$	Effectively computing $C(\mathbb{Q})$
Faltings (1983)	***Vojta (1990)***	***DGH; Kühne (2021)***	
✓	✓	✓	?

In many arithmetic problems, one needs to determine the rational points **explicitly**.

Fermat's Last Theorem: Wiles; Taylor–Wiles (1995)

If $x, y \in \mathbb{Q}$ satisfy $x^n + y^n = 1$ with $n \geq 3$,then the only solutions are $(1,0)$, $(0,1)$, and, for even n, also $(-1,0)$, $(0,-1)$.

After Faltings: finding the points

$C(\mathbb{Q})$ finite	Upper bound for $\#C(\mathbb{Q})$	$\#C(\mathbb{Q}) \leq f(g,r)$	Effectively computing $C(\mathbb{Q})$
Faltings (1983)	***Vojta (1990)***	***DGH; Kühne (2021)***	
✓	✓	✓	?

In many arithmetic problems, one needs to determine the rational points **explicitly**.

Fermat's Last Theorem: Wiles; Taylor–Wiles (1995)

If $x, y \in \mathbb{Q}$ satisfy $x^n + y^n = 1$ with $n \geq 3$,then the only solutions are $(1,0)$, $(0,1)$, and, for even n, also $(-1,0)$, $(0,-1)$.

A long history

In 1637, Fermat claimed to have a wonderful proof, but wrote that the margin was too small to contain it.

After Faltings: finding the points

$C(\mathbb{Q})$ finite	Upper bound for $\#C(\mathbb{Q})$	$\#C(\mathbb{Q}) \leq f(g,r)$	Effectively computing $C(\mathbb{Q})$
Faltings (1983)	***Vojta (1990)***	***DGH; Kühne (2021)***	
✓	✓	✓	?

In many arithmetic problems, one needs to determine the rational points **explicitly**.

Fermat's Last Theorem: Wiles; Taylor–Wiles (1995)

If $x, y \in \mathbb{Q}$ satisfy $x^n + y^n = 1$ with $n \geq 3$,then the only solutions are $(1,0)$, $(0,1)$, and, for even n, also $(-1,0)$, $(0,-1)$.

A long history

In 1637, Fermat claimed to have a wonderful proof, but wrote that the margin was too small to contain it. The eventual proof took **129 pages**.

After Faltings: finding the points

$C(\mathbb{Q})$ finite	Upper bound for $\#C(\mathbb{Q})$	$\#C(\mathbb{Q}) \leq f(g,r)$	Effectively computing $C(\mathbb{Q})$
Faltings (1983) ✓	*Vojta (1990)* ✓	*DGH; Kühne (2021)* ✓	?

In many arithmetic problems, one needs to determine the rational points **explicitly**.

Fermat's Last Theorem: Wiles; Taylor–Wiles (1995)

If $x, y \in \mathbb{Q}$ satisfy $x^n + y^n = 1$ with $n \geq 3$,then the only solutions are $(1,0)$, $(0,1)$, and, for even n, also $(-1,0)$, $(0,-1)$.

A long history

In 1637, Fermat claimed to have a wonderful proof, but wrote that the margin was too small to contain it. The eventual proof took **129 pages**.

Effective Mordell, conjectured by Szpiro

After Faltings: finding the points

$C(\mathbb{Q})$ finite	Upper bound for $\#C(\mathbb{Q})$	$\#C(\mathbb{Q}) \leq f(g,r)$	Effectively computing $C(\mathbb{Q})$
Faltings (1983)	*Vojta (1990)*	*DGH; Kühne (2021)*	
✓	✓	✓	?

In many arithmetic problems, one needs to determine the rational points **explicitly**.

Fermat's Last Theorem: Wiles; Taylor–Wiles (1995)

If $x, y \in \mathbb{Q}$ satisfy $x^n + y^n = 1$ with $n \geq 3$,then the only solutions are $(1,0)$, $(0,1)$, and, for even n, also $(-1,0)$, $(0,-1)$.

A long history

In 1637, Fermat claimed to have a wonderful proof, but wrote that the margin was too small to contain it. The eventual proof took **129 pages**.

Effective Mordell, conjectured by Szpiro

There exists an effectively computable constant $K(g) > 0$ such that, for every curve $C/\mathbb{Q}$ of genus $g \geq 2$ and every $P \in C(\mathbb{Q})$,

After Faltings: finding the points

$C(\mathbb{Q})$ finite	Upper bound for $\#C(\mathbb{Q})$	$\#C(\mathbb{Q}) \leq f(g,r)$	Effectively computing $C(\mathbb{Q})$
Faltings (1983)	*Vojta (1990)*	*DGH; Kühne (2021)*	
✓	✓	✓	?

In many arithmetic problems, one needs to determine the rational points **explicitly**.

Fermat's Last Theorem: Wiles; Taylor–Wiles (1995)

If $x, y \in \mathbb{Q}$ satisfy $x^n + y^n = 1$ with $n \geq 3$,then the only solutions are $(1,0)$, $(0,1)$, and, for even n, also $(-1,0)$, $(0,-1)$.

A long history

In 1637, Fermat claimed to have a wonderful proof, but wrote that the margin was too small to contain it. The eventual proof took **129 pages**.

Effective Mordell, conjectured by Szpiro

There exists an effectively computable constant $K(g) > 0$ such that, for every curve $C/\mathbb{Q}$ of genus $g \geq 2$ and every $P \in C(\mathbb{Q})$, $\hat{h}(P) \leq K(g) \cdot h_{\mathrm{Fal}}(\mathrm{Jac}(C))$.

After Faltings: finding the points

$C(\mathbb{Q})$ finite	Upper bound for $\#C(\mathbb{Q})$	$\#C(\mathbb{Q}) \leq f(g,r)$	Effectively computing $C(\mathbb{Q})$
Faltings (1983)	*Vojta (1990)*	*DGH; Kühne (2021)*	
✓	✓	✓	?

In many arithmetic problems, one needs to determine the rational points **explicitly**.

Fermat's Last Theorem: Wiles; Taylor–Wiles (1995)

If $x, y \in \mathbb{Q}$ satisfy $x^n + y^n = 1$ with $n \geq 3$,then the only solutions are $(1,0)$, $(0,1)$, and, for even n, also $(-1,0)$, $(0,-1)$.

A long history

In 1637, Fermat claimed to have a wonderful proof, but wrote that the margin was too small to contain it. The eventual proof took **129 pages**.

Effective Mordell, conjectured by Szpiro

There exists an effectively computable constant $K(g) > 0$ such that, for every curve $C/\mathbb{Q}$ of genus $g \geq 2$ and every $P \in C(\mathbb{Q})$, $\hat{h}(P) \leq K(g) \cdot h_{\mathrm{Fal}}(\mathrm{Jac}(C))$.

A largely open problem

Effective Mordell remains vastly open, though the Chabauty–Coleman–Kim circle of ideas gives practical methods for determining $C(\mathbb{Q})$ in many cases.

Thank you for listening