RESEARCH WITH A WIPER OF CIRCLE GLASS

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ИЗСЛЕДВАНИЯ С ЧИСТАЧКА НА КРЪГЛО СТЪКЛО

Abstract

Content is presented that is suitable for training in a STEM center. It is related to solving a practical task and creative work related to the use of technologies. Several computer models are provided with which research can be carried out and a result sufficient for practical needs can be reached. The computer models can be used in the Virtual Mathematics Laboratory, developed at the Institute of Mathematics and Informatics of the Bulgarian Academy of Sciences.

Keywords: *STEAM; Computer Model; Research Approach; Applied Task; AR; Creativity.*

INTRODUCTION

For mathematics education, the benefit of STEAM education is related to the understanding of applications of mathematics, with broader opportunities for the formation of research skills. The possibility of using various technologies in the STEM spaces created in Bulgarian schools is also important: 3D printing [1], [2], augmented reality [3], virtual reality [4], [5], [6], holograms [7], use of specialized software for mathematical research, as well as for visual arts, including with AI.

Here we will present a practical task for finding a solution with sufficient accuracy for which computer models are proposed. Work on the proposed content contributes both to the development of students' skills for independently creating computer models for solving practical tasks, and to the skill of using ready-made models.

THE TASK

On a flat glass in a frame in the shape of a circle with a radius of 10 units, a wiper similar to an automobile one should be placed - its movement is "end-to-end" and is limited only by the frame. If one end is fixed in the frame, what is the maximum area that can be cleaned?

The length of the wiper R can vary in the interval $[0; 2r]$, where r denotes the radius of the given circle, which in this case is 10 units. At $R=0$ and at $R=2r$ the area of the cleaned part is 0, and at some intermediate value its maximum is achieved.

COMPUTER MODELS

We will present several computer models with which one can conduct research and reach a result sufficient for practical needs. The computer models were made with GeoGebra [8]. They can be used in the Virtual Mathematics Laboratory, developed at the Institute of

Mathematics and Informatics of the Bulgarian Academy of Sciences at: https://cabinet.bg/index.php?contenttype=viewarticle&id=460 (last view: 01-08-2025).

COMPUTER MODEL WITH A POINT ON THE CIRCLE

A circle in *GeoGebra* can be constructed both using tools (for example, with a tool for constructing a circle with a center and a point on the circle, or with a tool for constructing a circle with a center and a radius), and by entering the corresponding equation. In this case, we construct the circle $x^2+y^2=100$. From the point of view of symmetry, it does not matter at which point on the corresponding circle the wiper will be attached, we choose this to be the point $B(0; -10)$. Point C is arbitrary on the circle and represents the other end of the wiper (Fig. 1). The sector BCC' shows the cleaned part of the given circle.

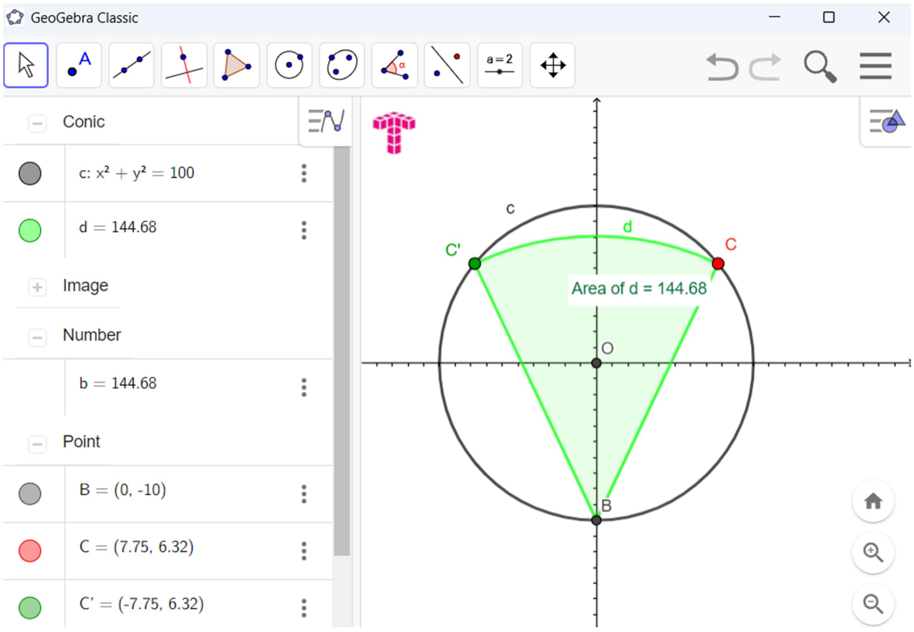
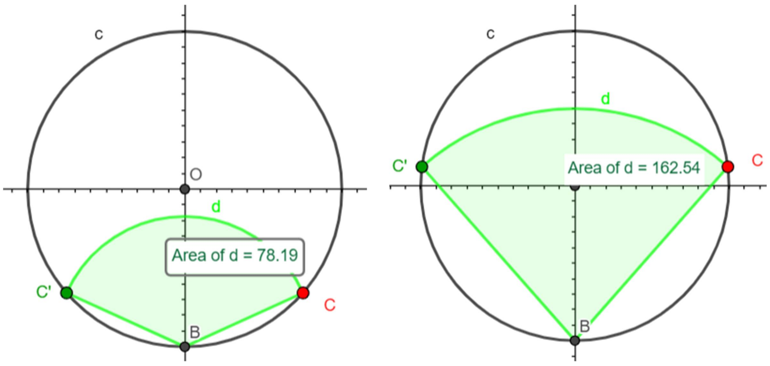


Fig. 1. Computer model with a point on the circle



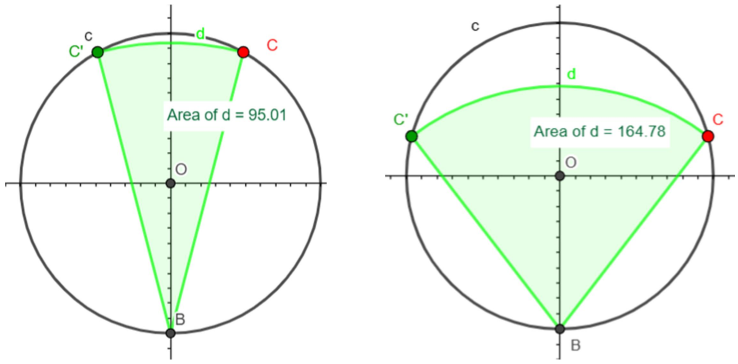


Fig. 2. Results for different positions of point C

The area of the green sector can be observed in the algebraic window, as well as, for convenience, displayed in the figure. Fig. 2 presents results for several different positions of point C, i.e. for different lengths of the wipers. It is noted that initially the area of the cleaned part increases, then begins to decrease. With several attempts, if necessary and using zoom, a sufficiently good approximation of the desired answer can be achieved.

Computer Model with a Slider Parameter

Usually it is much more convenient to achieve good accuracy using a slider parameter. In this computer model (Fig. 3) for the length of the wiper radius the slider parameter a is used, which varies in the interval [0; 2*r*].

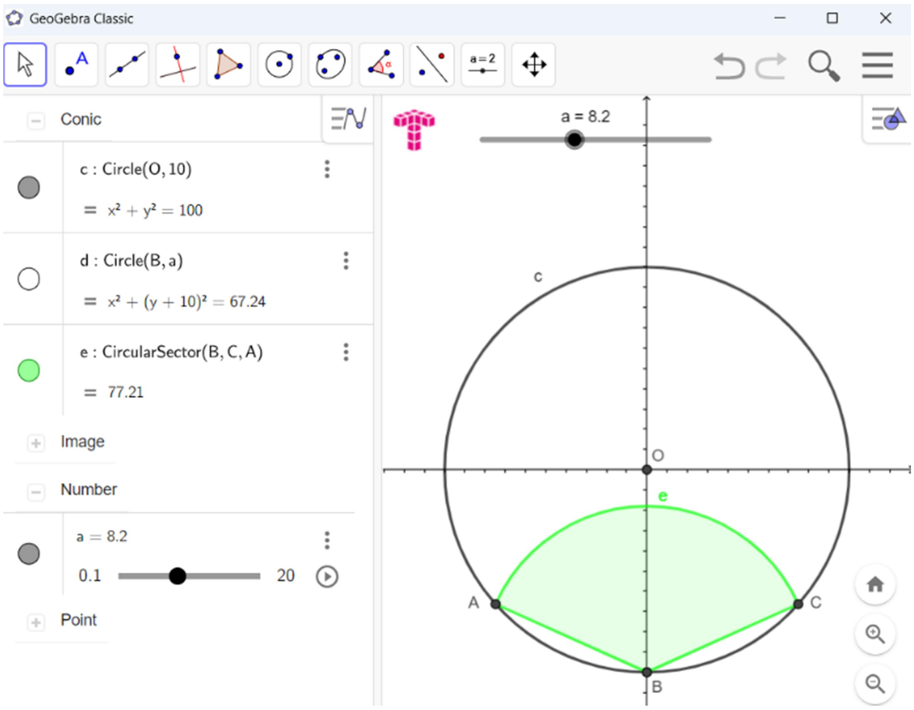


Fig. 3. Computer model with a slider parameter

In both the first and second models, it is appropriate to use the maximum command to obtain an answer:

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Maximise( <Dependent Number>, <Free Number> )  or
Maximise( <Dependent Number>, <Point on Path> )
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In the algebra window in Fig. 4, the result of the command is shown with accuracy to the hundredths $Maximise(e, a) = 15.88$. With this length of the wiper, the area of the cleaned part, accurate to hundredths, is 164.78.

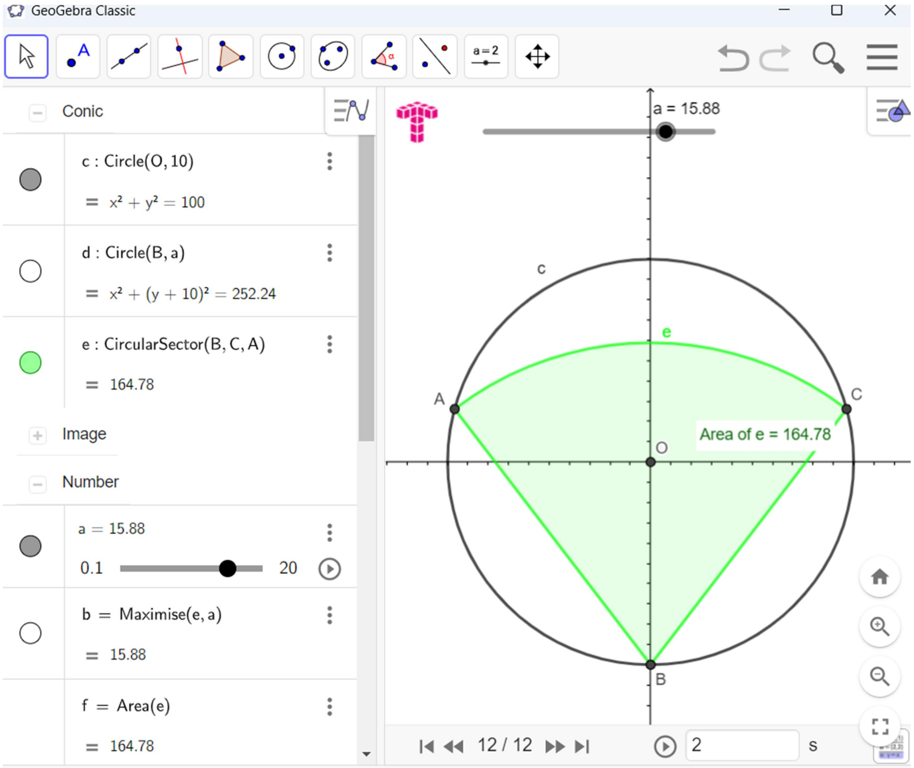

Fig. 4. Using the command to find the maximum

The investigation can continue by deriving the measure of the angle of the sector, the ratios of the radii of the given circle and sector, the ratios of their areas (Fig. 5).

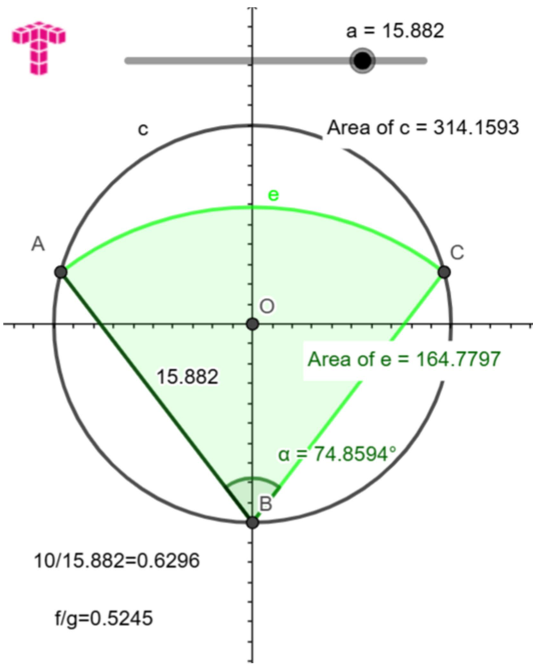

Fig. 5. Finding the relationship between lengths and areas

COMPUTER MODEL USING A FUNCTION GRAPH

Even if the researcher does not have knowledge of analytical study of a function for optimal values, the functional dependence can be used both to construct the graph in Goegebra and to find the optimal value in an interval using the corresponding commands. Thus, depending on the knowledge, with different accuracy and for different times, a solution with sufficient accuracy for practical needs can be reached. And from an educational point of view, with different age groups, as well as with students with different interests, such studies can be continued with different mathematical means.

Fig. 6 shows the graph of the function for the area of the sector through its angle and the result of using the command to find the maximum of a function in an interval Max(q(x),0,π).

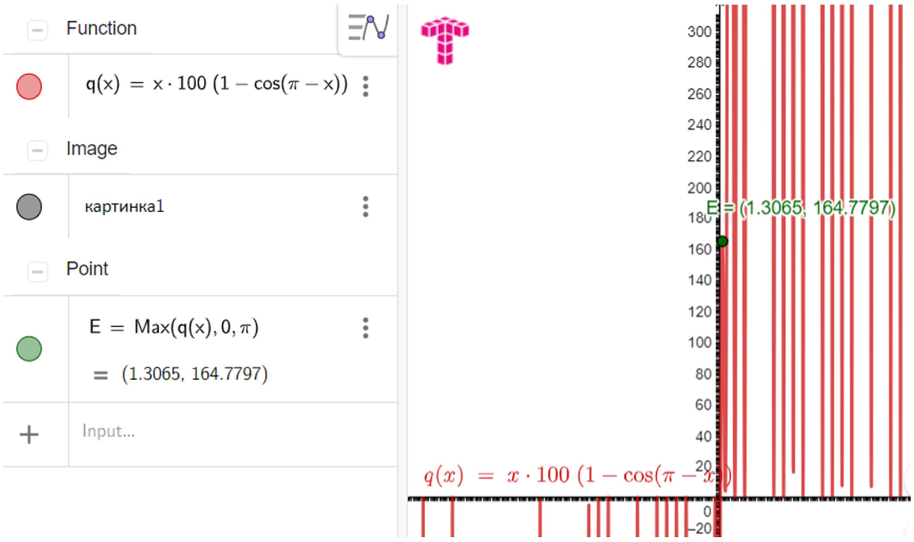

Fig. 6. Computer model using a function graph

The abscissa 1.3065 of point E is the radian measure of the angle of the sought sector with maximum area, and the ordinate 164.7797 is the area of this sector with an accuracy of ten thousandths.

To obtain the functional dependence $q(x) = x.100(1 - cos(\pi - x)$ the cosine theorem is used for the isosceles triangle with sides radius r of the given circle and base radius R of the sector $R^2 = 2r^2(1 - \cos(\pi - \alpha))$, as well as the formula for the area of a sector, when working in radians.

DISCUSSION AND CONCLUSION

The task in the general case is defined with accuracy up to similarity and finding the angle of the sector, or the ratio of the two radii, or the ratio of the areas of the circle and the sector are sufficient to solve the problem in the general case. The presence of a GeoGebra file allows the application of augmented reality to solve the problem for a specific case – by superimposing the model on the object (Fig. 7).

A task to create a picture or dynamic image with a wiper cleaning a round mirror, including using an artificial intelligence application, is one opportunity to continue working on the topic with an artistic element. Fig. 8 shows visualizations created using a variety of technological tools, including with an artificial intelligence.

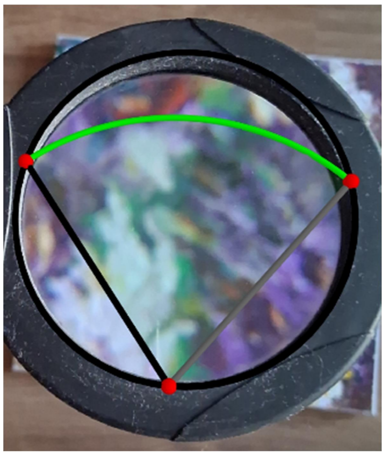

Fig. 7. Augmented Reality solution

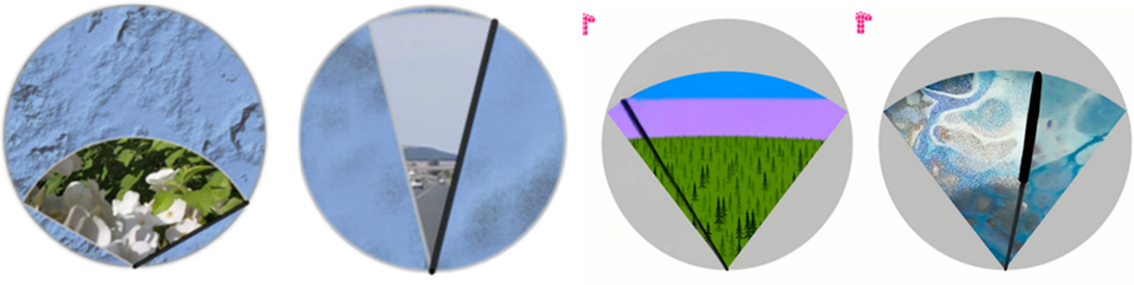

Fig. 8. Some visualizations

It is natural to continue the work on the topic by creating material models, changing some of the conditions in the task.

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