Realizing Complex Interval Functions in Computer Algebra Systems

Frithjof Blomquist
Bergische Universität Wuppertal,
Scientific Computing/Software Engineering,
Gaußstraße 20, D-42097 Wuppertal, Germany
blomquist@math.uni-wuppertal.de

Abstract

The computer algebra systems *Mathematica, Maple* and *MuPAD* provide the facility to calculate enclosures of codomains of elementary functions. In *Maple* these enclosures are implemented by the package `inpakX` [2].

In *Mathematica* a complete set of the real-valued elementary functions is available by use of the *Mathematica* function `Interval[]`, and enclosures of interval expressions including the elementary functions can be calculated with an accuracy of thousands of correct decimal digits.

Elementary complex-valued interval functions are not supported by the CA-systems, even though, especially in *Mathematica*, all necessary tools concerning the `Interval[{`r_1, r_2`}]` function are available. Thus, e.g. the separable functions like the exponential, sine and cosine functions and the hyperbolic sine and cosine functions are quite easy to realize. But also the complex-valued logarithm function can easily be implemented with the help of the interval functions $\sqrt{x^2+y^2}$ and `ArcTan[x,y]`, both available in *Mathematica*.

More complicated is the realization of the multi-valued functions such as the inverse sine $f(z) = \texttt{ArcSin}(z)$. If no singularity is contained in the rectangular input interval z, then the extreme values of the real and imaginary part of $\texttt{ArcSin}(z)$ lie on the boundary of z, and if the origin is not contained in z, then the extremal points are edges of the rectangular interval z. Thus, e.g. for calculating an enclosure of the real part

$$\Re(f(z)) := \texttt{ArcSin}\left(\frac{2\cdot x}{\sqrt{(x+1)^2+y^2}+\sqrt{(x-1)^2+y^2}}\right) =: r(x,y)\,,$$

$r(x,y)$ must be evaluated - using interval arithmetic - at the edges for the minimum and for the maximum, respectively [1, 3]. In this way enclosures of the complex-valued elementary functions can be realized for wide input intervals z using the already implemented interval tools of the CA-systems.

We have done a multiple precision implementation for a rather complete set of elementary functions based on so called staggered interval representations in C-XSC [1,3]. This implementation is freely available in source code. It may be used as a prototype implementation for realizing such packages in CA systems.

References:

[1] Blomquist, F., Hofschuster, W., Krämer, W.: A Modified Staggered Correction Arithmetic with Enhanced Accuracy and Very Wide Exponent

Range. Dagstuhl Seminar Proceedings 08021 - Numerical Validation in Current Hardware Architectures, 2008.

[**2**] Krämer, W.: intpakX - An Interval Arithmetic Package for Maple. Proceedings of the 12th GAMM-IMACS Symposium on Scientific Computing, Computer Arithmetic and Validated Numerics, SCAN 2006, IEEE Computer Society, ISBN 0-7695-2821-X, 2007.

[**3**] Blomquist, F.; Hofschuster, W.; Krämer, W.; Neher M.: *Complex Interval Functions in C-XSC*. Preprint BUW-WRSWT 2005/2, Scientific Computing/Software Engineering, University of Wuppertal, 2005.