Symbolic Versus Interval Rational Interpolation: The Problem of Unattainable Data

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The problem of the univariate interpolation of data, when one admits infinity in the independent variable as well as in the function value, is solved symbolically in its full generality by Van Barel and Bultheel [2]. A typical problem with rational interpolation is that of a so-called unattainable point, when the interpolation condition cannot be met by the rational interpolant of the specified degree. In their symbolic approach the authors solve this problem by returning a parameterized set of rational functions of higher degree without unattainable points.

However, in many applications observations are prone to imprecise measurements. A natural way for dealing with uncertainty in the data is by means of an uncertainty interval. When it is assumed that the uncertainty in the independent variable is negligible and that for each observation an uncertainty interval can be given which contains the (unknown) exact value, then we have shown in [1] how a rational function of lowest complexity can be obtained which intersects all uncertainty intervals. The problem is reduced to a quadratic programming problem of which the objective function is strictly convex. Compared to rational least squares approximation, which is inherently a nonlinear optimization problem where the objective function may have many local minima, this makes the interval formulation attractive. The proposed solution also solves the typical problem of unattainable data numerically.

References

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[2] M. Van Barel and A. Bultheel. A new formal approach to the rational interpolation problem. *Numerische Mathematik*, 62(1):87–122, 1992.