

Two-Bay Two-Story Frame Benchmark Model

Consider a FEM model of a two-bay two-story steel frame, as described in [Popova,E.,R.Iankov,Z.Bonev:Bounding the Response of Mechanical Structures with Uncertainties in all the Parameters.In R.L.Muhannah,R.L.Mullen (Eds): Proceedings of the NSF Workshop on Reliable Engineering Computing (REC), Savannah, Georgia USA, Feb. 22-24, 2006, pp.245-265.
<http://www.math.bas.bg/~epopova/papers/06recPopovaEtAl.pdf>]

The FEM model leads to a parametric linear system of dimension 18 involving rational dependencies between the uncertain parameters. Depending on which parameters are considered to be uncertain, we consider two problems defined below.

■ Problem 1. 13 uncertain parameters.

The data, saved in 3 ASCII files, can be downloaded from http://www.math.bas.bg/~epopova/papers/2Bay2Story13pars_matrix.dat, resp. [2Story13pars_vector.dat](http://www.math.bas.bg/~epopova/papers/2Story13pars_vector.dat) and [2Story13pars_pars110.dat](http://www.math.bas.bg/~epopova/papers/2Story13pars_pars110.dat). Therein the matrix is presented row by row enclosed in curly brackets, the row elements also enclosed in curly brackets.

The parametric matrix is presented below column by column, where the rows and the columns are labeled accordingly.

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In[16]:= TableForm[TakeColumns[mat13, 5] , TableHeadings->{Automatic, Automatic}]
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Out[16]//TableForm=
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	1	2	3	4	5
1	$\frac{Ab\ Eb}{6} + \frac{4\ Ec\ Ic}{9}$	0	$\frac{2\ Ec\ Ic}{3}$	$-\frac{Ab\ Eb}{6}$	0
2	0	$\frac{c\ Eb\ Ib+6\ Ac\ Ec\ (c+Eb\ Ib)}{18\ (c+Eb\ Ib)}$	$\frac{c\ Eb\ Ib}{6\ c+6\ Eb\ Ib}$	0	$-\frac{c\ Eb\ Ib}{18\ (c+Eb\ Ib)}$
3	$\frac{2\ Ec\ Ic}{3}$	$\frac{c\ Eb\ Ib}{6\ c+6\ Eb\ Ib}$	$\frac{c\ Eb\ Ib\ (2\ c+Eb\ Ib)}{3\ c^2+4\ c\ Eb\ Ib+Eb^2\ Ib^2} + \frac{4\ Ec\ Ic}{3}$	0	$-\frac{c\ Eb\ Ib}{6\ (c+Eb\ Ib)}$
4	$-\frac{Ab\ Eb}{6}$	0	0	$\frac{1}{9}\ (3\ Ab\ Eb + 4\ Ec\ Ic)$	0
5	0	$-\frac{c\ Eb\ Ib}{18\ (c+Eb\ Ib)}$	$-\frac{c\ Eb\ Ib}{6\ (c+Eb\ Ib)}$	0	$\frac{c\ Eb\ Ib+3\ Ac\ Ec\ (c+Eb\ Ib)}{9\ (c+Eb\ Ib)}$
6	0	$\frac{c\ Eb\ Ib}{6\ c+6\ Eb\ Ib}$	$\frac{c^2\ Eb\ Ib}{3\ c^2+4\ c\ Eb\ Ib+Eb^2\ Ib^2}$	$\frac{2\ Ec\ Ic}{3}$	0
7	0	0	0	$-\frac{Ab\ Eb}{6}$	0
8	0	0	0	0	$-\frac{c\ Eb\ Ib}{18\ (c+Eb\ Ib)}$
9	0	0	0	0	$\frac{c\ Eb\ Ib}{6\ c+6\ Eb\ Ib}$
10	$-\frac{4\ Ec\ Ic}{9}$	0	$-\frac{2\ Ec\ Ic}{3}$	0	0
11	0	$-\frac{Ac\ Ec}{3}$	0	0	0
12	$\frac{2\ Ec\ Ic}{3}$	0	$\frac{2\ Ec\ Ic}{3}$	0	0
13	0	0	0	$-\frac{4\ Ec\ Ic}{9}$	0
14	0	0	0	0	$-\frac{Ac\ Ec}{3}$
15	0	0	0	$\frac{2\ Ec\ Ic}{3}$	0
16	0	0	0	0	0
17	0	0	0	0	0
18	0	0	0	0	0

`In[17]:= TableForm[TakeColumns[mat13, {6, 10}], TableHeadings → {Automatic, {6, 7, 8, 9, 10}}]`

`Out[17]//TableForm=`

	6	7	8	9	10
1	0	0	0	0	$-\frac{4 \text{ Ec Ic}}{9}$
2	$\frac{c \text{ Eb Ib}}{6 c+6 \text{ Eb Ib}}$	0	0	0	0
3	$\frac{c^2 \text{ Eb Ib}}{3 c^2+4 c \text{ Eb Ib}+\text{Eb}^2 \text{ Ib}^2}$	0	0	0	$-\frac{2 \text{ Ec Ic}}{3}$
4	$\frac{2 \text{ Ec Ic}}{3}$	$-\frac{\text{Ab Eb}}{6}$	0	0	0
5	0	0	$-\frac{c \text{ Eb Ib}}{18 (c+\text{Eb Ib})}$	$\frac{c \text{ Eb Ib}}{6 c+6 \text{ Eb Ib}}$	0
6	$4 \left(\frac{c \text{ Eb Ib} (2 c+\text{Eb Ib})}{6 c^2+8 c \text{ Eb Ib}+2 \text{Eb}^2 \text{ Ib}^2} + \frac{\text{Ec Ic}}{3} \right)$	0	$-\frac{c \text{ Eb Ib}}{6 (c+\text{Eb Ib})}$	$\frac{c^2 \text{ Eb Ib}}{3 c^2+4 c \text{ Eb Ib}+\text{Eb}^2 \text{ Ib}^2}$	0
7	0	$\frac{\text{Ab Eb}}{6} + \frac{4 \text{ Ec Ic}}{9}$	0	$\frac{2 \text{ Ec Ic}}{3}$	0
8	$-\frac{c \text{ Eb Ib}}{6 (c+\text{Eb Ib})}$	0	$\frac{c \text{ Eb Ib}+6 \text{ Ac Ec} (c+\text{Eb Ib})}{18 (c+\text{Eb Ib})}$	$-\frac{c \text{ Eb Ib}}{6 (c+\text{Eb Ib})}$	0
9	$\frac{c^2 \text{ Eb Ib}}{3 c^2+4 c \text{ Eb Ib}+\text{Eb}^2 \text{ Ib}^2}$	$\frac{2 \text{ Ec Ic}}{3}$	$-\frac{c \text{ Eb Ib}}{6 (c+\text{Eb Ib})}$	$\frac{c \text{ Eb Ib} (2 c+\text{Eb Ib})}{3 c^2+4 c \text{ Eb Ib}+\text{Eb}^2 \text{ Ib}^2} + \frac{4 \text{ Ec Ic}}{3}$	0
10	0	0	0	0	$\frac{\text{Ab Eb}}{6} + \frac{8 \text{ Ec Ic}}{9}$
11	0	0	0	0	0
12	0	0	0	0	0
13	$-\frac{2 \text{ Ec Ic}}{3}$	0	0	0	$-\frac{\text{Ab Eb}}{6}$
14	0	0	0	0	0
15	$\frac{2 \text{ Ec Ic}}{3}$	0	0	0	0
16	0	$-\frac{4 \text{ Ec Ic}}{9}$	0	$-\frac{2 \text{ Ec Ic}}{3}$	0
17	0	0	$-\frac{\text{Ac Ec}}{3}$	0	0
18	0	$\frac{2 \text{ Ec Ic}}{3}$	0	$\frac{2 \text{ Ec Ic}}{3}$	0

`In[22]:= TableForm[TakeColumns[mat13, {11, 15}], TableHeadings → {None, {11, 12, 13, 14, 15}}]`

`Out[22]//TableForm=`

11	12	13	14	15
0	$\frac{2 E_c I_c}{3}$	0	0	0
$-\frac{A_c E_c}{3}$	0	0	0	0
0	$\frac{2 E_c I_c}{3}$	0	0	0
0	0	$-\frac{4 E_c I_c}{9}$	0	$\frac{2 E_c I_c}{3}$
0	0	0	$-\frac{A_c E_c}{3}$	0
0	0	$-\frac{2 E_c I_c}{3}$	0	$\frac{2 E_c I_c}{3}$
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	$-\frac{A_b E_b}{6}$	0	0
$\frac{c E_b I_b + 12 A_c E_c (c + E_b I_b)}{18 (c + E_b I_b)}$	$\frac{c E_b I_b}{6 c + 6 E_b I_b}$	0	$-\frac{c E_b I_b}{18 (c + E_b I_b)}$	$\frac{c E_b I_b}{6 c + 6 E_b I_b}$
$\frac{c E_b I_b}{6 c + 6 E_b I_b}$	$\frac{1}{3} \left(\frac{c E_b I_b (6 c + 3 E_b I_b)}{(c + E_b I_b) (3 c + E_b I_b)} + 8 E_c I_c \right)$	0	$-\frac{c E_b I_b}{6 (c + E_b I_b)}$	$\frac{c^2 E_b I_b}{3 c^2 + 4 c E_b I_b + E_b^2 I_b^2}$
0	0	$\frac{1}{9} (3 A_b E_b + 8 E_c I_c)$	0	0
$-\frac{c E_b I_b}{18 (c + E_b I_b)}$	$-\frac{c E_b I_b}{6 (c + E_b I_b)}$	0	$\frac{c E_b I_b + 6 A_c E_c (c + E_b I_b)}{9 (c + E_b I_b)}$	0
$\frac{c E_b I_b}{6 c + 6 E_b I_b}$	$\frac{c^2 E_b I_b}{3 c^2 + 4 c E_b I_b + E_b^2 I_b^2}$	0	0	$4 \left(\frac{c E_b I_b (2 c + E_b I_b)}{6 c^2 + 8 c E_b I_b + 2 E_b^2 I_b^2} + \frac{2 E_c I_c}{3} \right)$
0	0	$-\frac{A_b E_b}{6}$	0	0
0	0	0	$-\frac{c E_b I_b}{18 (c + E_b I_b)}$	$-\frac{c E_b I_b}{6 (c + E_b I_b)}$
0	0	0	$\frac{c E_b I_b}{6 c + 6 E_b I_b}$	$\frac{c^2 E_b I_b}{3 c^2 + 4 c E_b I_b + E_b^2 I_b^2}$

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In[19]:= TableForm[TakeColumns[mat13, {16, 18}], TableHeadings → {Automatic, {16, 17, 18}}]
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Out[19]//TableForm=
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	16	17	18
1	0	0	0
2	0	0	0
3	0	0	0
4	0	0	0
5	0	0	0
6	0	0	0
7	$-\frac{4 E_c I_c}{9}$	0	$\frac{2 E_c I_c}{3}$
8	0	$-\frac{A_c E_c}{3}$	0
9	$-\frac{2 E_c I_c}{3}$	0	$\frac{2 E_c I_c}{3}$
10	0	0	0
11	0	0	0
12	0	0	0
13	$-\frac{A_b E_b}{6}$	0	0
14	0	$-\frac{c E_b I_b}{18 (c + E_b I_b)}$	$\frac{c E_b I_b}{6 c + 6 E_b I_b}$
15	0	$-\frac{c E_b I_b}{6 (c + E_b I_b)}$	$\frac{c^2 E_b I_b}{3 c^2 + 4 c E_b I_b + E_b^2 I_b^2}$
16	$\frac{A_b E_b}{6} + \frac{8 E_c I_c}{9}$	0	0
17	0	$\frac{c E_b I_b + 12 A_c E_c (c + E_b I_b)}{18 (c + E_b I_b)}$	$-\frac{c E_b I_b}{6 (c + E_b I_b)}$
18	0	$-\frac{c E_b I_b}{6 (c + E_b I_b)}$	$\frac{1}{3} \left(\frac{c E_b I_b (6 c + 3 E_b I_b)}{(c + E_b I_b) (3 c + E_b I_b)} + 8 E_c I_c \right)$

The parametric right hand side vector is

$$\text{Out}[23] = \left\{ f_2, -3 w_1, -\frac{3 w_1}{1 + \frac{E_b I_b}{3 c}}, 0, -3 w_1 - 3 w_2, \frac{3 w_1}{1 + \frac{E_b I_b}{3 c}} - \frac{3 w_2}{1 + \frac{E_b I_b}{3 c}}, 0, -3 w_2, \frac{3 w_2}{1 + \frac{E_b I_b}{3 c}}, f_1, -3 w_3, -\frac{3 w_3}{1 + \frac{E_b I_b}{3 c}}, 0, -3 w_3 - 3 w_4, \frac{3 w_3}{1 + \frac{E_b I_b}{3 c}} - \frac{3 w_4}{1 + \frac{E_b I_b}{3 c}}, 0, -3 w_4, \frac{3 w_4}{1 + \frac{E_b I_b}{3 c}} \right\}$$

The parameters vary within the following intervals

```
{Eb → Interval[{2.0895*10^8, 2.1105*10^8}], Ib → Interval[{0.000230144, 0.0002324565}], Ab → Interval[{0.00840377, 0.00848823}],
Ec → Interval[{2.0895*10^8, 2.1105*10^8}], Ic → Interval[{0.0001917365, 0.0001936635}], Ac → Interval[{0.0130743, 0.0132057}],
c → Interval[{9.5*10^7, 1.05*10^8}], w1 → Interval[{28.5, 31.5}], w2 → Interval[{28.5, 31.5}],
w3 → Interval[{28.5, 31.5}], w4 → Interval[{28.5, 31.5}], f1 → Interval[{95.0, 105.0}], f2 → Interval[{95.0, 105.0}]}
```

The vector of the unknown variables is (dx1, dy1, theta1, dx2, dy2, theta2, dx3, dy3, theta3, ...). Some results are presented in the paper cited above.

■ Problem 2. 37 uncertain parameters.

Consider the same structure as above but assume that each structure element has properties varying independently within tolerance intervals.

The data, saved in 3 ASCII files, can be downloaded from http://www.math.bas.bg/~epopova/papers/2Bay2Story37pars_matrix.dat, resp. 2Story37pars_vector.dat and 2Story37pars_pars1.dat. Therein the matrix is presented row by row enclosed in curly brackets, the row elements also enclosed in curly brackets.

The parametric matrix is presented below column by column, where the rows and the columns are labeled accordingly.

```
In[29]:= TableForm[TakeColumns[mat37, 4] , TableHeadings->{Automatic, Automatic}]
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```
Out[29]//TableForm=
```

	1	2	3	4
1	$\frac{Ab1 Eb1}{6} + \frac{4 Ec1 Ic1}{9}$	0	$\frac{2 Ec1 Ic1}{3}$	$-\frac{Ab1 Eb1}{6}$
2	0	$\frac{c Eb1 Ib1 + 6 Ac1 Ec1 (c + Eb1 Ib1)}{18 (c + Eb1 Ib1)}$	$\frac{c Eb1 Ib1}{6 c + 6 Eb1 Ib1}$	0
3	$\frac{2 Ec1 Ic1}{3}$	$\frac{c Eb1 Ib1}{6 c + 6 Eb1 Ib1}$	$\frac{c Eb1 Ib1 (2 c + Eb1 Ib1)}{3 c^2 + 4 c Eb1 Ib1 + Eb1^2 Ib1^2} + \frac{4 Ec1 Ic1}{3}$	0
4	$-\frac{Ab1 Eb1}{6}$	0	0	$\frac{1}{18} (3 Ab1 Eb1 + 3 Ab2 Eb2 + 8 Ec2 Ic2)$
5	0	$-\frac{c Eb1 Ib1}{18 (c + Eb1 Ib1)}$	$-\frac{c Eb1 Ib1}{6 (c + Eb1 Ib1)}$	0
6	0	$\frac{c Eb1 Ib1}{6 c + 6 Eb1 Ib1}$	$\frac{c^2 Eb1 Ib1}{3 c^2 + 4 c Eb1 Ib1 + Eb1^2 Ib1^2}$	$\frac{2 Ec2 Ic2}{3}$
7	0	0	0	$-\frac{Ab2 Eb2}{6}$
8	0	0	0	0
9	0	0	0	0
10	$-\frac{4 Ec1 Ic1}{9}$	0	$-\frac{2 Ec1 Ic1}{3}$	0
11	0	$-\frac{Ac1 Ec1}{3}$	0	0
12	$\frac{2 Ec1 Ic1}{3}$	0	$\frac{2 Ec1 Ic1}{3}$	0
13	0	0	0	$-\frac{4 Ec2 Ic2}{9}$
14	0	0	0	0
15	0	0	0	$\frac{2 Ec2 Ic2}{3}$
16	0	0	0	0
17	0	0	0	0
18	0	0	0	0

```
In[3]:= TableForm[TakeColumns[mat37, {5, 7}], TableHeadings → {None, {5, 6, 7}}]
```

```
Out[3]//TableForm=
```

5	6	7
0	0	0
$-\frac{c \text{Eb1 Ib1}}{18 (c+\text{Eb1 Ib1})}$	$\frac{c \text{Eb1 Ib1}}{6 c+6 \text{Eb1 Ib1}}$	0
$-\frac{c \text{Eb1 Ib1}}{6 (c+\text{Eb1 Ib1})}$	$\frac{c^2 \text{Eb1 Ib1}}{3 c^2+4 c \text{Eb1 Ib1}+\text{Eb1}^2 \text{Ib1}^2}$	0
0	$\frac{2 \text{Ec2 Ic2}}{3}$	$-\frac{\text{Ab2 Eb2}}{6}$
$\frac{6 \text{Ac2 Ec2} (c+\text{Eb1 Ib1}) (c+\text{Eb2 Ib2})+c (c \text{Eb1 Ib1}+c \text{Eb2 Ib2}+2 \text{Eb1 Eb2 Ib1 Ib2})}{18 (c+\text{Eb1 Ib1}) (c+\text{Eb2 Ib2})}$	$-\frac{c^2 (\text{Eb1 Ib1}-\text{Eb2 Ib2})}{6 (c+\text{Eb1 Ib1}) (c+\text{Eb2 Ib2})}$	0
$-\frac{c^2 (\text{Eb1 Ib1}-\text{Eb2 Ib2})}{6 (c+\text{Eb1 Ib1}) (c+\text{Eb2 Ib2})}$	$\frac{1}{3} \left(\frac{c \text{Eb1 Ib1} (6 c+3 \text{Eb1 Ib1})}{(c+\text{Eb1 Ib1}) (3 c+\text{Eb1 Ib1})} + \frac{c \text{Eb2 Ib2} (6 c+3 \text{Eb2 Ib2})}{(c+\text{Eb2 Ib2}) (3 c+\text{Eb2 Ib2})} + 4 \text{Ec2 Ic2} \right)$	0
0	0	$\frac{\text{Ab2 Eb2}}{6} + \frac{4 \text{Ec3 Ic3}}{9}$
$-\frac{c \text{Eb2 Ib2}}{18 (c+\text{Eb2 Ib2})}$	$-\frac{c \text{Eb2 Ib2}}{6 (c+\text{Eb2 Ib2})}$	0
$\frac{c \text{Eb2 Ib2}}{6 c+6 \text{Eb2 Ib2}}$	$\frac{c^2 \text{Eb2 Ib2}}{3 c^2+4 c \text{Eb2 Ib2}+\text{Eb2}^2 \text{Ib2}^2}$	$\frac{2 \text{Ec3 Ic3}}{3}$
0	0	0
0	0	0
0	0	0
0	$-\frac{2 \text{Ec2 Ic2}}{3}$	0
$-\frac{\text{Ac2 Ec2}}{3}$	0	0
0	$\frac{2 \text{Ec2 Ic2}}{3}$	0
0	0	$-\frac{4 \text{Ec3 Ic3}}{9}$
0	0	0
0	0	$\frac{2 \text{Ec3 Ic3}}{3}$

```
In[32]:= TableForm[TakeColumns[mat37, {8, 11}], TableHeadings -> {None, {8, 9, 10, 11}}]
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Out[32]//TableForm=
```

8	9	10	11
0	0	$-\frac{4 \text{Ec1 Ic1}}{9}$	0
0	0	0	$-\frac{\text{Ac1 Ec1}}{3}$
0	0	$-\frac{2 \text{Ec1 Ic1}}{3}$	0
0	0	0	0
$-\frac{c \text{Eb2 Ib2}}{18 (c+\text{Eb2 Ib2})}$	$\frac{c \text{Eb2 Ib2}}{6 c+6 \text{Eb2 Ib2}}$	0	0
$-\frac{c \text{Eb2 Ib2}}{6 (c+\text{Eb2 Ib2})}$	$\frac{c^2 \text{Eb2 Ib2}}{3 c^2+4 c \text{Eb2 Ib2}+\text{Eb2}^2 \text{Ib2}^2}$	0	0
0	$\frac{2 \text{Ec3 Ic3}}{3}$	0	0
$\frac{c \text{Eb2 Ib2}+6 \text{Ac3 Ec3} (c+\text{Eb2 Ib2})}{18 (c+\text{Eb2 Ib2})}$	$-\frac{c \text{Eb2 Ib2}}{6 (c+\text{Eb2 Ib2})}$	0	0
$-\frac{c \text{Eb2 Ib2}}{6 (c+\text{Eb2 Ib2})}$	$\frac{c \text{Eb2 Ib2} (2 c+\text{Eb2 Ib2})}{3 c^2+4 c \text{Eb2 Ib2}+\text{Eb2}^2 \text{Ib2}^2} + \frac{4 \text{Ec3 Ic3}}{3}$	0	0
0	0	$\frac{1}{18} (3 \text{Ab3 Eb3} + 8 \text{Ec1 Ic1} + 8 \text{Ec4 Ic4})$	0
0	0	0	$\frac{c \text{Eb3 Ib3}+6 \text{Ac1 Ec1} (c+\text{Eb3 Ib3})+6 \text{Ac4 Ec4} (c+\text{Eb3 Ib3})}{18 (c+\text{Eb3 Ib3})}$
0	0	$\frac{2}{3} (-\text{Ec1 Ic1} + \text{Ec4 Ic4})$	$\frac{c \text{Eb3 Ib3}}{6 c+6 \text{Eb3 Ib3}}$
0	0	$-\frac{\text{Ab3 Eb3}}{6}$	0
0	0	0	$-\frac{c \text{Eb3 Ib3}}{18 (c+\text{Eb3 Ib3})}$
0	0	0	$\frac{c \text{Eb3 Ib3}}{6 c+6 \text{Eb3 Ib3}}$
0	$-\frac{2 \text{Ec3 Ic3}}{3}$	0	0
$-\frac{\text{Ac3 Ec3}}{3}$	0	0	0
0	$\frac{2 \text{Ec3 Ic3}}{3}$	0	0

```
In[35]:= TableForm[TakeColumns[mat37, {12, 13}], TableHeadings → {Automatic, {12, 13}}]
```

```
Out[35]//TableForm=
```

	12	13
1	$\frac{2 \text{Ec1 Ic1}}{3}$	0
2	0	0
3	$\frac{2 \text{Ec1 Ic1}}{3}$	0
4	0	$-\frac{4 \text{Ec2 Ic2}}{9}$
5	0	0
6	0	$-\frac{2 \text{Ec2 Ic2}}{3}$
7	0	0
8	0	0
9	0	0
10	$\frac{2}{3} (-\text{Ec1 Ic1} + \text{Ec4 Ic4})$	$-\frac{\text{Ab3 Eb3}}{6}$
11	$\frac{\text{c Eb3 Ib3}}{6 \text{c} + 6 \text{Eb3 Ib3}}$	0
12	$\frac{4}{3} \left(\frac{3 \text{c Eb3 Ib3} (2 \text{c} + \text{Eb3 Ib3})}{4 (3 \text{c}^2 + 4 \text{c Eb3 Ib3} + \text{Eb3}^2 \text{Ib3}^2)} + \text{Ec1 Ic1} + \text{Ec4 Ic4} \right)$	0
13	0	$\frac{1}{18} (3 \text{Ab3 Eb3} + 3 \text{Ab4 Eb4} + 8 \text{Ec2 Ic2} + 8 \text{Ec5 Ic5})$
14	$-\frac{\text{c Eb3 Ib3}}{6 (\text{c} + \text{Eb3 Ib3})}$	0
15	$\frac{\text{c}^2 \text{Eb3 Ib3}}{3 \text{c}^2 + 4 \text{c Eb3 Ib3} + \text{Eb3}^2 \text{Ib3}^2}$	$\frac{2}{3} (-\text{Ec2 Ic2} + \text{Ec5 Ic5})$
16	0	$-\frac{\text{Ab4 Eb4}}{6}$
17	0	0
18	0	0

```
In[4]:= TableForm[TakeColumns[mat37, {14, 15}], TableHeadings → {None, {14, 15}}]
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```
Out[4]//TableForm=
```

14	15
0	0
0	0
0	0
0	0
0	$\frac{2 \text{Ec2 Ic2}}{3}$
$-\frac{\text{Ac2 Ec2}}{3}$	0
0	$\frac{2 \text{Ec2 Ic2}}{3}$
0	0
0	0
0	0
0	0
$-\frac{\text{c Eb3 Ib3}}{18 (\text{c} + \text{Eb3 Ib3})}$	$\frac{\text{c Eb3 Ib3}}{6 \text{c} + 6 \text{Eb3 Ib3}}$
$-\frac{\text{c Eb3 Ib3}}{6 (\text{c} + \text{Eb3 Ib3})}$	$\frac{\text{c}^2 \text{Eb3 Ib3}}{3 \text{c}^2 + 4 \text{c Eb3 Ib3} + \text{Eb3}^2 \text{Ib3}^2}$
0	$\frac{2}{3} (-\text{Ec2 Ic2} + \text{Ec5 Ic5})$
$\frac{6 \text{Ac2 Ec2} (\text{c} + \text{Eb3 Ib3}) (\text{c} + \text{Eb4 Ib4}) + 6 \text{Ac5 Ec5} (\text{c} + \text{Eb3 Ib3}) (\text{c} + \text{Eb4 Ib4}) + \text{c} (\text{c Eb3 Ib3} + \text{c Eb4 Ib4} + 2 \text{Eb3 Eb4 Ib3 Ib4})}{18 (\text{c} + \text{Eb3 Ib3}) (\text{c} + \text{Eb4 Ib4})}$	$-\frac{\text{c}^2 (\text{Eb3 Ib3} - \text{Eb4 Ib4})}{6 (\text{c} + \text{Eb3 Ib3}) (\text{c} + \text{Eb4 Ib4})}$
$-\frac{\text{c}^2 (\text{Eb3 Ib3} - \text{Eb4 Ib4})}{6 (\text{c} + \text{Eb3 Ib3}) (\text{c} + \text{Eb4 Ib4})}$	$\frac{1}{3} \left(\frac{\text{c Eb3 Ib3} (6 \text{c} + 3 \text{Eb3 Ib3})}{(\text{c} + \text{Eb3 Ib3}) (3 \text{c} + \text{Eb3 Ib3})} + \frac{\text{c Eb4 Ib4} (6 \text{c} + 3 \text{Eb4 Ib4})}{(\text{c} + \text{Eb4 Ib4}) (3 \text{c} + \text{Eb4 Ib4})} + 4 \text{Ec2 Ic2} + 4 \text{Ec5 Ic5} \right)$
0	0
$-\frac{\text{c Eb4 Ib4}}{18 (\text{c} + \text{Eb4 Ib4})}$	$-\frac{\text{c Eb4 Ib4}}{6 (\text{c} + \text{Eb4 Ib4})}$
$\frac{\text{c Eb4 Ib4}}{6 \text{c} + 6 \text{Eb4 Ib4}}$	$\frac{\text{c}^2 \text{Eb4 Ib4}}{3 \text{c}^2 + 4 \text{c Eb4 Ib4} + \text{Eb4}^2 \text{Ib4}^2}$

```
In[37]:= TableForm[TakeColumns[mat37, {16, 18}], TableHeadings → {Automatic, {16, 17, 18}}]
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Out[37]//TableForm=
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	16	17	18
1	0	0	0
2	0	0	0
3	0	0	0
4	0	0	0
5	0	0	0
6	0	0	0
7	$-\frac{4 \text{Ec3 Ic3}}{9}$	0	$\frac{2 \text{Ec3 Ic3}}{3}$
8	0	$-\frac{\text{Ac3 Ec3}}{3}$	0
9	$-\frac{2 \text{Ec3 Ic3}}{3}$	0	$\frac{2 \text{Ec3 Ic3}}{3}$
10	0	0	0
11	0	0	0
12	0	0	0
13	$-\frac{\text{Ab4 Eb4}}{6}$	0	0
14	0	$-\frac{\text{c Eb4 Ib4}}{18 (\text{c} + \text{Eb4 Ib4})}$	$\frac{\text{c Eb4 Ib4}}{6 \text{c} + 6 \text{Eb4 Ib4}}$
15	0	$-\frac{\text{c Eb4 Ib4}}{6 (\text{c} + \text{Eb4 Ib4})}$	$\frac{\text{c}^2 \text{Eb4 Ib4}}{3 \text{c}^2 + 4 \text{c Eb4 Ib4} + \text{Eb4}^2 \text{Ib4}^2}$
16	$\frac{1}{18} (3 \text{Ab4 Eb4} + 8 \text{Ec3 Ic3} + 8 \text{Ec6 Ic6})$	0	$\frac{2}{3} (-\text{Ec3 Ic3} + \text{Ec6 Ic6})$
17	0	$\frac{\text{c Eb4 Ib4} + 6 \text{Ac3 Ec3} (\text{c} + \text{Eb4 Ib4}) + 6 \text{Ac6 Ec6} (\text{c} + \text{Eb4 Ib4})}{18 (\text{c} + \text{Eb4 Ib4})}$	$-\frac{\text{c Eb4 Ib4}}{6 (\text{c} + \text{Eb4 Ib4})}$
18	$\frac{2}{3} (-\text{Ec3 Ic3} + \text{Ec6 Ic6})$	$-\frac{\text{c Eb4 Ib4}}{6 (\text{c} + \text{Eb4 Ib4})}$	$\frac{4}{3} \left(\frac{3 \text{c Eb4 Ib4} (2 \text{c} + \text{Eb4 Ib4})}{4 (3 \text{c}^2 + 4 \text{c Eb4 Ib4} + \text{Eb4}^2 \text{Ib4}^2)} + \text{Ec3 Ic3} + \text{Ec6 Ic6} \right)$

The parametric right hand side vector is

$$\{f2, -3 * w1, (-3 * w1) / (1 + (\text{Eb1} * \text{Ib1}) / (3 * c)), 0, -3 * w1 - 3 * w2, (3 * w1) / (1 + (\text{Eb1} * \text{Ib1}) / (3 * c)) - (3 * w2) / (1 + (\text{Eb2} * \text{Ib2}) / (3 * c)), 0, -3 * w2, (3 * w2) / (1 + (\text{Eb2} * \text{Ib2}) / (3 * c)), f1, -3 * w3, (-3 * w3) / (1 + (\text{Eb3} * \text{Ib3}) / (3 * c)), 0, -3 * w3 - 3 * w4, (3 * w3) / (1 + (\text{Eb3} * \text{Ib3}) / (3 * c)) - (3 * w4) / (1 + (\text{Eb4} * \text{Ib4}) / (3 * c)), 0, -3 * w4, (3 * w4) / (1 + (\text{Eb4} * \text{Ib4}) / (3 * c))\}$$

The parameters vary within the following intervals

$\{Eb1 \rightarrow \text{Interval}[\{208950000, 211050000\}], Eb2 \rightarrow \text{Interval}[\{208950000, 211050000\}], Eb3 \rightarrow \text{Interval}[\{208950000, 211050000\}],$
 $Eb4 \rightarrow \text{Interval}[\{208950000, 211050000\}], Ib1 \rightarrow \text{Interval}[\{\frac{460287}{2000000000}, \frac{464913}{2000000000}\}], Ib2 \rightarrow \text{Interval}[\{\frac{460287}{2000000000}, \frac{464913}{2000000000}\}],$
 $Ib3 \rightarrow \text{Interval}[\{\frac{460287}{2000000000}, \frac{464913}{2000000000}\}], Ib4 \rightarrow \text{Interval}[\{\frac{460287}{2000000000}, \frac{464913}{2000000000}\}], Ab1 \rightarrow \text{Interval}[\{\frac{840377}{1000000000}, \frac{848823}{1000000000}\}],$
 $Ab2 \rightarrow \text{Interval}[\{\frac{840377}{1000000000}, \frac{848823}{1000000000}\}], Ab3 \rightarrow \text{Interval}[\{\frac{840377}{1000000000}, \frac{848823}{1000000000}\}], Ab4 \rightarrow \text{Interval}[\{\frac{840377}{1000000000}, \frac{848823}{1000000000}\}],$
 $Ec1 \rightarrow \text{Interval}[\{208950000, 211050000\}], Ec2 \rightarrow \text{Interval}[\{208950000, 211050000\}], Ec3 \rightarrow \text{Interval}[\{208950000, 211050000\}],$
 $Ec4 \rightarrow \text{Interval}[\{208950000, 211050000\}], Ec5 \rightarrow \text{Interval}[\{208950000, 211050000\}], Ec6 \rightarrow \text{Interval}[\{208950000, 211050000\}],$
 $Ic1 \rightarrow \text{Interval}[\{\frac{383473}{2000000000}, \frac{387327}{2000000000}\}], Ic2 \rightarrow \text{Interval}[\{\frac{383473}{2000000000}, \frac{387327}{2000000000}\}], Ic3 \rightarrow \text{Interval}[\{\frac{383473}{2000000000}, \frac{387327}{2000000000}\}],$
 $Ic4 \rightarrow \text{Interval}[\{\frac{383473}{2000000000}, \frac{387327}{2000000000}\}], Ic5 \rightarrow \text{Interval}[\{\frac{383473}{2000000000}, \frac{387327}{2000000000}\}], Ic6 \rightarrow \text{Interval}[\{\frac{383473}{2000000000}, \frac{387327}{2000000000}\}],$
 $Ac1 \rightarrow \text{Interval}[\{\frac{130743}{100000000}, \frac{132057}{100000000}\}], Ac2 \rightarrow \text{Interval}[\{\frac{130743}{100000000}, \frac{132057}{100000000}\}], Ac3 \rightarrow \text{Interval}[\{\frac{130743}{100000000}, \frac{132057}{100000000}\}],$
 $Ac4 \rightarrow \text{Interval}[\{\frac{130743}{100000000}, \frac{132057}{100000000}\}], Ac5 \rightarrow \text{Interval}[\{\frac{130743}{100000000}, \frac{132057}{100000000}\}], Ac6 \rightarrow \text{Interval}[\{\frac{130743}{100000000}, \frac{132057}{100000000}\}],$
 $c \rightarrow \text{Interval}[\{995000000, 1005000000\}], w1 \rightarrow \text{Interval}[\{\frac{597}{20}, \frac{603}{20}\}], w2 \rightarrow \text{Interval}[\{\frac{597}{20}, \frac{603}{20}\}],$
 $w3 \rightarrow \text{Interval}[\{\frac{597}{20}, \frac{603}{20}\}], w4 \rightarrow \text{Interval}[\{\frac{597}{20}, \frac{603}{20}\}], f1 \rightarrow \text{Interval}[\{\frac{199}{2}, \frac{201}{2}\}], f2 \rightarrow \text{Interval}[\{\frac{199}{2}, \frac{201}{2}\}]\}$