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ON SOME FINITE-DIMENSIONAL REPRESENTATIONS OF ARTIN BRAID GROUP*

Valentin V. Iliev

The author studies certain homomorphic images G of the Artin braid group on n strands in finite symmetric groups. Any permutation group G is an extension of the symmetric group on n letters by an appropriate abelian group. The extension G depends on an integer parameter $q \geq 1$, and splits if and only if 4 does not divide q. In the case when q is odd, all finite-dimensional irreducible representations of G are found, thus finding an infinite series of irreducible representations of the braid group.

The braid group B_n is defined by E. Artin in his article [1]. He gives two definitions: geometrical and algebraic. As an abstract group, B_n has a presentation on $n-1$ generators β_1, β_2, …, β_{n-1}, subject to the following braid relations

$$\beta_r\beta_s = \beta_s\beta_r,\ |r-s| \geq 2,$$

$$\beta_r\beta_s\beta_r = \beta_s\beta_r\beta_s,\ |r-s| = 1.$$

A group is said to be *n-braid-like* if it is a homomorphic image of the Artin braid group B_n.

The geometrical definition starts with introducing the braids, the elements of a group which turns out to be isomorphic to the algebraical braid group B_n. A *braid* σ consists of n descending strands (topological intervals) in the real 3-dimensional space $\mathbb{R}^3$ (the third coordinate axis being turned downward), subject to the following conditions:

(a) any strand joints a point of the form $(i,0,0)$ with a point of the form $(j,0,1)$ $1 \leq i,j \leq n$;

(b) the strands do not intersect each other;

(c) the braids are considered up to isotopy in $\mathbb{R}^3$.

The *product* of two braids σ and τ is a braid obtained by identifying the endpoint $(j,0,1)$ of σ with the corresponding endpoint $(j,0,0)$ of τ, and then scaling. The *unit* braid e consists of n strands connecting the point $(i,0,0)$ with the point $(i,0,1)$, $1 \leq i \leq n$. The *elementary* braid σ_s, $1 \leq s \leq n-1$, is a modification of the unit braid e. It consists of the strands that joint the points $(k,0,0)$ with $(k,0,1)$, $k = 1,\ldots,n$, $k \neq s, s+1$, the point $(s,0,0)$ with $(s+1,0,1)$, and the point $(s+1,0,0)$ with $(s,0,1)$, the last one being overgoing. In case the last one is undergoing, we obtain σ_s^{-1}. The elementary braids σ_s satisfy the braid relations and the mapping $\beta_s \mapsto \sigma_s$ can be extended to an isomorphism of the group B_n and the group generated by σ_s.

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Fig. 1. A real braid

The algebraic definition yields existence of the following two homomorphisms:

$$B_n \to \mathbb{Z},\ \beta_s \mapsto 1,\ 1 \le s \le n-1,$$

and

$$\nu\colon B_n \to S_n,\ \beta_s \mapsto (s, s+1),\ 1 \le s \le n-1,$$

where S_n is the symmetric group on n symbols $1, \dots, n$, and $(s, s+1)$ is the permutation that transposes s and $s+1$. The kernel of the first homomorphism is the commutator subgroup B_n^c of B_n, and the kernel P_n of the second homomorphism ν is said to be the *pure braid group.* Let $\mathbb{C}$ be the field of complex numbers and let $\mathbb{C}^*$ be its multiplicative subgroup of non-zero complex numbers. We obtain immediately the following two propositions:

Proposition 1. *One has* $\mathrm{Hom}(B_n, \mathbb{C}^*) \simeq \mathbb{C}^*$*, where the formulae* $\chi_c(\beta_s) = c$*,* $1 \le s \le n-1$*,* $c \in \mathbb{C}^*$*, exhaust all one-dimensional representations of* B_n*.*

Proposition 2. *Every irreducible finite-dimensional representation of* S_n *produces* via *composing with* ν *an irreducible finite-dimensional representation of* B_n*.*

In particular, the two one-dimensional representations of S_n, which govern the statistical behaviour of bosons (Bose-Einstein statistics) and fermions (Fermi-Dirac statistics), are members of the infinite set $\mathrm{Hom}(B_n, \mathbb{C}^*)$ of one-dimensional representations of B_n, which are responsible for statistical behaviour of abelian anions. The abelian anions are quasi-particles that "live" in two-dimensional spaces, and there are experimental data that they can be used for the construction of topological quantum computer (an idea of the Russian physicist Alexei Kitaev from 1997). The finite-dimensional representations of B_n of dimension ≥ 2 govern the statistical behaviour of non-abelian anions which also are ingredients of the proposed architectures for a topological quantum computer, but their existence is not experimentally confirmed yet.

Our aim is to show that for any integer $q \ge 1$ there exists a group $B_n(q)$ which is intermediate, i.e. $B_n \to B_n(q) \to S_n$, and still finite, and whose finite-dimensional representations can be found, in principle. In case q is an odd integer, the group $B_n(q)$ is the semi-direct product of S_n and the elementary abelian group $(\mathbb{Z}/q\mathbb{Z})^n$, and we use

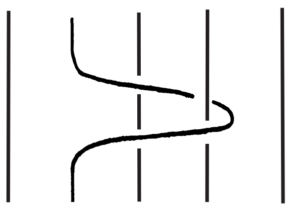

Fig. 2. Artin's braid σ

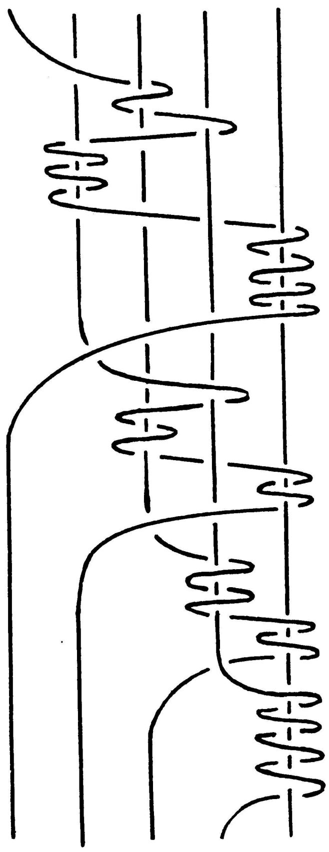

Fig. 3. Artin's braid τ

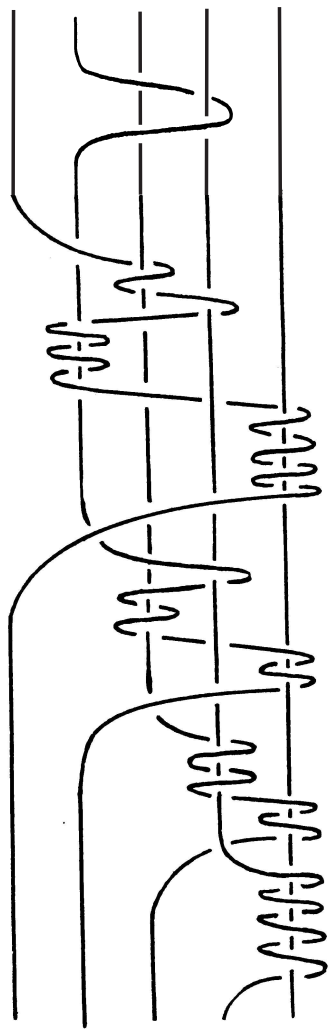

Fig. 4.Their product $\sigma\tau$

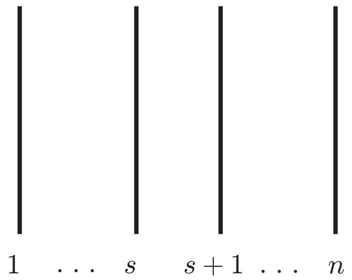

Fig. 5. The unit braid

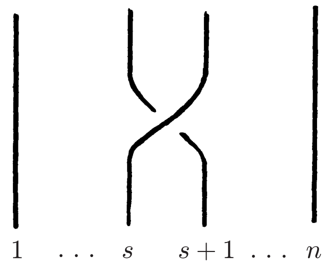

Fig. 6. The elementary braid σ_s

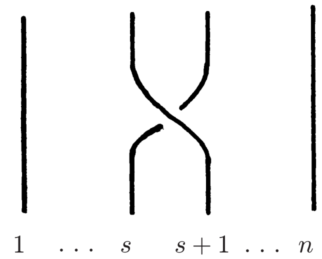

Fig. 7. The braid σ_s^{-1}

the method of "little groups" of Wigner and Mackey (see [4, Proposition 25]) in order to describe completely all irreducible representations of $B_n(q)$. Thus *via* lifting, we obtain a series of finite-dimensional irreducible representations of the Artin braid group B_n.

First, we introduce some notation. We define the symmetric group S_∞ as the group of all permutations of the set of positive integers, which fix all but finitely many elements. The symmetric group S_d is identified with the subgroup of S_∞, consisting of all permutations fixing any $k > d$. We denote by ω the injective endomorphism of S_∞, defined by the rule

$$(\omega(\sigma))(k) = \sigma(k-1) + 1,\ k \geq 2,\ (\omega(\sigma))(1) = 1.$$

We assume that d and n are integers, $d \geq 1$, $n \geq 3$, and, as usual, we identify the wreath product $S_d \wr S_n$ with the image of its natural faithful permutation representation ([3, 4.1.18]) and for each subgroup $W \leq S_d$ we identify the wreath product $W \wr S_n$ with its image *via* the above inclusion. In other words, $S_d \wr S_n$ is the group of all permutations of $1, \ldots, d, d+1, \ldots, 2d, 2d+1, \ldots, nd$, which permute the integer-valued intervals $[1, d], [d+1, 2d] \ldots, [(n-1)d+1, nd]$.

Let $\theta_s \in S_{nd}$, be the involutions

$$\theta_s = \begin{pmatrix} (s-1)d+1 & \cdots & sd & sd+1 & \cdots & (s+1)d \\ sd+1 & \cdots & (s+1)d & (s-1)d+1 & \cdots & sd \end{pmatrix} \in S_{nd},$$

$s = 1, \ldots, n-1$. We set $\Sigma_n = \langle \theta_1, \theta_2, \ldots, \theta_{n-1} \rangle \leq S_{nd}$. Then the direct product $W^{(n)} = W\omega^d(W) \ldots \omega^{(n-1)d}(W)$ is a normal subgroup of of the wreath product $W \wr S_n$ (its base group), Σ_n is a complement of $W^{(n)}$, and we have that $W \wr S_n$ is the semi-direct product of Σ_n by $W^{(n)}$: $W \wr S_n = W^{(n)} \cdot \Sigma_n$. The isomorphism $\Sigma_n \simeq S_n$ maps the involution θ_s onto the transposition $(s, s+1)$, $s = 1, \dots n-1$, and we identify both groups by this isomorphism.

In particular, for any $W \leq S_d$ the wreath product $W \wr S_2$ is the semi-direct product of $\Sigma_2 = \langle \theta \rangle$ by its base group $W^{(2)}$, where $\theta = \theta_1$. The left coset $\theta W^{(2)}$ of $W^{(2)}$ in $W \wr S_2$ consists of permutations of $[1, 2d]$ that map $[1, d]$ onto $[d+1, 2d]$.

Let $W \leq S_d$ be a permutation group, and let $p(W)$ be the number of conjugacy classes of W. The action of W on itself *via* conjugation can be used in order to prove

Lemma 3. *The number of ordered pairs of elements of the group W, whose components commute, is equal to $|W|p(W)$.*

A pair of permutations η, ζ from S_∞ is said to be *braid-like* if $\eta\zeta \neq \zeta\eta$, and $\eta\zeta\eta = \zeta\eta\zeta$.

Theorem 4. *Let $W \leq S_d$ be a permutation group, and let $\sigma \in \theta W^{(2)}$, $\sigma = \theta\varsigma_1\omega^d(\varsigma_2)$, where $\varsigma_1, \varsigma_2 \in W$. Then, the following three statements are equivalent:*

(i) *the pair of permutations σ, $\omega^d(\sigma)$ is braid-like;*

(ii) *there exists a permutation $\tau \in W$ such that*

$$\sigma^2 = \tau\omega^d(\tau); \tag{1}$$

(iii) *the permutations ς_1 and ς_2 commute.*

Let $\sigma \in \theta(S_d)^{(2)}$, where $\theta = \theta_1 \in \Sigma_n$. The permutations

$$\sigma_1 = \sigma,\ \sigma_2 = \omega^d(\sigma), \dots, \sigma_{n-1} = \omega^{(n-2)d}(\sigma)$$

are from the wreath product $S_d \wr S_n$. Let $B_n(\sigma)$ be the subgroup of $S_d \wr S_n$, generated by these permutations, i.e.

$$B_n(\sigma) = \langle \sigma_1, \sigma_2, \dots, \sigma_{n-1} \rangle.$$

If $W \leq S_d$ is a permutation group and $\sigma \in \theta W^{(2)}$, then $B_n(\sigma)$ is a subgroup of the wreath product $W \wr S_n$.

We suppose that the pair of permutations σ, $\omega^d(\sigma)$, where $\sigma \in \theta S_d \omega^d(S_d)$, is braid-like, and let $\sigma^2 = \tau\omega^d(\tau)$, $\tau \in S_d$.

The intersection $BW_n(\sigma) = B_n(\sigma) \cap W^{(n)}$ is a normal subgroup of $B_n(\sigma)$. In particular, if $\tau \in W$, and $\langle \tau \rangle$ is the cyclic group generated by τ, then $A_n(\sigma) = B_n(\sigma) \cap \langle \tau \rangle^{(n)}$ is an abelian normal subgroup of both $B_n(\sigma)$ and $BW_n(\sigma)$.

Theorem 5. *Let $W \leq S_d$ be a permutation group. Let us suppose that $\sigma \in \theta W^{(2)}$, and that the pair of permutations σ, $\omega^d(\sigma)$ is braid-like. Let $\sigma^2 = \tau\omega^d(\tau)$, and let q be the order of $\tau \in W$. Then,*

(i) *the map $\theta_s \mapsto \sigma_s \bmod BW_n(\sigma)$, $s = 1, \dots, n-1$, can be extended to an isomorphism $S_n \to B_n(\sigma)/BW_n(\sigma)$;*

(ii) *one has $BW_n(\sigma) = A_n(\sigma)$, and $A_n(\sigma)$ is an abelian group, isomorphic to*

$$\underbrace{\mathbb{Z}/q\mathbb{Z} \coprod \cdots \coprod \mathbb{Z}/q\mathbb{Z}}_{n-1 \text{ times}} \coprod \mathbb{Z}/q_2\mathbb{Z} \tag{2}$$

where $q = q_2\delta$, and δ is the greatest common divisor of q and 2;

(iii) *the group $B_n(\sigma)$ is an extension of the symmetric group S_n by the abelian group (2):*

$$0 \longrightarrow \mathbb{Z}/q\mathbb{Z} \coprod \cdots \coprod \mathbb{Z}/q\mathbb{Z} \coprod \mathbb{Z}/q_2\mathbb{Z} \xrightarrow{\mu} B_n(\sigma) \xrightarrow{\pi} S_n \longrightarrow 1; \tag{3}$$

(iv) *the extension* (3) *splits if and only if* 4 *does not divide* q.

Corollary 6. (i) *The abelian group* $A_n(\sigma)$ *does not depend on the permutation* σ*, but only on the number* q*;*

(ii) *if* 4 *does not divide* q*, then the group* $B_n(\sigma)$ *does not depend on the permutation* σ *but only on the number* q.

Let ι_s be the restriction of the conjugation by σ_s, $s = 1, \dots, n-1$, on the normal subgroup (2). Then ι_s are involutions of this abelian group.

Proposition 7. *The monodromy homomorphism*

$$m\colon S_n \to \mathrm{Aut}(\mathbb{Z}/q\mathbb{Z} \coprod \cdots \coprod \mathbb{Z}/q\mathbb{Z} \coprod \mathbb{Z}/q_2\mathbb{Z}),\ \theta_s \mapsto \iota_s,$$

that corresponds to the extension (3) *is injective.*

From now on we assume that the order q of the permutation $\tau \in S_d$ from equation (1) is odd. Corollary 6 yields that we can denote by $A_n(q)$ and $B_n(q)$ the groups $A_n(\sigma)$ and $B_n(\sigma)$, respectively. In this case the group $B_n(q)$ is isomorphic to the semi-direct product of S_n by the elementary abelian group $A_n(q) = (\mathbb{Z}/q\mathbb{Z})^n$. Moreover, $A_n(q)$ has a structure of S_n-module, given by the monodromy homomorphism m from Proposition 7. The group $A_n(q)$ can also be considered as a free $\mathbb{Z}/q\mathbb{Z}$-module of rank n with basis $e_1 = (1, 0, \dots, 0)$, $e_2 = (0, 1, \dots, 0)$, …, $e_n = (0, 0, \dots, 1)$. We identify the group $A_n(q)$ with its multiplicatively written version $\langle\tau\rangle^{(n)}$ *via* the rule $x_1e_1 + \cdots + x_ne_n \mapsto \tau^{x_1}\omega^d(\tau^{x_2}) \dots \omega^{(n-1)d}(\tau^{x_n})$. The involution $m(\theta_s) = \iota_s$, $s = 1, \dots, n-1$, acts on the basis (e_i) by the rule $\iota_s e_s = e_{s+1}$, and $\iota_s e_r = e_r$ for $r \neq s, s+1$. Therefore, if $\zeta \in S_n$, then we have

$$\zeta \cdot x = (x_{\zeta^{-1}(1)}, \dots, x_{\zeta^{-1}(n)})$$

for any $x = x_1e_1 + \cdots + x_ne_n \in A_n(q)$. The contragredient action of the symmetric group S_n on the dual $\mathbb{Z}/q\mathbb{Z}$-module $A_n(q)^*$ is given by the formulae

$$\zeta \cdot b = (b_{\zeta^{-1}(1)}, \dots, b_{\zeta^{-1}(n)})$$

for any $b = b_1x_1 + \cdots + b_nx_n \in A_n(q)^*$.

Let P_n be the set of all partitions of n and let $P_{\leq q;n} \subset P_n$ be the subset consisting of all $\lambda \in P_n$ with length not exceeding q. Any linear form $b = (b_1, \dots, b_n) \in A_n(q)^*$ defines a family of non-negative numbers $(\ell_k^{(b)})_{k \in \mathbb{Z}/(q)}$, where $\ell_k^{(b)} = |\{i \in [1, n] \mid b_i = k\}|$. This family, after ordering from largest to smallest, produces a partition $\lambda^{(b)} = (\lambda_1^{(b)}, \lambda_2^{(b)}, \dots) \in P_n$. The map $A_n(q)^* \to P_n$, $b \mapsto \lambda^{(b)}$, is S_n-equivariant and let $t\colon S_n \backslash A_n(q)^* \to P_n$ be its factorization.

Let us fix a primitive qth root of unity $\varepsilon \in \mathbb{C}^*$. Let $X = \mathrm{Hom}(A_n(q), \mathbb{C}^*)$ be the group of characters of the irreducible representations of the group $A_n(q)$. The group X consists of all maps

$$\chi = \chi_b\colon A_n(q) \to \mathbb{C}^*,\ \chi_b(x) = \varepsilon^{b(x)},$$

where $b \in A_n(q)^*$, and the map $A_n(q)^* \to X$, $b \mapsto \chi_b$, is a group isomorphism. If $\chi = \chi_b$, then the rule $\zeta \cdot \chi = \chi_{\zeta \cdot b}$ defines an action of the symmetric group S_n on the group X. We denote by $\overline{\chi}$ the S_n-orbit of the character $\chi \in X$. *Via transport de structure*, using t, we obtain a map

$$t_X\colon S_n \backslash X \to P_n,\ \overline{\chi} \mapsto \lambda^{(b)},$$

and its image coincides with $P_{\leq q;n}$. For any S_n-orbit $\overline{b} \in t^{-1}(\lambda)$, $\lambda \in P_{\leq q;n}$, we choose a representative b with stabilizer $S_\lambda = S_{\lambda_1} \times S_{\lambda_2} \times \ldots$, and denote the corresponding representative χ_b of $\overline{\chi_b} \in t_X^{-1}(\lambda)$ by $\chi_{b;\lambda}$. Moreover, let $D_\lambda \leq B_n(q)$ be the semi-direct product of the Young subgroup $S_\lambda \leq S_n$ by the abelian group $A_n(q)$: $D_\lambda = A_n(q) \cdot S_\lambda$. Let $\mu_i = (\mu_{i1}, \mu_{i2}, \ldots)$ be a partition of λ_i, $i = 1, 2, \ldots$, and let $[\mu_i]$ be the corresponding irreducible representation of the group S_{λ_i}. The family $(\gamma_{\mu_1,\mu_2,\ldots} = [\mu_1] \otimes [\mu_2] \otimes \cdots)$ consists of all irreducible representations of S_λ. Let $\hat{\gamma}_{\mu_1,\mu_2,\ldots}$ be the composition of $\gamma_{\mu_1,\mu_2,\ldots}$ with the canonical surjective homomorphism $D_\lambda \to S_\lambda$. Since each character $\chi_{b;\lambda}$ has stabilizer S_λ, it can be extended to a one-dimensional character of the group D_λ, which we denote by the same letter.

Let $m_\lambda(x_1, x_2, \ldots)$ be the monomial symmetric function, corresponding to the partition λ, and let $p(n) = p(S_n) = |P_n|$.

Theorem 8. (i) *The induced representations*

$$\mathrm{Ind}_{D_\lambda}^{B_n(q)}(\chi_{b;\lambda} \otimes \hat{\gamma}_{\mu_1,\mu_2,\ldots}),$$

where $\overline{b} \in t^{-1}(\lambda)$, $\lambda \in P_{\leq q;n}$, *are irreducible, pairwise non-isomorphic, and each irreducible representation of the group* $B_n(q)$ *has this form;*

(ii) *the group* $B_n(q)$ *has exactly*

$$\sum_{\lambda \in P_n} p(\lambda_1)p(\lambda_2)\ldots m_\lambda(\underbrace{1,\ldots,1}_{q \text{ times}}, 0, \ldots)$$

irreducible representations.

Example 9. Let $d = 3$, $n = 3$, $\tau = (123)$. Then, $q = 3$, $A(3) = (\mathbb{Z}/(3))^3$, $B_3(3) = A(3) \cdot S_3$. Moreover, $P_3 = \{(3), (2,1), (1^3)\}$, and $P_{\leq 3;3} = P_3$. The inverse images $t^{-1}(\lambda)$ of the partitions $\lambda \in P_3$ *via* the map

$$t\colon S_3\backslash A_3(3)^* \to P_3,\ \overline{b} \mapsto \lambda^{(b)},$$

are

$$t^{-1}((3)) = \{\overline{(0,0,0)}, \overline{(1,1,1)}, \overline{(2,2,2)}\},$$

$$t^{-1}((2,1)) = \{\overline{(0,0,1)}, \overline{(0,0,2)}, \overline{(1,1,0)}, \overline{(1,1,2)}, \overline{(2,2,0)}, \overline{(2,2,1)}\},$$

and

$$t^{-1}((1^3)) = \{\overline{(0,1,2)}\}.$$

Further, we obtain

$$\chi_{(0,0,0);(3)}(x) = 1,\ \chi_{(1,1,1);(3)}(x) = \varepsilon^{x_1+x_2+x_3},\ \chi_{(2,2,2);(3)}(x) = \varepsilon^{2x_1+2x_2+2x_3},$$

$$\chi_{(0,0,1);(2,1)}(x) = \varepsilon^{x_3},\ \chi_{(0,0,2);(2,1)}(x) = \varepsilon^{2x_3},\ \chi_{(1,1,0);(2,1)}(x) = \varepsilon^{x_1+x_2},$$

$$\chi_{(1,1,2);(2,1)}(x) = \varepsilon^{x_1+x_2+2x_3},\ \chi_{(2,2,0);(2,1)}(x) = \varepsilon^{2x_1+2x_2},$$

$$\chi_{(2,2,1);(2,1)}(x) = \varepsilon^{2x_1+2x_2+x_3},\ \chi_{(0,1,2);(1^3)}(x) = \varepsilon^{x_2+2x_3}.$$

The irreducible representations of the stabilizer S_3 are $[(3)]$, $[(1^3)]$, and $[2,1]$ (2-dimensional). Thus taking into account that $D_{(3)} = B_3(3)$, we obtain nine irreducible representations of the group $B_3(3)$, produced by the stabilizer S_3 – six one-dimensional, and three 2-dimensional:

$$\chi_{(0,0,0);(3)} \otimes \widehat{\gamma}_{(3)},\ \chi_{(0,0,0);(3)} \otimes \widehat{\gamma}_{(1^3)},\ \chi_{(0,0,0);(3)} \otimes \widehat{\gamma}_{(2,1)},$$

$$\chi_{(1,1,1);(3)} \otimes \widehat{\gamma}_{(3)},\ \chi_{(1,1,1);(3)} \otimes \widehat{\gamma}_{(1^3)},\ \chi_{(1,1,1);(3)} \otimes \widehat{\gamma}_{(2,1)},$$

$$\chi_{(2,2,2);(3)} \otimes \widehat{\gamma}_{(3)},\ \chi_{(2,2,2);(3)} \otimes \widehat{\gamma}_{(1^3)},\ \chi_{(2,2,2);(3)} \otimes \widehat{\gamma}_{(2,1)}.$$

The stabilizer $S_2 \times S_1$ has two irreducible representations: $[(2)] \otimes [(1)]$, and $[(1^2)] \otimes [(1)]$. Thus, we obtain the following twelve 3-dimensional irreducible representations of the group $B_3(3)$:

$$\mathrm{Ind}_{D_{(2,1)}}^{B_3(3)}(\chi_{(0,0,1);(2,1)} \otimes \widehat{\gamma}_{(2),(1)}),\ \mathrm{Ind}_{D_{(2,1)}}^{B_3(3)}(\chi_{(0,0,1);(2,1)}\widehat{\gamma}_{(1^2),(1)}),$$

$$\mathrm{Ind}_{D_{(2,1)}}^{B_3(3)}(\chi_{(0,0,2);(2,1)} \otimes \widehat{\gamma}_{(2),(1)}),\ \mathrm{Ind}_{D_{(2,1)}}^{B_3(3)}(\chi_{(0,0,2);(2,1)}\widehat{\gamma}_{(1^2),(1)}),$$

$$\mathrm{Ind}_{D_{(2,1)}}^{B_3(3)}(\chi_{(1,1,0);(2,1)} \otimes \widehat{\gamma}_{(2),(1)}),\ \mathrm{Ind}_{D_{(2,1)}}^{B_3(3)}(\chi_{(1,1,0);(2,1)}\widehat{\gamma}_{(1^2),(1)}),$$

$$\mathrm{Ind}_{D_{(2,1)}}^{B_3(3)}(\chi_{(1,1,2);(2,1)} \otimes \widehat{\gamma}_{(2),(1)}),\ \mathrm{Ind}_{D_{(2,1)}}^{B_3(3)}(\chi_{(1,1,2);(2,1)}\widehat{\gamma}_{(1^2),(1)}),$$

$$\mathrm{Ind}_{D_{(2,1)}}^{B_3(3)}(\chi_{(2,2,0);(2,1)} \otimes \widehat{\gamma}_{(2),(1)}),\ \mathrm{Ind}_{D_{(2,1)}}^{B_3(3)}(\chi_{(2,2,0);(2,1)}\widehat{\gamma}_{(1^2),(1)}),$$

$$\mathrm{Ind}_{D_{(2,1)}}^{B_3(3)}(\chi_{(2,2,1);(2,1)} \otimes \widehat{\gamma}_{(2),(1)}),\ \mathrm{Ind}_{D_{(2,1)}}^{B_3(3)}(\chi_{(2,2,1);(2,1)}\widehat{\gamma}_{(1^2),(1)}).$$

The trivial stabilizer $S_1 \times S_1 \times S_1$ has one irreducible representation — the unit representation, and, moreover, $D_{(1^3)} = A_3(3)$, so we obtain one more 6-dimensional irreducible representation of the group $B_3(3)$:

$$\mathrm{Ind}_{A_3(3)}^{B_3(3)}(\chi_{(0,1,2);(1^3)}).$$

We note that $6.1^2 + 3.2^2 + 12.3^2 + 1.6^2 = 3^3 3! = |B_3(3)|$.

Remark 10. The complete proofs of the results from this paper will be published elsewhere.

Remark 11. Most of the figures in the text are taken from Artin's paper [2].

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Valentin V. Iliev
Section of Algebra
Institute of Mathematics and Informatics
Bulgarian Academy of Sciences
Acad. G. Bonchev Str., Bl. 8
1113 Sofia, Bulgaria
e-mail: viliev@math.bas.bg

ВЪРХУ НЯКОИ КРАЙНОМЕРНИ ПРЕДСТАВЯНИЯ НА ГРУПАТА НА АРТИН НА ПЛИТКИТЕ

Валентин В. Илиев

Авторът изучава някои хомоморфни образи G на групата на Артин на плитките върху n нишки в крайни симетрични групи. Получените пермутационни групи G са разширения на симетричната група върху n букви чрез подходяща абелева група. Разширенията G зависят от един целочислен параметър $q \geq 1$ и се разцепват тогава и само тогава, когато 4 не дели q. В случая на нечетно q са намерени всички крайномерни неприводими представяния на G, а те от своя страна генерират безкрайна редица от неприводими представяния на групата на плитките.