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IDENTIFICATION OF MECHANICAL PARAMETERS IN MATHEMATICAL MODEL OF HUMAN ABDOMINAL FASCIA*

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Uniaxial relaxation tests on 14 specimens from umbilical fascia were performed. Time dependant mechanical behaviour of human abdominal fascia was modeled applying the non-linear theory proposed by Maxwell-Gurevich-Rabinovich (MGR). The mechanical parameters for relaxed samples of human umbilical fascia were determined and compared according to direction of loading. It was shown that the initial viscosity parameter η^* and the parameter m^*, which reflects strain rate influence on stress, vary in wide interval not only among samples from different donors but also between samples from one donor. The detailed time course of the apparent viscosity $\eta(t)$ and viscous deformation $\varepsilon(t)$ was calculated. It was established that the results about viscosity and viscous deformation depend on directions of sample's loading.

1. Introduction. Fascia is a connective tissue membrane, which surrounds all organs, muscles, bones and nerve fibers [1]. Abdominal fascia is a part of the abdominal wall, formed of two layers of undulating collagen and elastic fibers, separated by adipose tissue [2]. There are some hypothesis concerning developing of hernia and one of these hypothesis proposed that changed mechanical properties of human abdominal fascia play a main role in the process of herniation. The influence that mechanical properties of abdominal fascia play in hernia development had been studied for the first time from Minns et all who used tensile tests [3]. Punching tests, stress relaxation experiments and elasticity measurements have been also applied to determine the mechanical properties of abdominal fascia [3, 4, 5].

Abdominal fascia was also studied using mathematical modeling which have added to knowledge and understanding of tissue behavior. The review of literature showed that the information about mathematical modeling of visco-elastic properties of human abdominal fascia is scarce. Like most soft biological tissues the abdominal fascia has to be considered as a visco-elastic solid. The most popular theory used for modeling the visco-elastic behaviour of this type of biological tissues is that proposed by Fung [6]. The quasi-linear visco-elastic theory of Fung was applied to relaxation process of abdominal fascia, but it was shown that the behaviour of some fascia samples could not be modeled with Fung's theory [7]. The theoretical description of the mechanical behavior of fascia was also done by Zeng et al., who used power function to fit the stress-strain curves of

***Key words:** human fascia, mathematical model.

the nasal fascia and compare the coefficients found [8]. Another theory, which can be used for mathematical modeling of visco-elastic properties of soft biological tissues, is the theory of Maxwell-Gurevich-Rabinovich (MGR) which was successfully applied for describing the relaxation behavior of the human small intestine [9,10].

The aim of this work is to identified the model parameters in mathematical model of human abdominal fascia, based on nonlinear MGR theory and to reveal the detailed time course of the apparent viscosity and viscous deformation according to directions of loading.

2. Experimental methods. The fascia specimens were extracted from the umbilical region. They were taken during autopsy of cadavers within 12 hours of death, released of fat and immersed in physiological solution. Tests were performed immediately after cutting and realized at ambient temperature $(21 \pm 2)\,^{\circ}\mathrm{C}$ [11].

The specimens with dimensions (10×70) mm were tested. The samples were oriented and cut parallel to the collagen fibers (direction L1) and perpendicular to it (direction L2). Uniaxial relaxation tests on 14 specimens from umbilical fascia were performed. The investigation was done using computer equipped testing machine FU1000/E. The experiments were performed at 1.26 mm/sec rate of elongation. Strips were elongated to 5% of their initial length.

The initial length of the specimens was measured after preconditioning of the specimens. The specimens were subjected to a few cycles of loading and unloading until repeatable mechanical performance was obtained. Thus we reached a steady state called a "preconditioned state" [6]. During relaxation tests the extension of the specimen was kept constant while the load was recorded with time. From the obtained experimental load-time curves the Lagrangian stress σ (tensile force per unit undeformed cross-sectional area) was calculated. The experimental data were presented as stress-time relationships.

3. A method for the determination of visco-elastic parameters based on MGR theory. The main assumption of this model is that the instantaneous nonlinear elastic and time-dependant viscous responses are independent. Thus in the theory, the total strain ε is a sum of an elastic strain e and viscous strain ε^* [9].

$$\varepsilon = e + \varepsilon^*. \tag{1}$$

The following assumptions are made also: at the beginning of relaxation initial strain $\varepsilon = \mathrm{const} = \varepsilon_0$ and there is no volumetric strain, so the compressibility coefficient $\gamma^* = 0$.

Stress relaxation is governed by the following differential equation

$$\frac{d\sigma}{dt} = -\frac{E}{\eta_0^*} f^* \exp\left\{ \left(\frac{1}{3}\gamma^* \sigma(t) + |f^*| \right) / m^* \right\} \tag{2}$$

where η_0^* is the initial coefficient of viscosity, m^* is the parameter, which show the dependence between the relaxation process and the strain rate. Function f^* is defined as:

$$f_i^* = \left(1 + \frac{E_\infty}{E}\right) \sigma_i(t) - E_\infty \varepsilon_0 \tag{3}$$

E_∞ is the visco-elastic modulus when the relaxation process finishes:

$$E_\infty = \frac{\sigma_{\min}}{\varepsilon^*_{\max}} \tag{4}$$

$\sigma_{\min}$ is the values of stress when relaxation is fully completed, $\varepsilon^*_{\max}$ is the maximal values of viscous strain, E is the initial elastic modulus defined as:

$$E = \frac{\sigma_0}{\varepsilon_0}. \tag{5}$$

Here ε_0 is the initial deformation of the sample, σ_0 is the initial stress at $t = 0$. The solution of the differential equation (2) is

$$t = \frac{\eta_0^*}{E + E_\infty}\left[-E_i(-\xi^*) + E_i(-\xi_0)\right] \tag{6}$$

where E_i is the exponential integral function:

$$-E_i(x) = -\int\limits_{\infty}^{x} \frac{e^{-x}}{x} dx, \quad x > 0. \tag{7}$$

$$-E_i(-x) = -C + \ln\frac{1}{x} - \sum_{n=1}^{\infty} \frac{(-x)^n}{n!n}, \quad x > 0,\ C = 0.57721\ldots$$

The values of the argument ξ^* are calculated according to (8):

$$\xi_i^* = \frac{f_i^*}{m^*} = \left|\left(1 + \frac{E_\infty}{E}\right)\sigma_i(t) - E_\infty\varepsilon_0\right| \Big/ m^* \tag{8}$$

The model includes seven parameters, namely: experimentally recorded initial stress σ_0, initial strain ε_0, compressibility coefficient γ^*, local elasticity modulus at the beginning of relaxation process E and visco-elastic modulus when the relaxation process is completedE_∞, initial viscosity coefficient η_0^*, which is a constant not depending on the time and a strain rate modulus m^*, which influences tissue viscosity through the relationship (9).

$$\eta^*(t) = \eta_0^* \exp\left(\frac{E_\infty\varepsilon^* - \sigma(t)}{m^*}\right) \tag{9}$$

The initial viscosity coefficient η_0^* depends on the structure of the material and the temperature. In case of isothermal process η_0^* is a constant. The relationship (9) between the viscosity of the material and stress applied is defined in [9] after numerous experiments.

From all seven parameters only two material parameters η_0^* and m^* have to be found. The other five parameters were determined from experimental curves. These parameters can be calculated using suitable numerical procedure. The following objective function F was proposed :

$$F = \left\{\sum_{i=1}^{N} \left[\sigma_i^{\text{teor}}(\eta_0^*, m^*) - \sigma_i^{\text{exper}}\right]^2 / N\right\}^{1/2} \tag{10}$$

$\sigma_i^{\text{theor}}(\eta_0^*, m^*)$ is the predicted stress computed from the model, σ_i^{exper} is the experimental stress relaxation data. The parameters η_0^* and m^*were determined such that the function F reached its minimum.

4. Results and discussion. The experimental stress-time curves were divided to the following groups – UFL1 (7 samples) and UFL2 (7 samples). The model parameters were found by the method described in the previous Section. The results show that values of all parameters in transverse direction are smaller than those in longitudinal direction, which confirm the orthotropy of the material determined in our preliminary

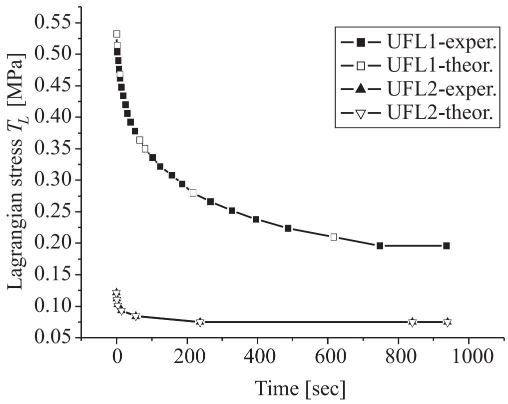

Fig. 1. Stress relaxation of median curves for UFL1 and UFL2. The obtained values of the objective function for presented samples are $F_{\mathrm{UFL1}} = 0.5\%$ and $F_{\mathrm{UFL2}} = 0.25\%$

experimental results [12]. The initial viscosity parameter η_0^* and parameter m^* vary in a wide range [193 -63 822] MPa.sec for η_0^* and [0.014-0.472] MPa for m^*. The maximum values of viscous deformation ε^* are between 1.4%–4.9%. (Not shown).

The median samples from the groups UFL1 and UUFL2 were chosen and presented in Fig. 1. Stress relaxation exhibits a decaying exponential form like other soft biological tissues.

The values of η_0^* for median UFL1 sample changes more than fivefold, while the values of parameter m^* changes eightfold (See Table 1 where the values of model parameters for median curves and absolute median deviation (AMD) are listed).

Table 1. The values of the model parameters for median curves from UFL1 and UFL2

Type	ε_0	$\varepsilon^*_{\max}$	E [MPa]	E_∞ [MPa]	η_0^* [MPa.sec]	m^* [MPa]
UFL1	0.046 ± 0.004	0.032 ± 0.005	10.64 ± 12.1	6.21 ± 62	15013 ± 5320	0.163 ± 0.130
UFL2	0.043 ± 0.001	0.016 ± 0.004	2.82 ± 0.47	4.51 ± 1.6	2047 ± 2304	0.024 ± 0.02

Fig. 2 presents the calculated time courses of the apparent viscosity $\eta^*(t)$ and viscous deformation $\varepsilon^*(t)$ for the chosen median samples shown in Fig. 1. The maximum values of viscosity η^*, varies in interval 2047–15000 [MPa.sec]. The maximum values of viscous deformation ε^* are between 1.6%–3.2%. The viscous deformation ε^* and viscosity sharply increase in first part of relaxation process and than tend to some asymptotic value.

The differences of the values of m^* and η^* for both direction L1 and L2 could be explained according to the physical meaning of the parameters. m^* has dimension of stress and depends inversely on the strain rate. Thus the smaller values of m^*, correspond to a pronounced sensitivity to the strain rate while the higher values of m^* indicate less strain rate sensitivity. This fact was proved during experimental investigation of intestines and probably is valid for fascia too. This hypothesis, however, needs additional experimental investigations. In addition the values of $m^*$and η^* reflect the tissue composition of investigated samples which is very different according to direction of loading that is why

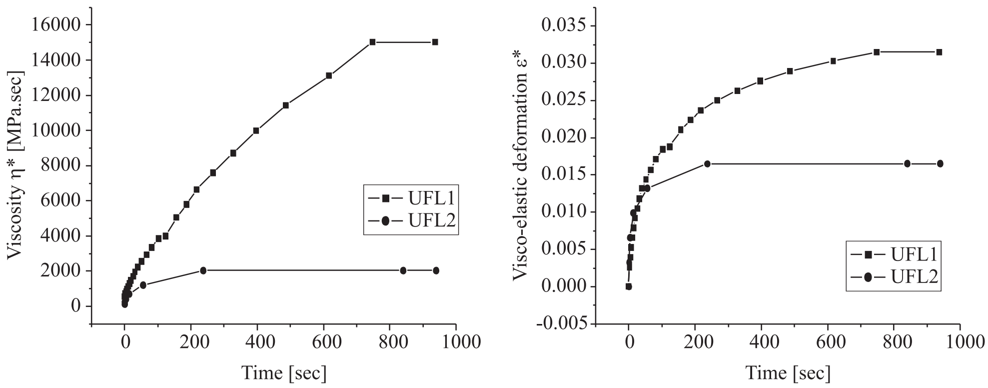

Fig. 2. Calculated time courses of the apparent viscosity $\eta^*(t)$ and viscous deformation $\varepsilon^*(t)$

$m^*$and η^* change in so broad range. The visco-elastic properties in transverse direction are characterized by values of parameters in the same order as those of intestines – about 3100 [MPa.sec] for η_0^* and 0,03[MPa] for m^* [10]. Probably the knowledge about mechanical properties of intestines could be used in prediction of mechanical behavior of fascia.

The visco-elastic theory of MGR comprises larger amount of effects: material compressibility γ^*, initial, or more correctly apparent viscosity η_0^* of the material, time course of the apparent viscosity $\eta^*(t)$, influence of strain rate on relaxation process m^*.

4. Conclusion. The obtained results show that, based on one-dimensional relaxation experiments, the MGR theory gives basic characteristics of the visco-elastic behaviour like time dependence of the apparent viscosity and the viscous deformation. The initial viscosity parameter η_0^* and the coefficient m^*, vary in wide interval not only among samples from different donors but also between samples from one donor. The detailed time course of the apparent viscosity, viscous deformation and comparison of the results about the stress relaxation process based on directions were done. The time courses of the apparent viscosity η(t) and viscous deformation $\varepsilon(t)$ give the possibility to assess the contribution of the viscous effects to the mechanical behavior of the human abdominal fascia.

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ИДЕНТИФИКАЦИЯ НА ПАРАМЕТРИТЕ В МАТЕМАТИЧЕН МОДЕЛ НА ЧОВЕШКА КОРЕМНА ФАСЦИЯ

Миглена Г. Кирилова-Донева

Едномерен експеримент на релаксация беше извършен с 14 образци от човешка пъпна фасция. Механичното поведение на фасцията по време на релаксация беше моделирано прилагайки нелинейната теория на Максвел-Гуревич-Рабинович. Параметрите на модела за изследваните образци бяха определени и стойностите им бяха сравнени в зависимост от посоката на натоварване на образците по време на експеримента. Установено бе, че стойностите на началния вискозитет η_0^* и на параметъра m^*, който се влияе от скоростта на деформация на материала се изменят в много широки граници не само за образци от различни донори, но и за образци от един донор. В резултат от прилагането на модела бе изчислено изменението на вискозитета и вискозната деформация на материала по време на релаксацията. Бе показано, че изменението на вискозитета и вискозната деформация зависи от посоката на натоварване на образците.