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EQUILIBRISTIC PANDISCIPLINARY APPROACH TO  
TECHNOLOGY ENHANCED LEARNING

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In this paper, we describe the core ideas of the equilibristic and pandisciplinary approaches in Technology Enhanced Learning (TEL) and the way they could be combined into a single approach. The goal is to provide means for education, which support both multi- and interdisciplinary approaches. A set of exemplary educational materials is described along with a short discussion about their potential benefits. The work described in this paper is an ongoing research and development, which has a relation with several international and national projects. The practical application of the educational materials will be carried out within these projects.

**1. Introduction.** Technology Enhanced Learning is a relatively new concept and there is no definition of it that is universally accepted. The role of technology in education has been a main research and exploration topic of several European projects (e.g. Kaleidoscope, PROLEARN, STELLAR, Share.TEC, TARGET).

A key outcome of using technology in education is the possibility of effective co-learning of disciplines. It is a common practice for educational institutions to offer multidisciplinary courses, which mix topics and approaches for otherwise distinct disciplines. In some cases multidisciplinary efforts evolve into interdisciplinary, allowing the sharing of approaches and techniques across various disciplines [10]. However, the design of suitable teaching materials is not as straightforward as it is for multidisciplinary subjects.

**2. Pandisciplinary Approach.** Most of the recent technologies and their corresponding professions and educational subjects emerge at the intersection of relatively close domains. For example, *Mathematics* and *Computer Science* share a lot of concepts and provide a productive foundation for crossing disciplines. Pandisciplinarity, on the other hand, covers many disciplines, which are not necessarily close [9].

One of the approaches to measure how close two disciplines are, is based on the taxonomical distance between them.

If we use the *Knowledge Area* branch of the Teacher Education Ontology [1] defined by the Share.TEC project (see Figure 1) we will find that *Mathematics* is close to *Computer Science* as they have a common predecessor. At the same time *Animation* and *Geometry* are placed in different categories in the taxonomy. It is possible to arrange disciplines in different ways using other academic or scientific criteria. The ontology in Figure 1 classifies topics in 9 categories, while other systems use more categories and shallower taxonomies. The *Joint Academic Coding System (JACS)* in UK uses slightly less than

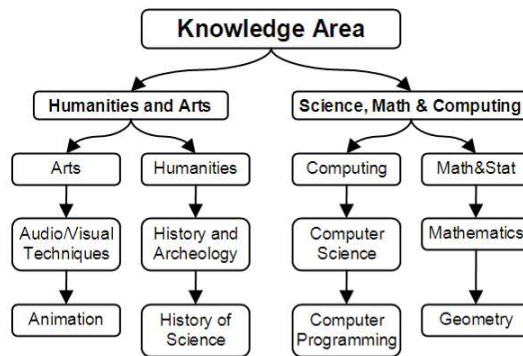


Fig. 1. The collection of 3D virtual devices

20 top-level categories, the *Australian and New Zealand Standard Research Classification (ANZSRC)* lists 22 top-level categories, while the *Classification of Instructional Programs* of the National Center for Education Statistics (USA) has 53 top-level categories.

Different taxonomies produce different and sometimes impractical results in respect to finding how diverse a set of disciplines is. The newly emerging research fields create numerous cross-links within the taxonomies that cannot fit in their tree-like structure. Additionally, this brings some confusion in classification. For example, *Mechanics* is classified under *Engineering (H140)* in JACS, under *Physical Sciences (0203)* in ANZSRC, and under *Science, Mathematics and Computing (4)* in TEO. It may happen that a set of disciplines is diverse in one taxonomy, but is contained in a single category in another taxonomy.

In contrast, the pandisciplinary approach covers a much larger set of disciplines, which are unlikely to exist in a single category in any (or most) of the known taxonomies.

The teaching materials, described further on in this paper, cover disciplines such as: *Geometry, Computer Graphics, Computer Science, Mechanics, Engineering, Art, Film production, History of Science, Linguistics*. Being just an example set, they are not bound to just these disciplines, but can provide potentially unabridged linkage with all disciplines.

**3. Equilibristic Approach.** The implementation of several disciplines poses interesting challenges. The multidisciplinary approach deals with multiple disciplines, which are kept separate, while the interdisciplinary one blends or integrates them. Both approaches have a large number of benefits and are commonly applied in TEL. A limitation, but not a disadvantage, of many teaching and learning materials is that they are precisely focused on predefined disciplines.

The equilibristic approach overcomes this limitation by allowing the same teaching material to be used mono-, multi-, inter- or pandisciplinary depending on the educational goals. The equilibristic approach is not about maintaining equilibrium, but about controlling it. An educator, using the same set of materials, can treat a discipline as dominant in some classes, as subordinate in other classes, or put them in any desired proportion. Thus, the same educational software can be reused in many disciplines. This greater degree of freedom comes at a price – often the development of such materials is



harder and requires a lot of imagination and out-of-the-box thinking. Also, not many teachers are willing to adopt approaches that are not traditional [8].

**4. Exemplary Educational Materials.** This section briefly describes a small set of educational materials, whose purpose is to support technology enhanced equilibristic pandisciplinary approach in education. This set is a component of an on-going development related to using Virtual Reality in the classroom. The set currently includes a collection of virtual mathematical mechanisms, a computer-generated animated 3D film and a teacher-and-user-friendly library for exploratory research. Future extensions to the set will include a conceptual graph of geometrical curves and a book of mathematical problems, based on the 3D film.

The design of the teaching materials was focused on ideas that will completely utilize all materials and reach their maximal *educational density*, which is sometimes defined as “the amount of learning that takes place per unit of time” [7]. However, in the context of our work, the educational density determines *how much* (not *how many*) of the materials have educational potential. A typical TEL-related material has educational value at a functional level – i.e. this value is reached while using the material. The proposed set of materials goes beyond this point, by allowing students to study and explore the fabrics of the materials. Thus, the educational potential exists not only at a functional, but also at a structural level.

The collection of virtual mechanisms comprises more than 60 mathematical devices (see Figure 2). All models support this educational duality – they can be used just as applications, but they can also be templates to start with. This relates to broader goal of making students not mere consumers, but also producers.

Initially the collection was created to support students in *Geometry* and *Computer science* courses by providing easy to rationalize geometrical constructions. Some of the virtual models represent distinct perspectives of the same mathematical concept revealing both variant and invariant properties of the concept [6].

The models could be used as *glass-boxes*, viz. they are available in two mutually completing formats – as executable programs and as standalone animations. The animations are designed to raise the curiosity and to provide food for reflection, whereas the *programs could be considered as materialized model-like hypotheses* about various mathematical constructions.

The glass-box model allows students to see what is inside every model and how its components are defined and managed. In this way they are encouraged to modify the specific construction and invent other models [4].

Let us consider just one of the virtual models – the mechanism for drawing alysoids (see Figure 3). Its use as an educational object can span over several disciplines covering multiple topics within each one of them, e.g. *Mathematics* (parametric equations, projective geometry, exponential and hyperbolic trigonometric functions); *Physics* (force distribution, gravity, application of alysoids, mechanics, linkages, linear and angular speed, reflection); *Engineering and architecture* (suspension bridges, arcs, domes and igloos, perspectives, machine design); *History of science* (different names of the same curve, mistakes of famous scientists – e.g. Galileo wrongly believed, that the alysoid is a parabola); *Biology* (binocular vision, depth perception, colour perception); *Computer science* (object-oriented programming, algorithms, computer graphics and animation, visual effects, grammars and programming languages) and *Information technologies* (working

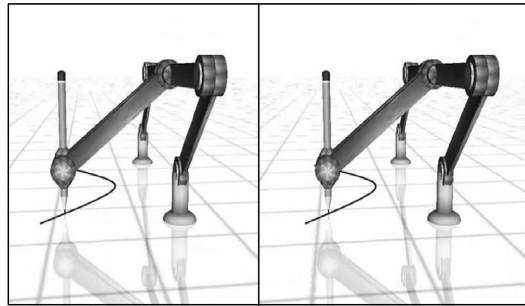


Fig. 3. Stereoscopic image of Alysoidograph for cross-eye viewing

with different applications, preparing and publishing own content, software licenses and rights, information seeking and retrieval).

The development of the animation often reaches areas outside science. Using appropriate colors and textures, defining realistic proportions, balancing the overall visual appearance are components of making artistic artifacts. Except for the video collection, the set of materials contains a short film about different ways of constructing ellipses (see Figure 4). The film has an artistic value by demonstrating several techniques used in film production. Additionally, being computer-generated, its source code is available and students can learn about camera movement, synchronizing sound with motion, timeline management, etc.

Although the film can be treated as a purely artistic artifact, it is loaded with educational potential. A book of mathematical problems will emerge from the film. The goal of the book is to present problems related to *Mathematics, Physics, Applied sciences, Art* and *History*. It will also provide ideas for practical exploration of conical sections.

The collection of mechanisms, the film and the library are software programs and modules implemented in ELICA (<http://elica.net>). This is a research programming environment, which has equilibristic pandisciplinarity embedded as are core design. It supports procedural, functional and object-oriented programming, the language can be flavored to behave like functional LISP, logical PROLOG, procedural C, and stack-based FORTH, and this is implemented in a minimalistic core of less than a dozen of reserved words. This simple core is unaware of basic things like addition of numbers, yet on top of it, it is possible to build interactive virtual reference.

The primary use of ELICA is to help the building of educational software [5]. It is

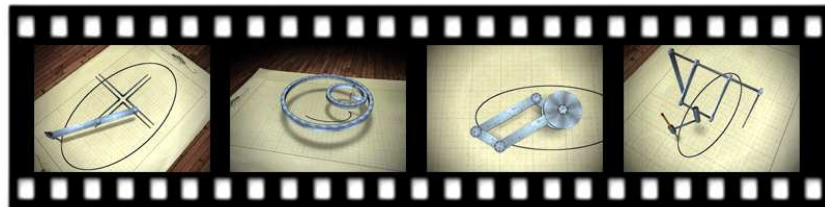


Fig. 4. Frames from the computer-generated film “Ellipses ...”

used in Sofia University for *Computer Graphics* [2] and *Languages and Environments for Education* courses. It is also used in several European projects related to TEL [3]. ELICA provides visual attractiveness and interactivity that are crucial components of the initial student motivation. The developed educational materials are predominantly 3D and they provide consistent look-and-feel as the mainstream entertainment products.

**5. Relations to Other Projects.** The work described in this paper is related to research and development carried in several international and national projects. The *ELICA (Educational Logo Interface for Creative Activities)* project started more than ten years ago. Some of the latest improvements are imposed by specific requirements of the software library for virtual constructions like support of reflections and generation of stereoscopic animations.

The EC-funded *InnoMathEd*<sup>1</sup> (*Innovations in Mathematics Education on European Level*) project emphasizes on the inquiry based learning. The goal of the project is to give students the chance to deepen their mathematical understanding and to acquire key competences essential for lifelong learning. Several teaching and learning materials in this project are developed with ELICA. The video collection is also included in the project.

The FP7 *TARGET*<sup>2</sup> (*Transformative, Adaptive, Responsive and Engaging Environment*) project focuses on “to research, analyse, and develop a new genre of Technology Enhanced Learning environment that supports rapid competence development of individuals”. The educational materials described in this paper may be converted into learning assets that “live” in the TARGET’s knowledge ecosystem.

The FP7 *Share.TEC*<sup>3</sup> (*Sharing Digital Resources in the Teaching Education Community*) project supports the teacher education community through sharing relevant educational resources. The project collects information about relevant resources and classifies it according to a dedicated ontology. ELICA and some of the teaching materials described in this paper are used in courses for students that will become teachers in Mathematics and Informatics. Resources, developed in relation to these courses will be “harvested” by the Share.TEC server and their metadata will be included in the global European-wide digital repository.

**6. Future Plans.** The work presented in this paper presents the current state of an on-going research in the area of equilibristic multidisciplinary and interdisciplinary application of Technology Enhanced Learning. Some of the teaching and learning materials, like the computer-generated film and the collection of virtual mechanisms are already fully developed. Others, like the user library and the Book of problems are in the process of initial implementation.

The main milestones in the future development of the materials are to provide teacher’s guides, exemplary lesson plans and lessons, which will be evaluated with test groups of students. Additionally the materials will be “packed” into separate modules representing properly tagged individual learning objects that are accessible by international digital repositories like the one provided by Share.TEC. Some of the learning objects could be converted into learning stories in the knowledge ecosystem of TARGET.

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<sup>1</sup>InnoMathEd project site, <http://www.math.uni-augsburg.de/de/prof/dida/innomath/>

<sup>2</sup>TARGET project site, <http://www.reachyourtarget.org/>

<sup>3</sup>Share.TEC project site, <http://www.share-tec.eu/>

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## РАВНОВЕСЕН ПАНДИСЦИПЛИНАРЕН ПОДХОД КЪМ УЧЕНЕ, ПОДПОМОГНАТО ОТ ТЕХНОЛОГИИТЕ

Павел Бойчев

В този доклад са описани основните идеи на равновесния и пандисциплинарния подходи при учене подпомогнато от технологии, а също така и как те могат да се обединят в единен подход. Целта е да се предоставят на образованието средства, които поддържат както мулти- така и интердисциплинарни подходи. Описани са примерни образователни материали и накратко са дискутирани техните потенциални предимства. Описаната в доклада работа е текущо изследване и разработка, имаща връзка с няколко интернационални и национални проекта. Практическото приложение на предложените материали ще бъде осъществено в рамките на тези проекти.