МАТЕМАТИКА И МАТЕМАТИЧЕСКО ОБРАЗОВАНИЕ, 2026
MATHEMATICS AND EDUCATION IN MATHEMATICS, 2026
Proceedings of the Fifty-Fifth Spring Conference
of the Union of Bulgarian Mathematicians
Tryavna, Bulgaria, April 5–9, 2026

HARNESSING THE POWER OF PRIMES: AN EDUCATIONAL GAME APPROACH TO PRIME NUMBERS IN MIDDLE SCHOOL

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This study introduces an innovative, game-based approach to teaching specific abstract concepts in Number Theory (prime numbers, the Fundamental Theorem of Arithmetic, and divisibility rule) to middle school students by immersing them in a simplified cryptographic world. The author developed two original educational board games: "The White Hat and the Ancient Key" and "Hacker Duel," which use custom components (cards representing "doors" with composite numbers and "keys" with prime factors) to make the principles tangible. The first game is a cooperative and strategic exercise in prime factorization that models the concept of asymmetric complexity which is central to cryptography, while the second is a competitive duel focusing on distinguishing between common and different prime factors to understand the least common multiple (LCM) and greatest common divisor (GCD). Preliminary observations from implementation in Bulgarian schools suggest that this method effectively improves students' computational skills, encourages active use of divisibility rules, and fosters proactive mathematical thinking about fundamental concepts such as the uniqueness of factorization, demonstrating that educational board games offer a promising, innovative enrichment to traditional mathematics instruction.

Keywords: educational game, serious game, prime numbers, factorization, game-based learning

ОВЛАДЯВАНЕ СИЛАТА НА ПРОСТИТЕ ЧИСЛА: ОБРАЗОВАТЕЛЕН ИГРОВИ ПОДХОД КЪМ ПРОСТИТЕ ЧИСЛА В ПРОГИМНАЗИАЛЕН ЕТАП

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https://doi.org/10.55630/mem.2026.55.232-241
2020 Mathematics Subject Classification: 97F60.

Статията представя новаторски, базиран на игри подход за преподаване на специфични абстрактни концепции в Теорията на числата (прости числа, Основна теорема на аритметиката и признаци за делимост) на ученици в прогимназиален етап, чрез въвеждането им в опростен криптографски свят. Авторката разработва две образователни настолни игри: „Бялата шапка и древният ключ“ и „Хакерски дуел“, които използват специално създадени компоненти (карти, представляващи „врати“ със съставни числа и „ключове“ с прости множители), за да направят изучаваните концепции осезаеми. Първата игра е кооперативно и стратегическо упражнение по разлагане на прости множители, което моделира концепцията за асиметрична сложност – централна за криптографията. Втората игра е състезателен дуел, фокусиран върху разграничаването между общи и различни прости множители за усвояване на най-малко общо кратно (НОК) и най-голям общ делител (НОД). Предварителните наблюдения от прилагането на метода в български училища сочат, че той ефективно подобрява изчислителните умения на учениците, насърчава активното използване на признаците за делимост и развива проактивно математическо мислене върху фундаментални концепции като единствеността на разлагането. Това доказва, че образователните настолни игри предлагат обогатяване на традиционния подход в обучението по математика.

Ключови думи: образователна игра, сериозна игра, прости числа, разлагане на прости множители, обучение базирано на игри

Introduction

Mathematics education in school setting often encounters difficulties in connecting abstract concepts to the real world. This problem is particularly pronounced in teaching elements of Number Theory, such as prime numbers, the Fundamental Theorem of Arithmetic, and divisibility rules. Core concepts such as *prime numbers* and *factorization* appear disconnected from their practical application. Cryptography is the natural domain for the application of these concepts. However, because it remains an unfamiliar area to students of this age, this limits the opportunities for effective learning. This article presents a novel approach to overcoming this difficulty by using game components and a simplified cryptographic world to introduce students to a practical application of number theory. This approach, based on the principle that the game experience can compensate for a lack of life experience, is implemented in two original educational board games – “The White Hat and the Ancient Key” and “Hacker Duel”.

Problem Description

The initiation of learning elements of Number Theory varies across different curricula. In Bulgaria, the study of number theory begins in the fifth grade (12-13 years old) with prime numbers, the Fundamental Theorem of Arithmetic, and divisibility rules for 2, 3, and 5. In the Cambridge IGCSE syllabi [6], prime numbers, as well as other number theory concepts, are usually taught to students in lower or upper secondary school age (14-16 years old). In the Common Core State Standards for Mathematics, the study of prime numbers is in 4th grade (10-11 years old).

As a result of studying the foundations of number theory, students are expected to be able to factor a natural number into prime factors, know the divisibility rules for 2, 3,

and 5, and find the least common multiple (LCM) and greatest common divisor (GCD) of two or three numbers.

Since cryptography is unfamiliar to most students and somewhat to teachers at the lower secondary level, we propose an approach that connects an abstract mathematical concept with this unfamiliar area to explain its application. This approach is of particular importance for the development of mathematics education as a whole, as the problem of finding real world applications of science concepts has other similar manifestations.

The proposed approach is based on creating game components and a simplified world for board games. Thus, through different games, students can quickly gain practical experience, and this can be used to make the connection between abstraction and its application.

Attempts to Use Educational Games to Explain Complex Scientific Concepts

Several attempts have been described previously to present complex scientific concepts through educational games.

1. Related to studying quantum mechanics in high school: Quantum Race; Particle in a Box; Quantum TiqTaqToe [2], [3];
2. Related to genetics and evolution theory in lower and upper secondary school: Radix Endeavour [4];
3. Related to prime numbers: Prime Climb

The main principle followed is that the game world and the game rules should lead to the exploration of scientific abstraction through game experiences. This is also the main difference between scientific simulations used in education and the game world used in educational games.

Various board games exist that are related to the study of cryptography. The objective of the cryptographic world described in this article is to provide a natural environment for practicing specific mathematical algorithms associated with the prime factorization of natural numbers.

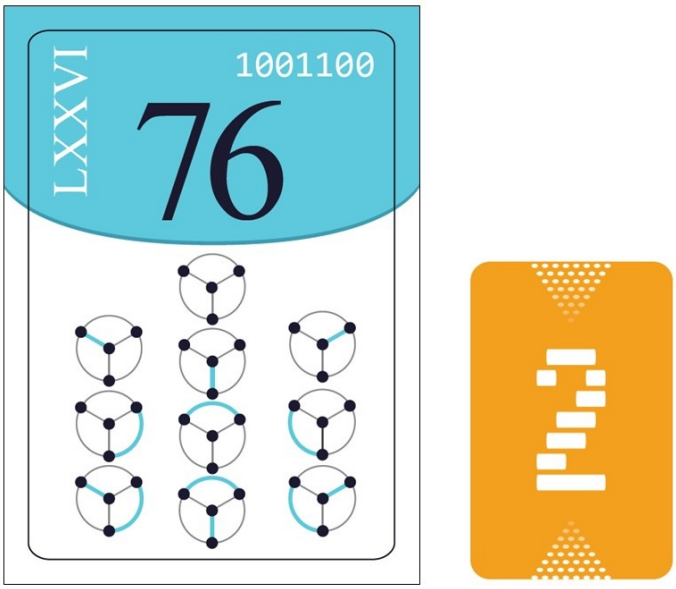
Developing a Simplified Cryptographic World and Game Components for It

Game Components: These consist of two sets of cards with numbers on them, respectively called *doors* and *keys,* which differ in size and design (Figure 1).

Figure 1 Door card 76 and key card 2

In the specific implementation, there are 36 doors with two-digit numbers on them. The images on the doors are related to various mathematical problems. Players rarely manage to guess what that problem might be. These design elements provoke players to ask the question: "What is this?" This proactive questioning by the players allows the teacher/mentor to answer this question and thus conduct a discussion in a broader mathematical context. The doors also have the door number written in Roman numerals and in the binary numeral system. These elements are usually recognized by the players.

The keys are selected to correspond to the prime factors of the numbers on the doors. For example, for a door with number 69, we have two keys labelled – 3 and



23, respectively. The number of keys depends on the selection of doors. In the specific implementation, there are 96 keys with only prime numbers. Some of the keys are coloured yellow and others green. This design element aims to provoke a question related to Mersenne numbers. This could lead to a discussion related to the discovery of prime numbers.

The duration of the games mainly depends on the number of doors. The number of doors itself is not fixed. In the specific implementation with 36 doors, the duration of the games is within one school hour (45 minutes). This facilitates their use as a learning tool. The teacher can reduce the number of participating doors and accordingly the keys. Thus, the playing time of the game will be reduced if the teacher determines that such a change is necessary.

Simplified Cryptographic World: The game world has a simple set of rules by which it operates. Players try to open the doors using the keys. A door can be unlocked if and only if keys whose product is equal to the number on that door are placed on it (Figure 2).

Figure 2: Door card 69 and the key cards 3 and 23 that unlock it

Bayta Girl's World: The game world is connected to a girl named Bayta from this world. Players are introduced to Bayta's world through a short story. This is a world where people use secrets from the past to build the future. But evil forces have taken over the Ministry of the Future and locked secrets important to humanity behind 36 doors.

Different games in this game world and with these game components are created by assigning different missions to the players.

Based on this world, various games have been developed. They have passed the stages of conceptual model and prototype and are currently used in schools at the lower and upper secondary levels in Bulgaria. This allows for future studies of its impact in the

Door Keys that unlock the door

educational environment.

Description of the Educational Game "The White Hat and the Ancient Key" for Studying Prime Numbers, Divisibility Rules, and the Fundamental Theorem of Arithmetic

The game is detailed in the publication „The game approach in building a general mathematical common sense [5],” while a brief video outlining the rules can be viewed here: `https://youtu.be/gkOCrOa-PDM`

Educational Goals of the Game:

Approaches to teaching the factorization of a natural number into prime factors vary. The game is compatible with all of them, and therefore its use does not depend on the chosen teaching method for the topic. The educational goals of the game are as follow:

1. Explain the key role of prime numbers in the set of natural numbers as fundamental building blocks.
2. Encourage students to actively use the divisibility rules for 2, 3, and 5.
3. Give the teacher a tool to demonstrate operations with asymmetric complexity – the foundation of modern cryptography.
4. Give the teacher a tool to show an application of the Fundamental Theorem of Arithmetic and prime numbers in cryptography.
5. Improve the student's computational skills regarding the factorization of two-digit numbers into prime factors.

Observations on Achieving the Educational Goals:

Although systematic studies on the game's effectiveness have not yet been conducted, we share our first observations of its being used in a class setting.

1. During the game, the question naturally arises as to whether a door cannot be unlocked by a different combination of keys. The emergence of this question is also described in other situations. This question allows for commenting on the unique factorization of a natural number into prime factors, up to the order. Since the keys are only prime numbers, the unlocking of each door happens with a single

combination of keys. When this question arises, the fact that there are only prime numbers among the keys is usually also commented on. This emphasizes the role of prime numbers as the building blocks of natural numbers and the role of the Fundamental Theorem of Arithmetic as ensuring a unique "recipe" for creating every number.

2. In the course of the game, players are convinced that when searching for suitable keys for a given door it is easier to use the divisibility rules. This provokes curiosity about the rule's definition. Observations show that the divisibility rules for 2 and 5 are used very intuitively. The divisibility rule for 3 is used if it has been recently studied. Otherwise, players remember that such a rule exists but need time or help to recall it.
3. During the game, the question often arises: "Are there other divisibility rules?" Since the doors have only two-digit numbers, division by 11 is easily discovered and used, although students might encounter difficulties when needing to precisely define the divisibility rule for 11 for numbers larger than two digits.
4. At the beginning of the game, the question arises whether the use of a calculator is permissible. Players are convinced that the use of a calculator to ease the game makes sense only in combination with a divisibility rule. This explains the asymmetric complexity of the operations of multiplication and factoring a number into prime factors.
5. The game helps develop the players' computational abilities, as well as train short-term memory. Teachers report that even students who perform poorly on computational tasks tend to calculate mentally during the game.
6. The game has low competitiveness and a high degree of social interaction. It is interesting for "Socializers" type players (The Bartle classification [1]), who, according to Bartle's model, represent about 80% of all players.

This game was the first in a series based on the developed card sets. Several teachers have implemented it in their classrooms, and the results of their observations are currently being summarized and analyzed. At the Az Sam Balgarche Private Secondary School in Varna, the game has been in use since 2023. Our colleague there introduced supplementary worksheets of her own design to accompany the game.

At Cosmos International School, lessons based on the game have been conducted since 2023; the faculty reports that this approach enables them to increase student motivation in mastering the relevant curriculum material.

A demonstration lesson for teachers, focusing on the game's potential to introduce students to the world of cryptography, was held on October 11, 2025, with the support of Science on Stage – Europe (https://www.science-on-stage.eu/news/workshop-quantum-science-and-technology-sofia).

Furthermore, on January 13, 2026, a demonstration session featuring the game 'The White Hat and the Ancient Key' was conducted with teachers and members of the Regional Management of Education (RUE) – Razgrad at the Exarch Joseph Foreign Language Grammar School in Razgrad. The subsequent observations from this session are also pending analysis.

The Role of the Teacher: Before the game, the teacher organizes the players into groups and explains the rules. During the game, the teacher answers questions related to the rules of the game. After the game, the teacher can use all the situations that arose during the game to outline the important problems whose solutions the players discovered. This can be done by asking the following exemplary questions, both from the teacher and if formulated by the students during the game:

1. Could a door be unlocked with two different combinations of keys? (No, because the keys are only prime numbers, and every number is factored into prime factors in a unique way.)
2. What is this due to? (To the fact that all keys are prime numbers and the Fundamental Theorem of Arithmetic holds.)
3. Which is easier – to check if the keys unlock a door or to determine which keys would unlock a door? (The first is multiplication and is simpler, the second is factorization and is more complex.)
4. What can help us find suitable keys? (Divisibility rules.)

Developing the Educational Game "Hacker Duel" for Studying Least Common Multiple and Greatest Common Divisor

Players' Mission: To elect the best "white hats" for freeing the secrets by winning more keys.

Game Rules: The game is played by two players. They take turns playing first and second on each turn. At the start of the game, each player receives 6 doors. The rest of the doors are in a face-down pile. All keys are placed on the table face-up. The player whose turn it is selects one of their doors and places it on the table face-up and challenges the other player to attack the door. The other player selects one of their doors and answers the challenge, in turn challenging the first player to attack their door. Each player finds the keys for the opponent's door. A winning pair of keys is any pair that can be formed from two identical keys, one from the unlocking of the first door and the second from the unlocking of the second door. The second player wins the keys from the winning pairs, and the first player takes all the remaining keys (Figure 4). The two doors are removed from the game. Each player draws a new door from the door card pile.

End of the Game: The game ends when the doors run out. The player who has collected more keys wins.

Interaction Between Players: The game is strategic.

Number of Players: 2

Game Duration: The game lasts between 40 and 50 minutes. The teacher can reduce the duration by reducing the number of doors.

Educational Goals of the Game:

Approaches to teaching the algorithms for finding the least common multiple (LCM) and greatest common divisor (GCD) vary. The game is compatible with all of them, and therefore its use does not depend on the chosen teaching method for the topic.

1. Students to make a clear distinction between the common and different prime factors of two numbers.

Figure 3: Game play situation "Hacker Duel"

2. Students to improve their computational skills for the factorization of two-digit numbers into prime factors.
3. The teacher to demonstrate the importance of coprime numbers in cryptography.
4. The teacher to encourage students to actively use the divisibility rules for 2, 3, and 5.

Observations on Achieving the Educational Goals:

Although systematic studies on the game's effectiveness have not yet been conducted, observations made during the game's use can be shared.

1. The game is highly competitive. It is of interest to "Killers" type players according to Bartle's model, because of this high competitiveness. The game is also of interest to "Explorers" type players because of the possibility to discuss optimal strategies when combining doors.
2. Observations find that players quickly arrive at the idea of coprime numbers and ask questions about effective ways to find them.

Additional Opportunities for Designing Games Based on the Game Components

The game components are designed to work with natural numbers and their representation as a product of prime factors. Given the connection between this representation and the reduction of common fractions to normal form, a game similar to "Hacker Duel" can be constructed, which develops in a different game world and demonstrates the fundamental property of common fractions to be expanded or normalized by multiplication or division by the same number in the numerator and denominator. This would be the main rule in a new game world.

A new game based on the „door cards“ is currently under development, focusing specifically on understanding the principles of constructing Roman numerals. This game is still in the design phase.

A comprehensive STEM project has been developed based on these cards and their underlying concepts. The objective of the project is for students to create their own cards using physical models. During the project activities, students are required to complete the elements on the 'door cards' (including decimal, binary, and Roman notation for a selected number) and choose an image associated with that door. As a subsequent step, students must create „keys“ for their chosen number, determining the appropriate color based on whether the number is a Mersenne number or not.

Throughout this project, students investigate various mathematical properties of the number assigned to their door and provide a visualization of a property of their choice. Upon completion, all student-created cards are compiled into a single deck, which is then used to play „The White Hat and the Ancient Key.“ Data and results from the implementation of these projects are currently being collected and are pending further analysis.

The binary representation of numbers featured on the 'door cards' can be utilized to design educational games that demonstrate efficient computational algorithms. These include exponentiation by squaring (the square-and-multiply algorithm) and multiplication via bitwise shifting (binary multiplication)

Observations on the Effect of Different Design Elements of the Game Components on Player Behaviour

Door Cards:

1. Each door card has a number that is written in Roman notation and in the binary numeral system. This design element is recognized by players independently, without outside help.
2. The images on the door cards are related to various number patterns, set theory, combinatorics. It turns out that many lower secondary school teachers find it difficult to answer questions related to these mathematical topics. This taught us to add a short guide on the topics related to the design in the future. Currently, this problem is solved with short posts on the game's website, which provide an explanation of how the corresponding number sequence or problem arises and some applications.
3. An image related to a mathematical concept as a design element can prove extremely useful for the horizontal advancement of students in different areas of mathematics. I consider the addition of supplementary educational materials related to the emerging questions to be a significant factor in increasing general mathematical literacy. Further studies are needed on the influence of different design elements on general mathematical literacy and the increase of curiosity. Very often, questions related to the images on the door cards arise during the game and might potentially be distracting. Practice shows that in such a case, the teacher should postpone the answer to this question for the discussion after the game.

Key Cards:

Each key card (except the special keys 0 and 1) is in one of two colours – yellow or green – corresponding to whether the prime number depicted on it is a Mersenne prime or not. This design element often generates interest and provides an opportunity for

teachers/mentors to discuss the issue of the impossibility of giving an effective algorithm for generating the next prime number in an efficient way. This is an occasion to conduct a discussion about the Sieve of Eratosthenes, and from there, about other problems whose solution is a sieve-type algorithm (e.g., lucky numbers, through a sieve similar to the Sieve of Eratosthenes but without preserving positions). For this design element, it is also necessary to add supplementary materials for teachers so that they can freely discuss these interesting problems related to natural numbers.

Conclusion

The presented educational games "The White Hat and the Ancient Key" and "Hacker Duel" demonstrate an effective method for overcoming the traditional challenges in teaching number theory. By creating a simplified cryptographic world, the games successfully connect abstract mathematical concepts such as prime numbers, the Fundamental Theorem of Arithmetic, and divisibility rules with a tangible game experience. Observations show that this approach not only improves students' computational skills but also provokes curiosity and proactive thinking about fundamental mathematical questions that often remain misunderstood. Teachers, in turn, can use the discussions that arise during the game to reinforce knowledge and expand students' general mathematical literacy. In the future, systematic research is needed to confirm the effectiveness of these games, but the experience so far in Bulgarian schools shows that the use of board games in the educational process is a promising and innovative method for teaching mathematics.

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